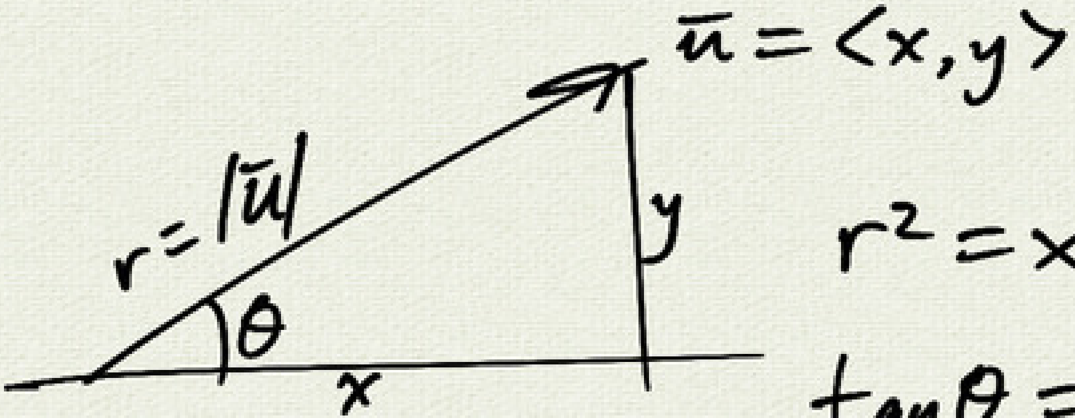


$r^2 = x^2 + y^2$

$\underbrace{\tan\theta}$ $= \dfrac{y}{x}$

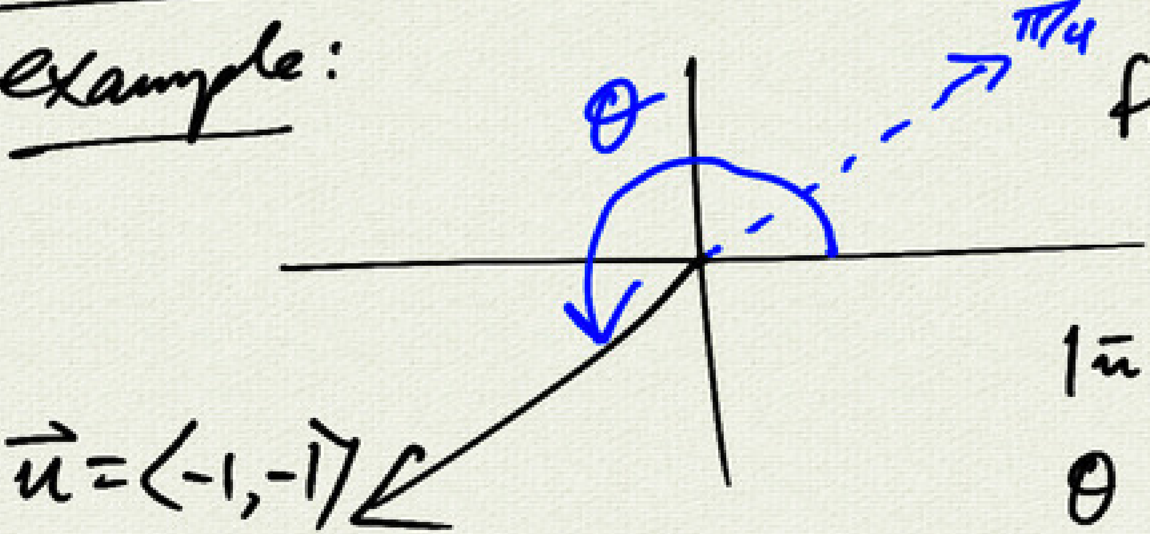
$\boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$ not quite

$x = r\cos\theta$

$y = r\sin\theta$

---

example:



find magnitude & direction

$|\bar{u}| = \sqrt{2}$

θ

$\tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(1)$

$= \pi/4$

$\theta = \frac{\pi}{4} + \underbrace{\pi}$

3.2 Dot Product

2 basic operations: $\bar{u} = \langle x_1, y_1 \rangle$

$\bar{v} = \langle x_2, y_2 \rangle$

① addition

$\bar{u} + \bar{v} = \langle x_1 + x_2, y_1 + y_2 \rangle$

② scalar multiplication $k\bar{u} = \langle kx_1, ky_1 \rangle$

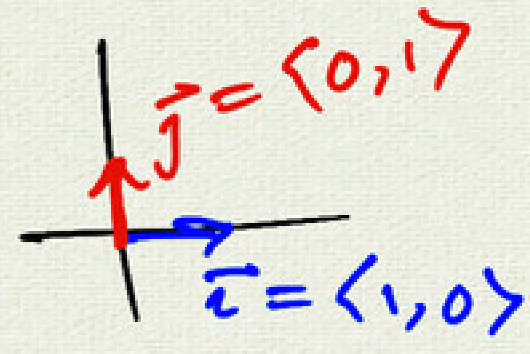
---

dot product "scalar product"

$$\underset{\text{vector}}{\bar{u}} \cdot \underset{\text{vector}}{\bar{v}} = \underbrace{x_1 x_2 + y_1 y_2}_{\text{not a vector (just a number)}}$$

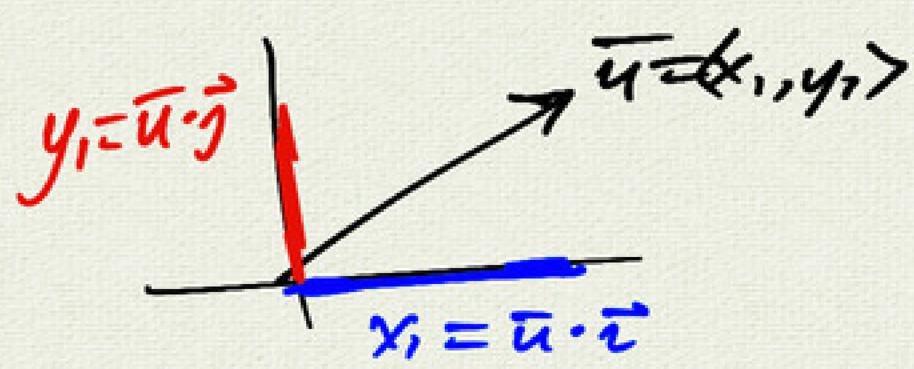
$\bar{0} = \langle 0, 0 \rangle$

$\bar{u} \cdot \vec{0} = \langle x_1, y_1 \rangle \cdot \langle 0, 0 \rangle$
$= 0$



$\bar{u} \cdot \vec{i} = \langle x_1, y_1 \rangle \cdot \langle 1, 0 \rangle$
$= x_1$

$\bar{u} \cdot \vec{j} = y_1$



$\bar{u} \cdot \bar{u} = \langle x_1, y_1 \rangle \cdot \langle x_1, y_1 \rangle$
$= x_1^2 + y_1^2$
$= |\bar{u}|^2$

$$\boxed{\bar{u} \cdot \bar{u} = |\bar{u}|^2}$$

$\vec{i} \cdot \vec{i} = 1 = \vec{j} \cdot \vec{j}$

<u>properties:</u>

$$\bar{u} = \langle x_1, y_1 \rangle$$
$$\bar{v} = \langle x_2, y_2 \rangle$$

$$\bar{u} \cdot \bar{v} = \bar{v} \cdot \bar{u} \qquad (= x_1 x_2 + y_1 y_2)$$
$$(= x_2 x_1 + y_2 y_1)$$

commutative

$$\bar{u} \cdot (\bar{v} + \bar{w}) = \bar{u} \cdot \bar{v} + \bar{u} \cdot \bar{w}$$

distributive

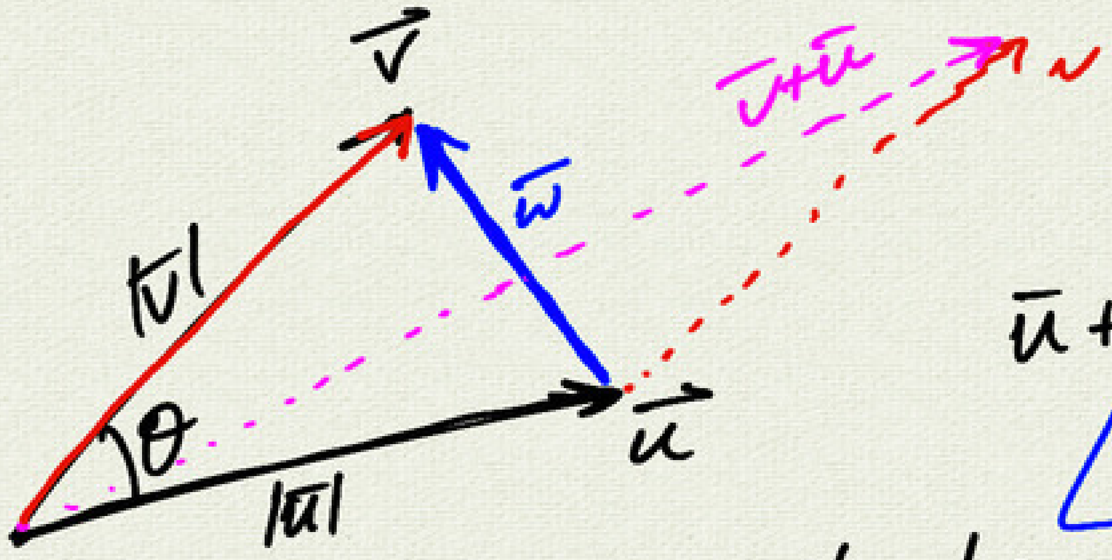
$$\langle x_1, y_1 \rangle \cdot \langle x_2 + x_3, y_2 + y_3 \rangle = x_1 (x_2 + x_3) + y_1 (y_2 + y_3)$$
$$= \ldots$$

(finish for challenge)

$$(\bar{u} + \bar{z}) \cdot (\bar{v} + \bar{w})$$

FOIL

$$= \underset{F}{\bar{u} \cdot \bar{v}} + \underset{O}{\bar{u} \cdot \bar{w}} + \underset{I}{\bar{z} \cdot \bar{v}} + \underset{L}{\bar{z} \cdot \bar{w}}$$



$\bar{u} + \bar{w} = \bar{v}$

$$\boxed{\bar{w} = \bar{v} - \bar{w}}$$

check: $\bar{u} + (\bar{v} - \bar{u}) = \bar{v}$ ✓

$$\boxed{|\bar{v}-\bar{u}|^2 = |\bar{u}|^2 + |\bar{v}|^2 - 2\boxed{|\bar{u}||\bar{v}|\cos\theta}}$$

Law of Cosines:
$c^2 = a^2 + b^2 - 2ab\cos C$

or use FOIL:

$$|\bar{v}-\bar{u}|^2 = (\bar{v}-\bar{u})\cdot(\bar{v}-\bar{u})$$

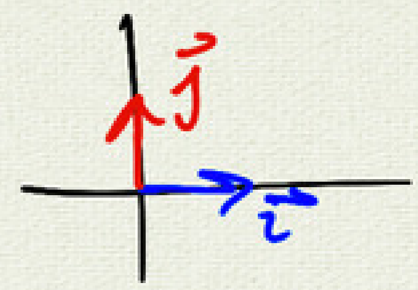
$$= \underbrace{\bar{v}\cdot\bar{v}}_{|\bar{v}|^2} - \bar{v}\cdot\bar{u} - \bar{u}\cdot\bar{v} + \underbrace{\bar{u}\cdot\bar{u}}_{|\bar{u}|^2}$$

$$\boxed{|\bar{v}-\bar{u}|^2 = |\bar{u}|^2 + |\bar{v}|^2 - 2\boxed{\bar{u}\cdot\bar{v}}}$$

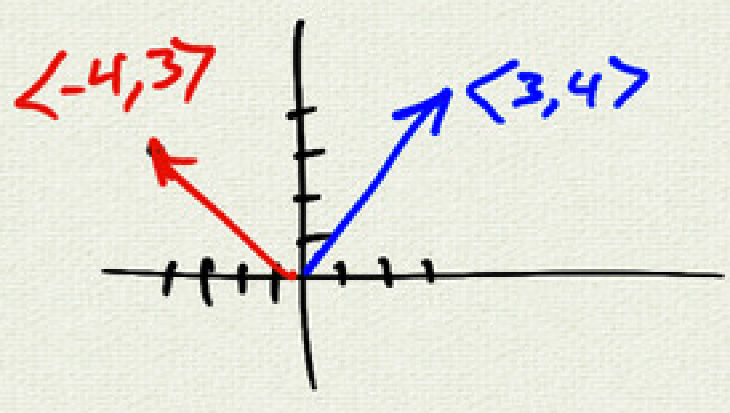
$$\Longrightarrow \boxed{\bar{u}\cdot\bar{v} = |\bar{u}||\bar{v}|\cos\theta}$$

another definition (or characterization)

$$\vec{i}\cdot\vec{j} = \langle 1,0\rangle\cdot\langle 0,1\rangle = 0$$



$$\langle 3,4\rangle\cdot\langle -4,3\rangle = 0$$



<u>def:</u>

$\bar{u}, \bar{v}$ <u>orthogonal</u> if $\bar{u}\cdot\bar{v} = 0$

$$\bar{u}\cdot\bar{v} = |\bar{u}||\bar{v}|\cos\theta$$

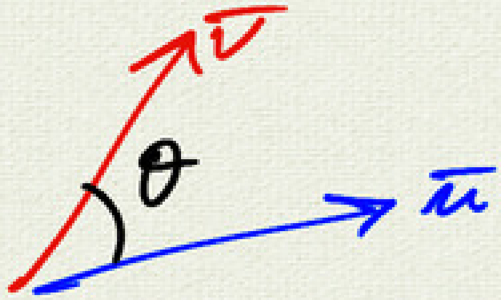
$$\bar{u}\cdot\bar{v} = 0 \longrightarrow \underbrace{|\bar{u}|}\,\underbrace{|\bar{v}|}\,\underbrace{\cos\theta} = 0$$

$\longrightarrow |\bar{u}| = 0$ or $|\bar{v}| = 0$ or $\cos\theta = 0$

$\theta = \frac{\pi}{2}$

perpendicular

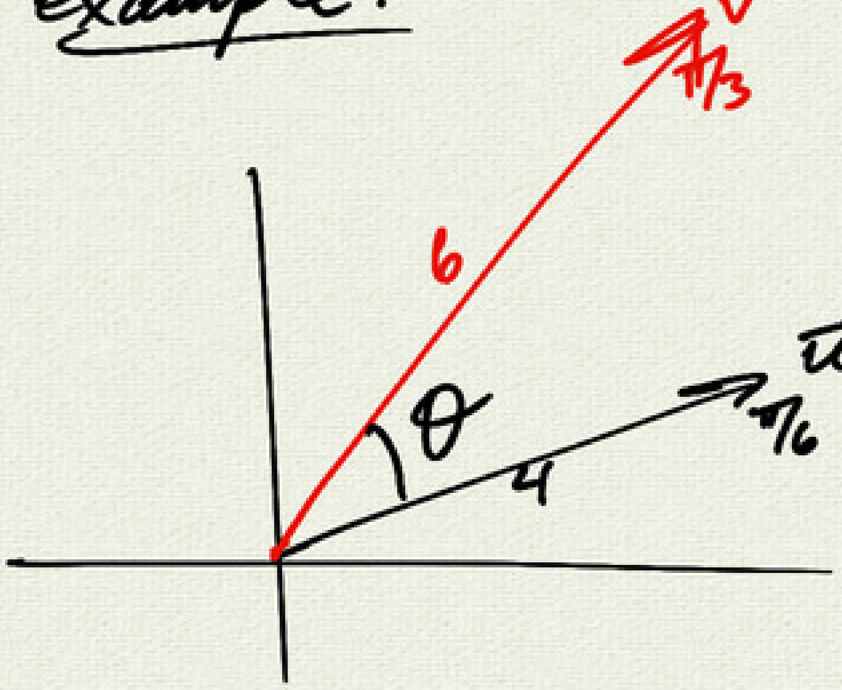
$$\bar{u} \cdot \bar{v} = |\bar{u}||\bar{v}|\cos\theta$$



$$\Rightarrow \cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$$

example:

$$\bar{v} = 6\left\langle \tfrac{1}{2}, \tfrac{\sqrt{3}}{2} \right\rangle = \langle 3, 3\sqrt{3} \rangle$$



$$\bar{u} = 4\left\langle \tfrac{\sqrt{3}}{2}, \tfrac{1}{2} \right\rangle = \langle 2\sqrt{3}, 2 \rangle$$

find θ (angle between $\bar{u}$ and $\bar{v}$)

$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$$

$\bar{u} = \langle 2\sqrt{3}, 2 \rangle$

$\bar{v} = \langle 3, 3\sqrt{3} \rangle$

$$\bar{u}\cdot\bar{v} = \langle 2\sqrt{3}, 2\rangle \cdot \langle 3, 3\sqrt{3}\rangle$$
$$= 6\sqrt{3} + 6\sqrt{3}$$
$$= 12\sqrt{3}$$

$|\bar{u}| = 4$

$|\bar{v}| = 6$

$$|\bar{u}|^2 = (2\sqrt{3})^2 + 2^2$$
$$= 4\cdot 3 + 4$$
$$= 16$$
$$|\bar{u}| = 4$$

$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$$
$$= \frac{12\sqrt{3}}{4\cdot 6}$$
$$= \sqrt{3}/2$$

$$\boxed{\theta = \frac{\pi}{6}}$$