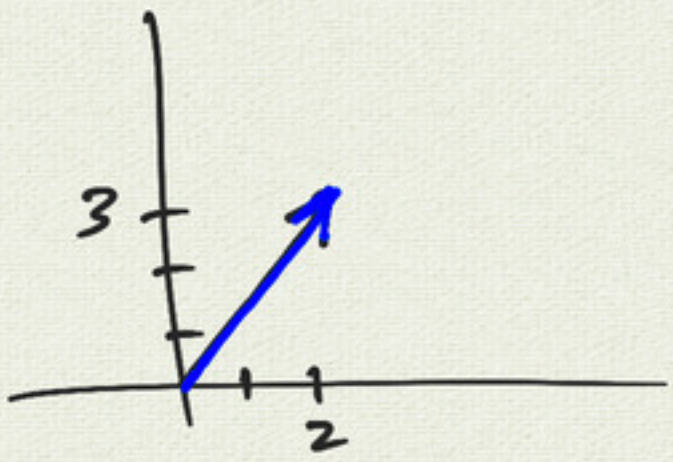
3.1 Vectors

vector

$\bar{u} = \langle x, y \rangle$

$\vec{u}, \bar{u}, u$

$\langle 2, 3 \rangle$



2 operations: $\bar{u} = \langle x_1, y_1 \rangle$

$\bar{v} = \langle x_2, y_2 \rangle$

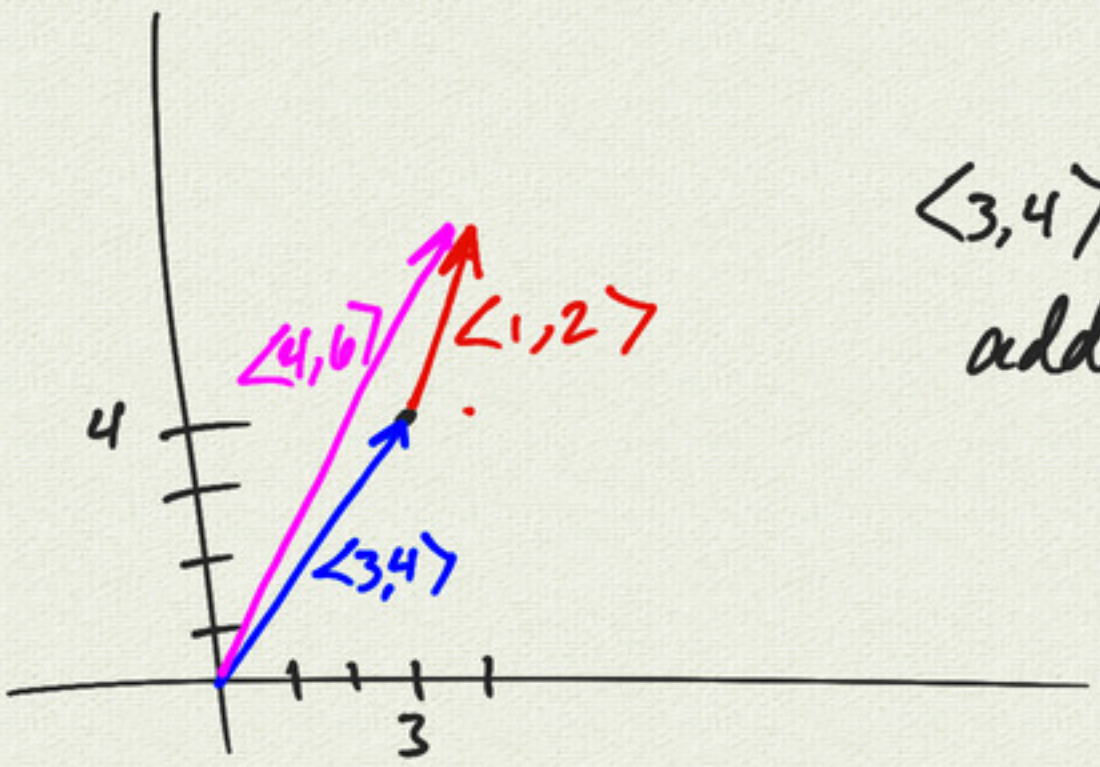
(1) $\bar{u} + \bar{v} = \langle x_1 + x_2, y_1 + y_2 \rangle$ addition

example: $\langle 3, 4 \rangle + \langle 1, 2 \rangle = \langle 4, 6 \rangle$

(2) Scalar multiplication

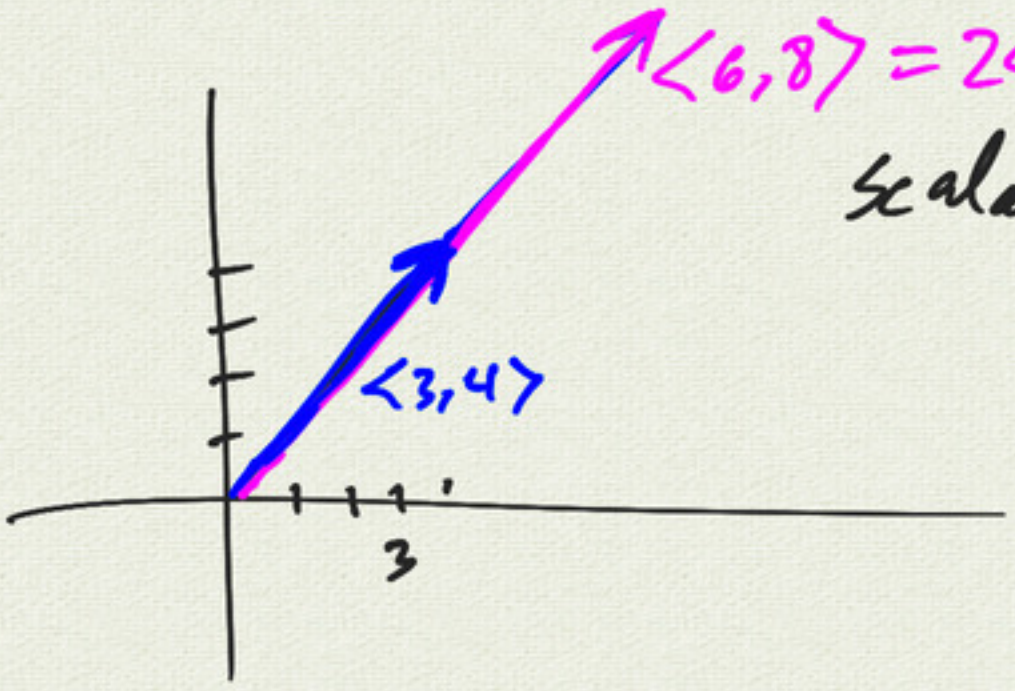
$k\bar{u} = \langle kx_1, ky_1 \rangle$ $(k \in \mathbb{R})$

example: $2\langle 3, 4 \rangle = \langle 6, 8 \rangle$

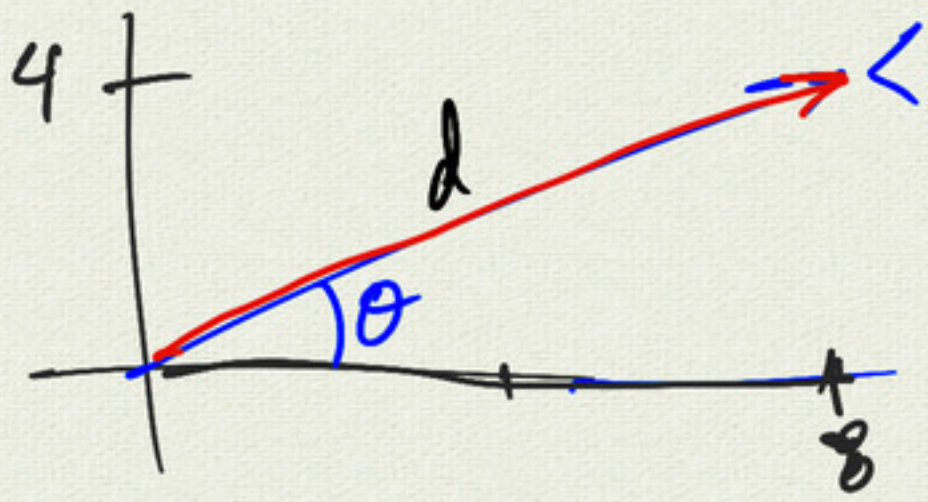


$\langle 3, 4 \rangle + \langle 1, 2 \rangle = \langle 4, 6 \rangle$

addition



scalar multiplication

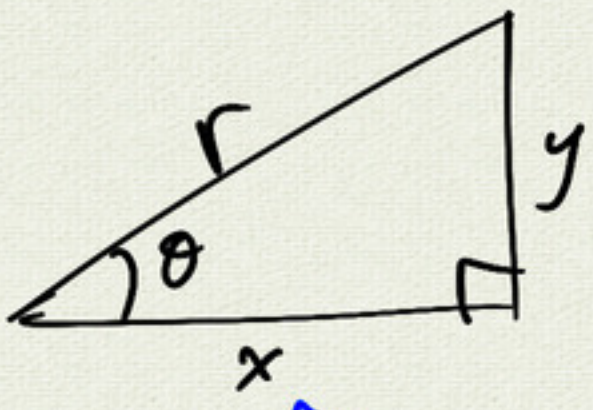


$\langle 8,4\rangle$ component form

magnitude (length)
direction

$d^2 = 8^2 + 4^2$
$= 64 + 16$
$= 80$
$d = \sqrt{80} = 4\sqrt{5}$ magnitude

$\tan\theta = \frac{4}{8} = \frac{1}{2}$
$\theta = \tan^{-1}\left(\frac{1}{2}\right)$ (use calculator)
direction



$\cos\theta = \frac{x}{r}$

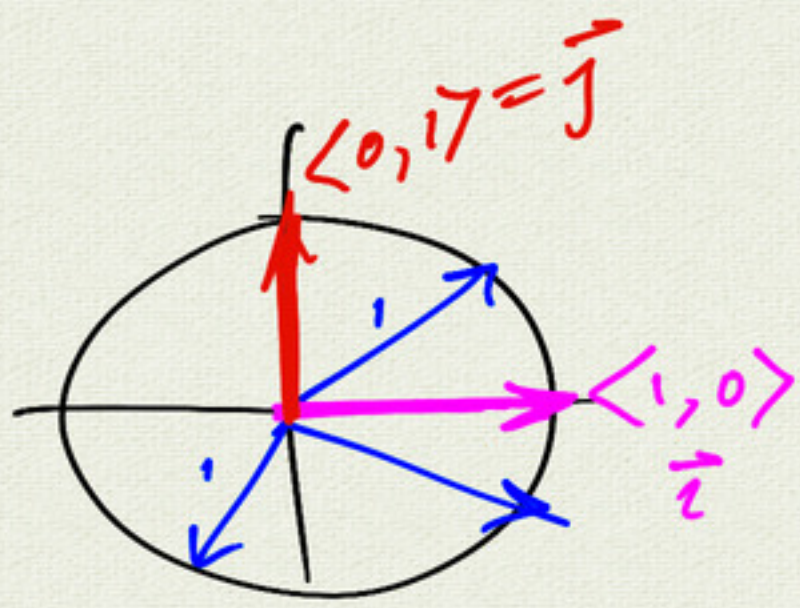
$r^2 = x^2 + y^2$	$x = r\cos\theta$
$\tan\theta = \frac{y}{x}$	$y = r\sin\theta$

$x = r\cos\theta$

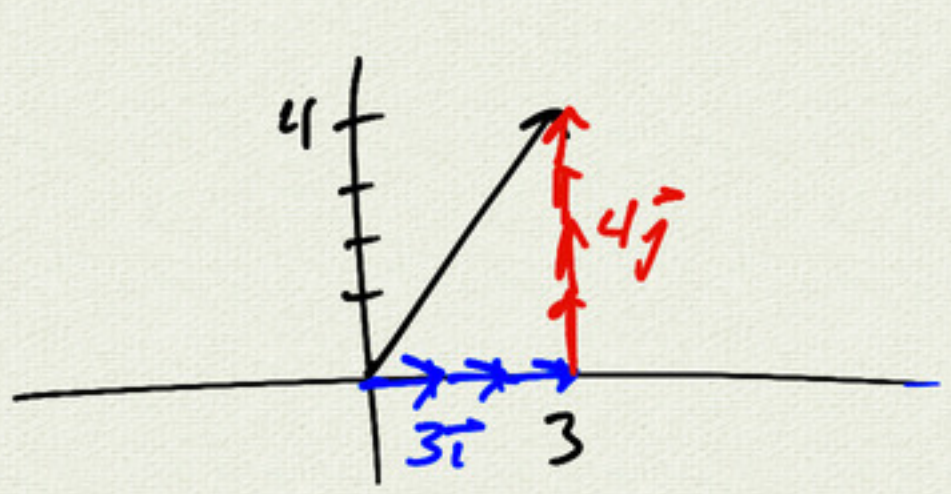
notation: $\bar{u} = \langle x, y\rangle$

magnitude $|\bar{u}| = \sqrt{x^2+y^2}$

unit vector: $|\bar{u}| = 1$

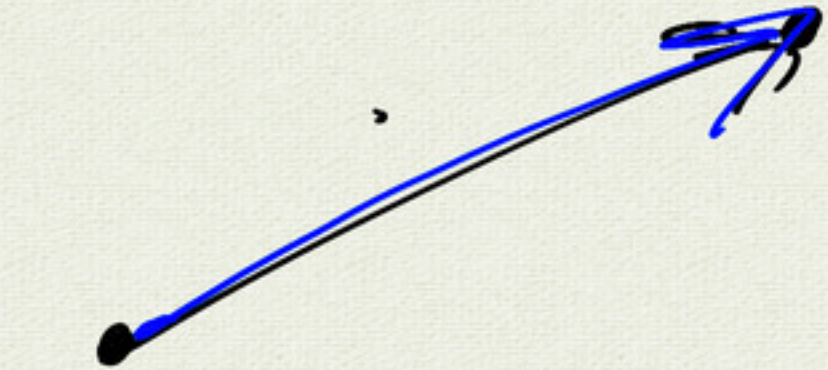


$\bar{u} = 3\vec{i} + 4\vec{j}$
$= 3\langle 1,0\rangle + 4\langle 0,1\rangle$
$= \langle 3,0\rangle + \langle 0,4\rangle$
$= \langle 3,4\rangle$



$|\vec{i}| = 1 = |\vec{j}|$
$|\bar{u}| = |\langle 3,4\rangle| = 5$

any vector $\langle x,y\rangle = x\vec{i} + y\vec{j}$
can be written as a linear combination of $\vec{i}$ and $\vec{j}$



$(3,-3)$

$\langle x_2 - x_1 , y_2 - y_1 \rangle$

$\langle 3-(-4), -3-(-2) \rangle$

$\langle 7, -5 \rangle$