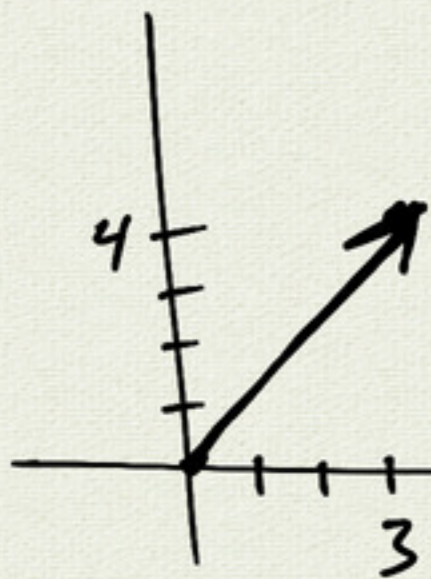


$\vec{s}_1 = \langle 5, 0 \rangle$

# 3.1 Vectors

$\langle 3, 4 \rangle$  ← vector  
 $\vec{u} = \langle x, y \rangle$   
 $\vec{u}$   
 $u$   
notation



2 operations:

①  $\vec{u} = \langle x_1, y_1 \rangle$   
 $\vec{v} = \langle x_2, y_2 \rangle$

$$\vec{u} + \vec{v} = \langle x_1 + x_2, y_1 + y_2 \rangle$$

addition

example:

$$\langle 3, 4 \rangle + \langle 1, 3 \rangle = \langle 4, 7 \rangle$$

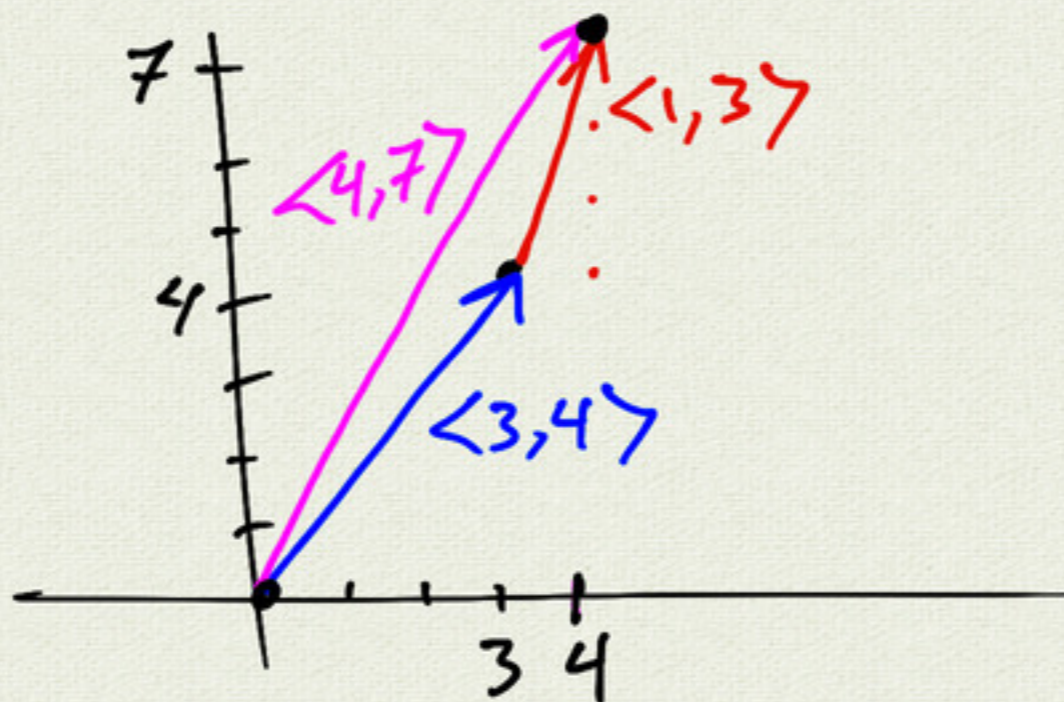
② scalar multiplication

$$k\vec{u} = \langle kx_1, ky_1 \rangle$$

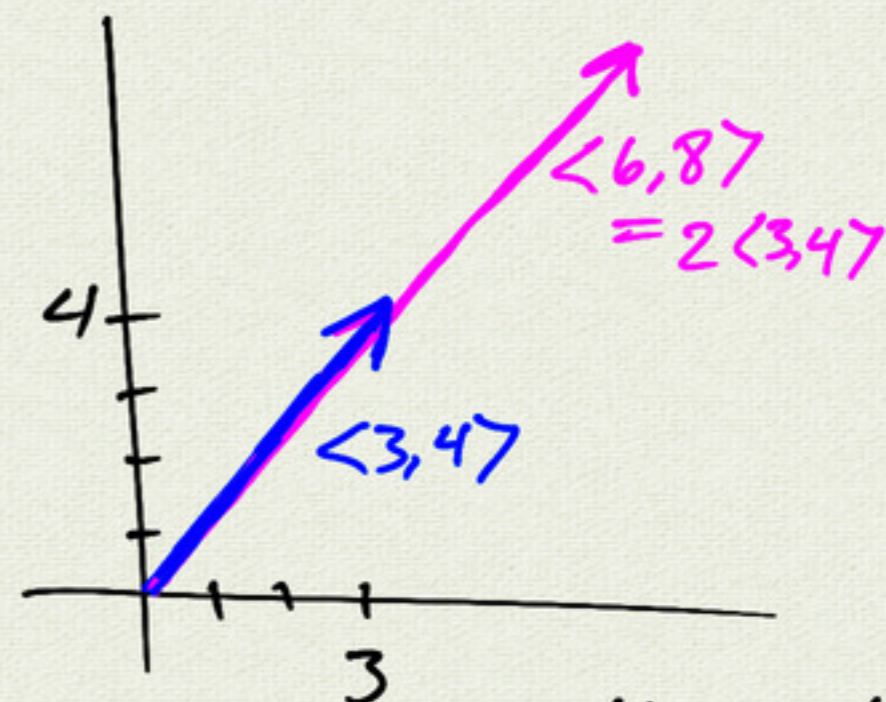
Scalar = real number  
 $k \in \mathbb{R}$

example:

$$2\langle 3, 4 \rangle = \langle 6, 8 \rangle$$



addition

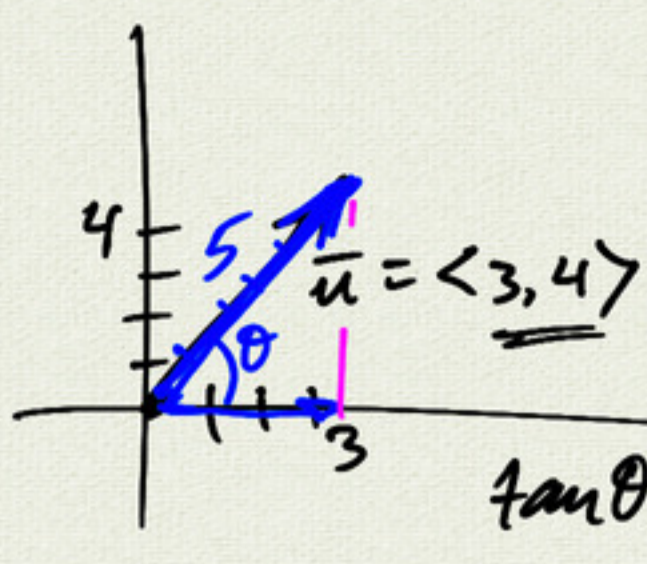


scalar multiplication

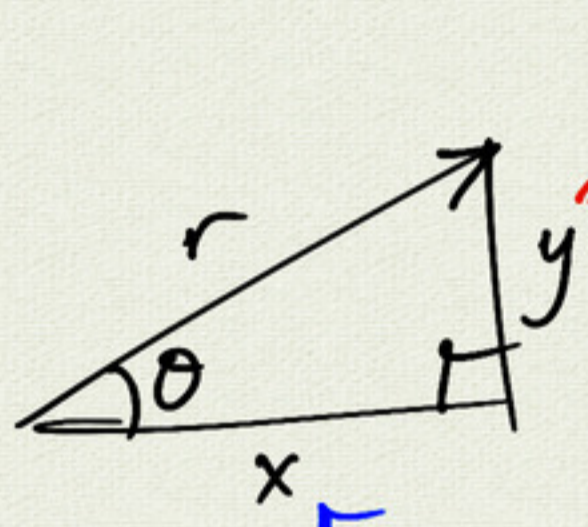
$\vec{u} = \langle 3, 4 \rangle$  component form  $\langle x, y \rangle$

$|\vec{u}| = 5$  magnitude (= length)

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$  direction



$\tan \theta = \frac{4}{3}$



$y = r \sin \theta$

$r^2 = x^2 + y^2$

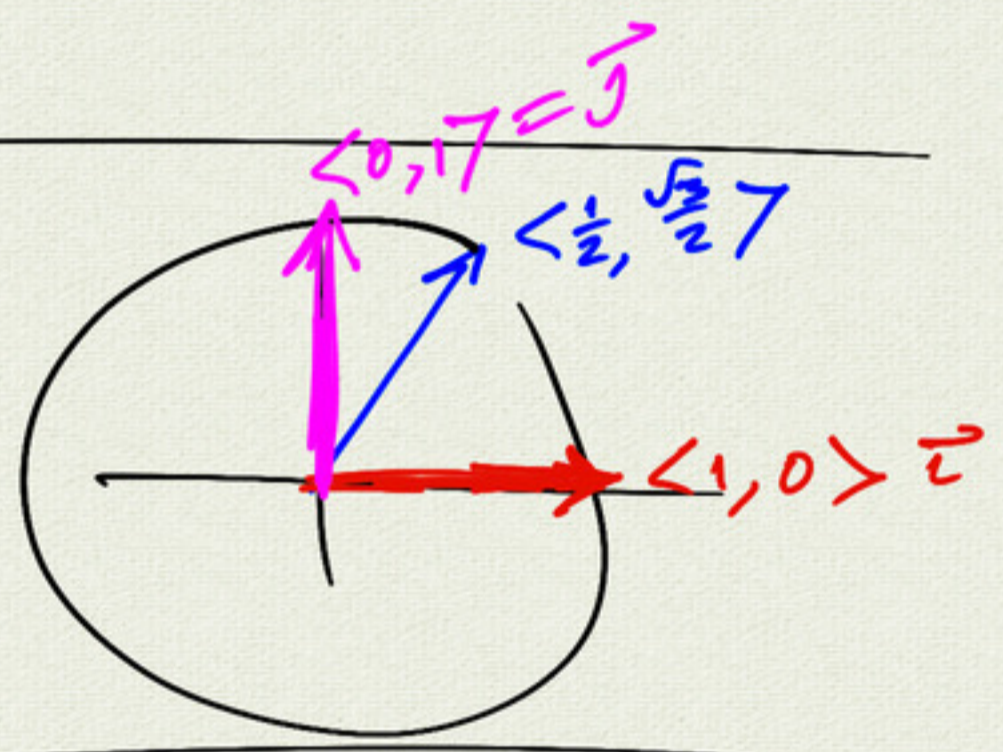
$\tan \theta = \frac{y}{x}$

$x = r \cos \theta$

$y = r \sin \theta$

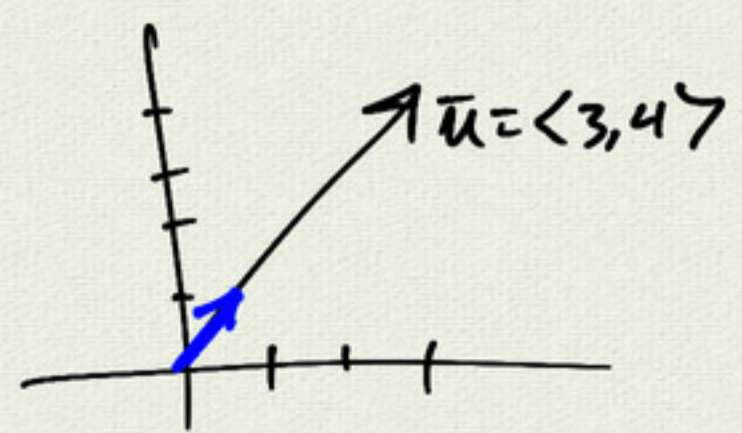
$\cos \theta = \frac{x}{r}$   
 $x = r \cos \theta$

unit vector  $|\vec{u}| = 1$   
magnitude



example:  $\vec{u} = \langle 3, 4 \rangle$

find unit vector in the same direction as  $\vec{u}$



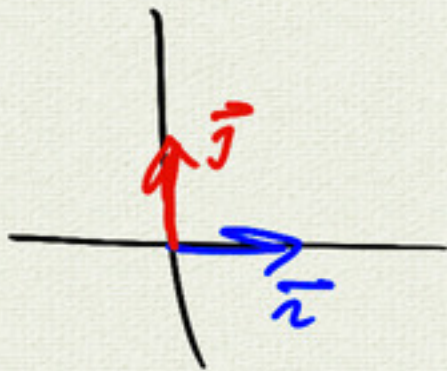
$\frac{\vec{u}}{|\vec{u}|} = \left(\frac{1}{|\vec{u}|}\right) \vec{u} = \frac{1}{5} \langle 3, 4 \rangle$

scalar mult =  $\langle \frac{3}{5}, \frac{4}{5} \rangle$

check:  $\sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} \stackrel{?}{=} 1$

$= \sqrt{\frac{9}{25} + \frac{16}{25}}$   
 $= 1 \checkmark$

$$\begin{aligned}\vec{u} &= 3\vec{i} + 4\vec{j} \\ &= 3\langle 1, 0 \rangle + 4\langle 0, 1 \rangle \\ &= \langle 3, 0 \rangle + \langle 0, 4 \rangle \\ &= \langle 3, 4 \rangle\end{aligned}$$



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$$\langle x, y \rangle = x\vec{i} + y\vec{j}$$

any vector can be written  
as "linear combination"  
of  $\vec{i}$  and  $\vec{j}$

