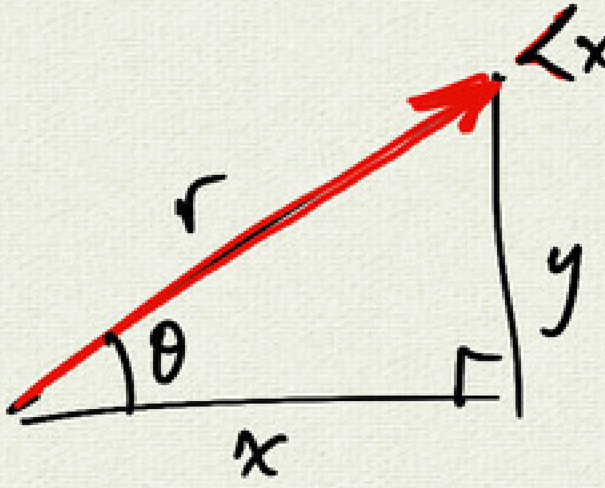
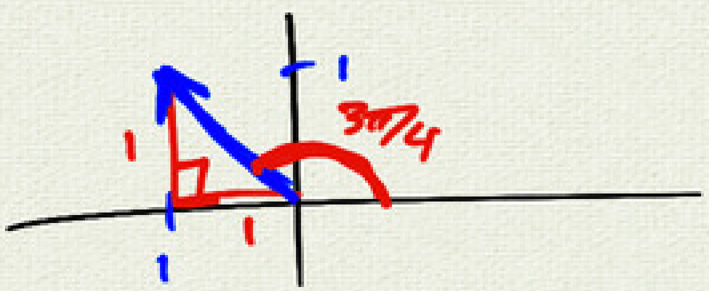


$\langle x,y\rangle$ component form $\Longleftrightarrow$ magnitude + direction

$$x = r\cos\theta \qquad r^2 = x^2 + y^2$$

$$y = r\sin\theta \qquad \tan\theta = \frac{y}{x}$$

$\bar{u} = \langle -1, 1\rangle \Longrightarrow$ find magnitude $|\bar{u}|$ and direction θ

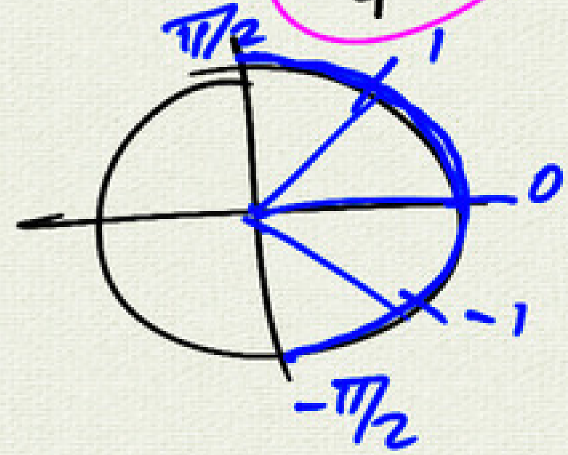


$$|u| = \sqrt{2} \qquad \left(= \sqrt{(-1)^2 + 1^2} = \sqrt{2}\right)$$

$$\boxed{\tan\theta = \frac{y}{x}} = \frac{1}{-1} = -1$$

$$\boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)} \;?$$

$$\tan^{-1}(-1) = \frac{-\pi}{4}$$



$+\pi$

$$\theta = \frac{3\pi}{4}$$

3.2 Dot Product

basic operations: $\bar{u} = \langle x_1, y_1 \rangle$

$\bar{v} = \langle x_2, y_2 \rangle$

① addition $\quad \bar{u} + \bar{v} = \langle x_1 + x_2, y_1 + y_2 \rangle$

② scalar multiplication

$$k\bar{u} = \langle kx_1, ky_1 \rangle$$

---

dot product: $\underbrace{\bar{u}}_{\text{vector}} \cdot \underbrace{\bar{v}}_{\text{vector}} = \underbrace{x_1x_2 + y_1y_2}_{\text{real number}}$

$\bar{0} = \langle 0, 0 \rangle$ zero vector

$\vec{i} = \langle 1, 0 \rangle$

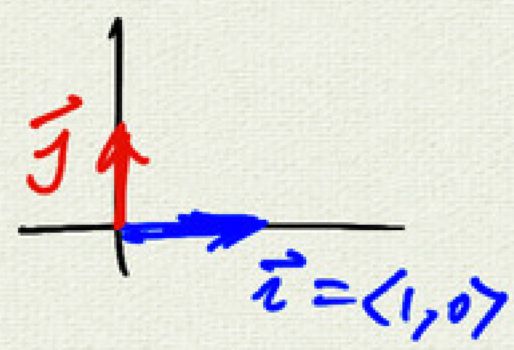
$\vec{j} = \langle 0, 1 \rangle$

$\bar{u} \cdot \bar{0} = \langle x_1, y_1 \rangle \cdot \langle 0, 0 \rangle = 0$

$\bar{u} \cdot \bar{v} = \bar{v} \cdot \bar{u}$ commutative

$\bar{u} \cdot \vec{i} = \langle x_1, y_1 \rangle \cdot \langle 1, 0 \rangle = x_1$

$\bar{u} \cdot \vec{j} = \langle x_1, y_1 \rangle \cdot \langle 0, 1 \rangle = y_1$



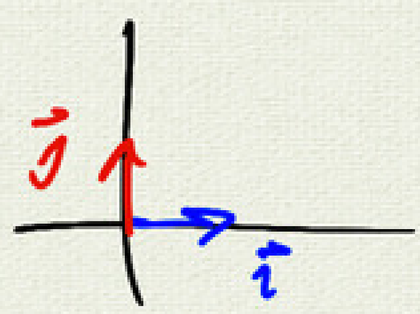
$\bar{u} \cdot \bar{u} = \langle x_1, y_1 \rangle \cdot \langle x_1, y_1 \rangle = x_1^2 + y_1^2 = |\bar{u}|^2$

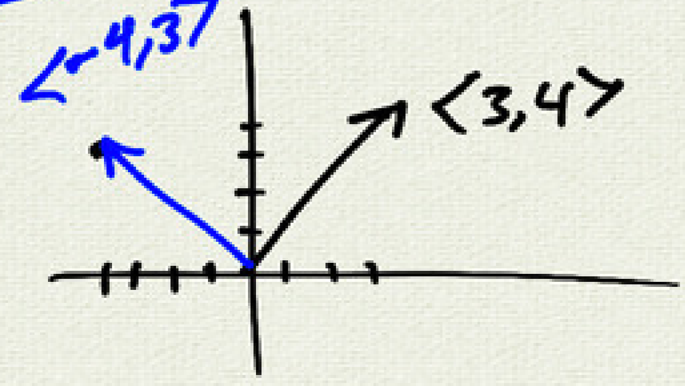
$$\boxed{\bar{u} \cdot \bar{u} = |\bar{u}|^2}$$

$\vec{i} \cdot \vec{i} = 1 = \vec{j} \cdot \vec{j}$

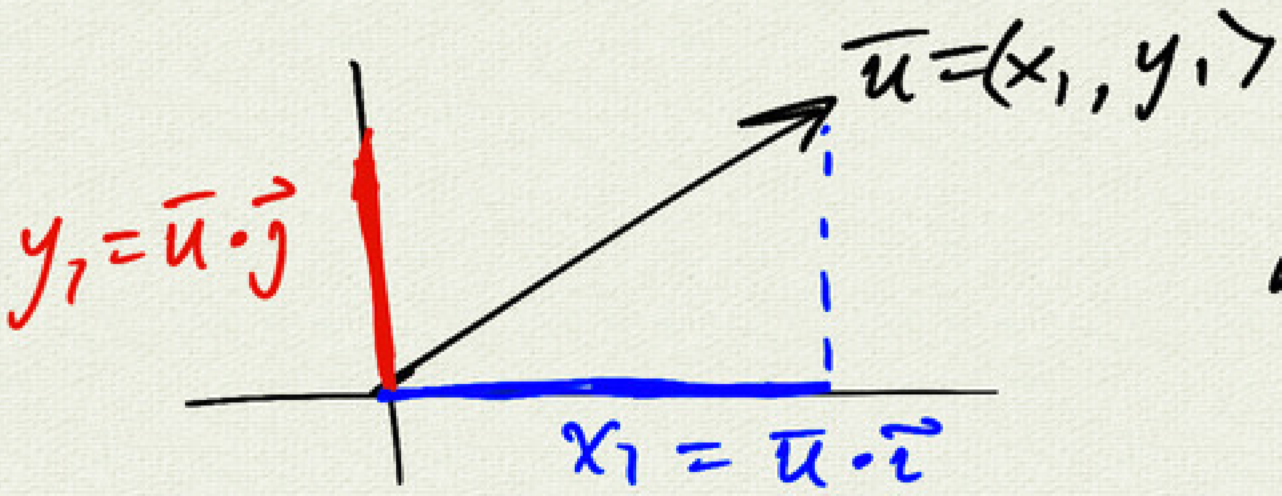
$\vec{i} \cdot \vec{j} = \langle 1, 0 \rangle \cdot \langle 0, 1 \rangle = 0$

$\langle 3, 4 \rangle \cdot \langle -4, 3 \rangle = 0$





$\bar{u} \cdot \bar{v} = 0$ orthogonal (definition)



dot product $\approx$ "projection"

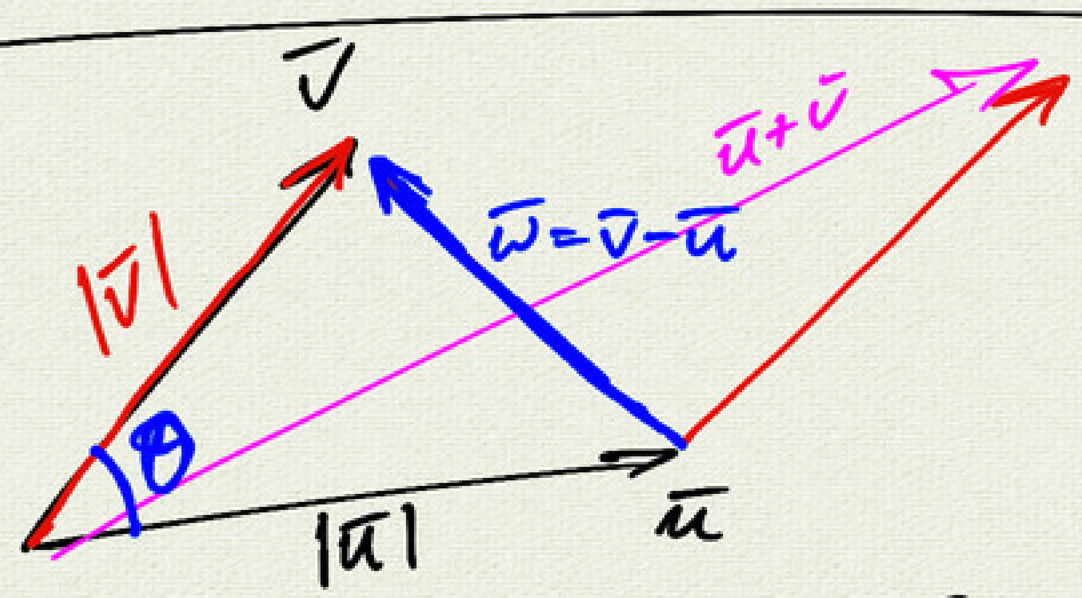
$\bar{u}\cdot(\bar{v}+\bar{w}) = \bar{u}\cdot\bar{v} + \bar{u}\cdot\bar{w}$ distributive

$\langle x_1, y_1\rangle \cdot \langle x_2+x_3, y_2+y_3\rangle = \ldots$ (challenge: prove)

---

distributive $\Rightarrow$ FOIL

$$(\bar{u}+\bar{z})\cdot(\bar{v}+\bar{w}) = \underset{F}{\bar{u}\cdot\bar{v}} + \underset{O}{\bar{u}\cdot\bar{w}} + \underset{I}{\bar{z}\cdot\bar{v}} + \underset{L}{\bar{z}\cdot\bar{w}}$$



$\bar{u}+\bar{w}=\bar{v}$

$\Rightarrow \bar{w}=\bar{v}-\bar{u}$

$c^2 = a^2+b^2-2ab\cos\theta$

$$\boxed{|\bar{v}-\bar{u}|^2 = |\bar{u}|^2 + |\bar{v}|^2 - 2|\bar{u}||\bar{v}|\cos\theta}$$ (Law of Cosines)

also:

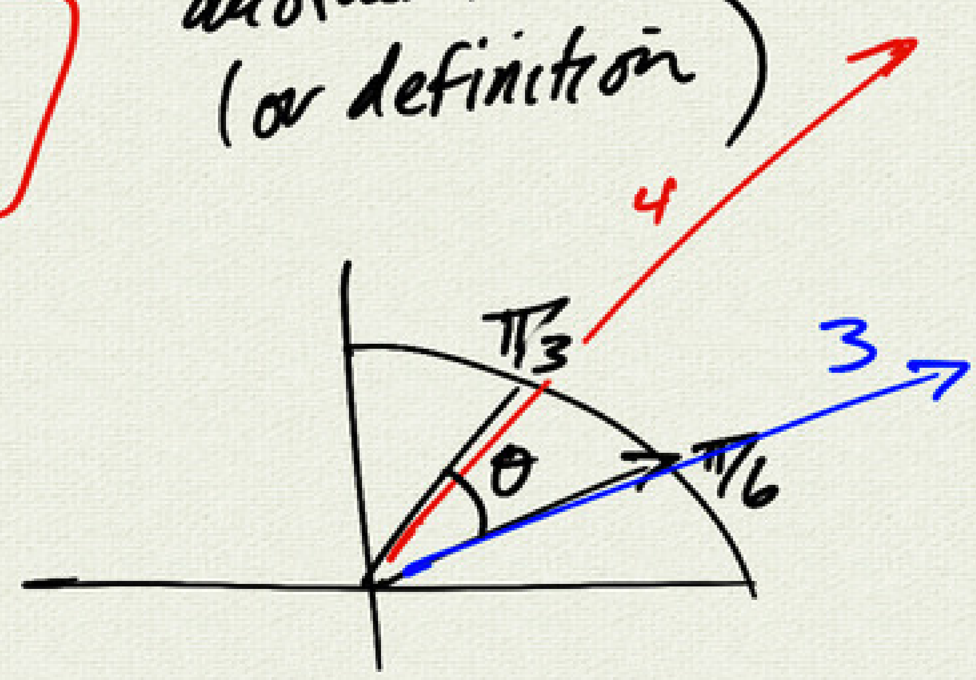
$$|\bar{v}-\bar{u}|^2 = (\bar{v}-\bar{u})\cdot(\bar{v}-\bar{u})$$

$$= \underbrace{\bar{v}\cdot\bar{v}}_{|v|^2} - \bar{v}\cdot\bar{u} - \bar{u}\cdot\bar{v} + \underbrace{\bar{u}\cdot\bar{u}}_{|u|^2}$$

$$\boxed{|\bar{v}-\bar{u}|^2 = |\bar{u}|^2 + |\bar{v}|^2 - 2\bar{u}\cdot\bar{v}}$$

$$\Rightarrow \boxed{\bar{u}\cdot\bar{v} = |\bar{u}||\bar{v}|\cos\theta}$$

another interpretation (or definition)



example:

$$\bar{u} = 3\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2}\right\rangle = \left\langle \frac{3\sqrt{3}}{2}, \frac{3}{2}\right\rangle$$

$$\bar{v} = 4\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2}\right\rangle = \langle 2, 2\sqrt{3}\rangle$$

find angle between $\bar{u}$ and $\bar{v}$: $\cos\theta = \dfrac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$

$$\bar{u}\cdot\bar{v} = \left\langle \frac{3\sqrt{3}}{2}, \frac{3}{2}\right\rangle \cdot \langle 2, 2\sqrt{3}\rangle = 3\sqrt{3}+3\sqrt{3} = 6\sqrt{3}$$

$|\bar{u}| = 3$

$|\bar{v}| = 4$

$$= \frac{6\sqrt{3}}{3\cdot 4} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

$$\boxed{\bar{u}\cdot\bar{v} = |\bar{u}||\bar{v}|\cos\theta}$$

$\bar{u}, \bar{v}$ orthogonal $\Rightarrow \bar{u}\cdot\bar{v} = 0$

$$\underbrace{|\bar{u}|}\,\underbrace{|\bar{v}|}\,\underbrace{\cos\theta} = 0$$

$\Rightarrow |\bar{u}| = 0$ or $|\bar{v}| = 0$

or $\cos\theta = 0$
$\theta = \pi/2$] perpendicular

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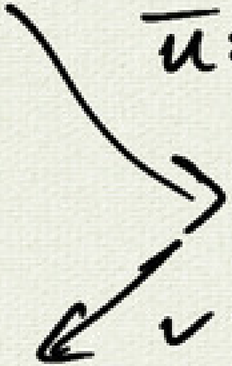

$\bar{u} = \langle 3, -2 \rangle$

$v = \langle -1, -1 \rangle$

find: $\bar{u} + \bar{v}$

$\bar{u} - \bar{v}$

$2\bar{u}$