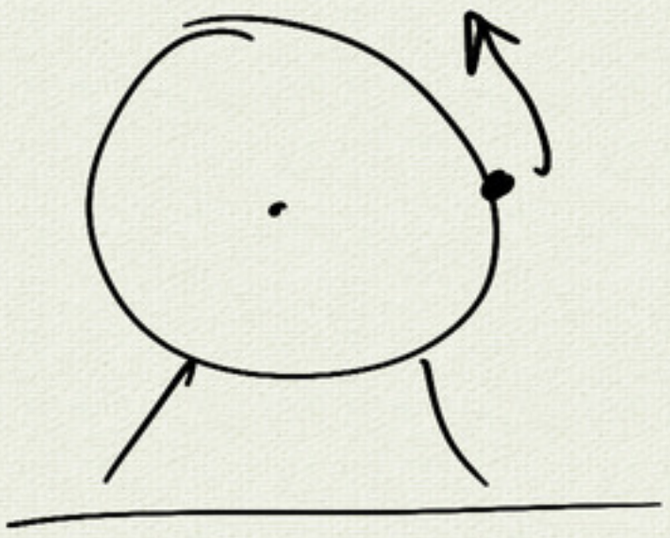
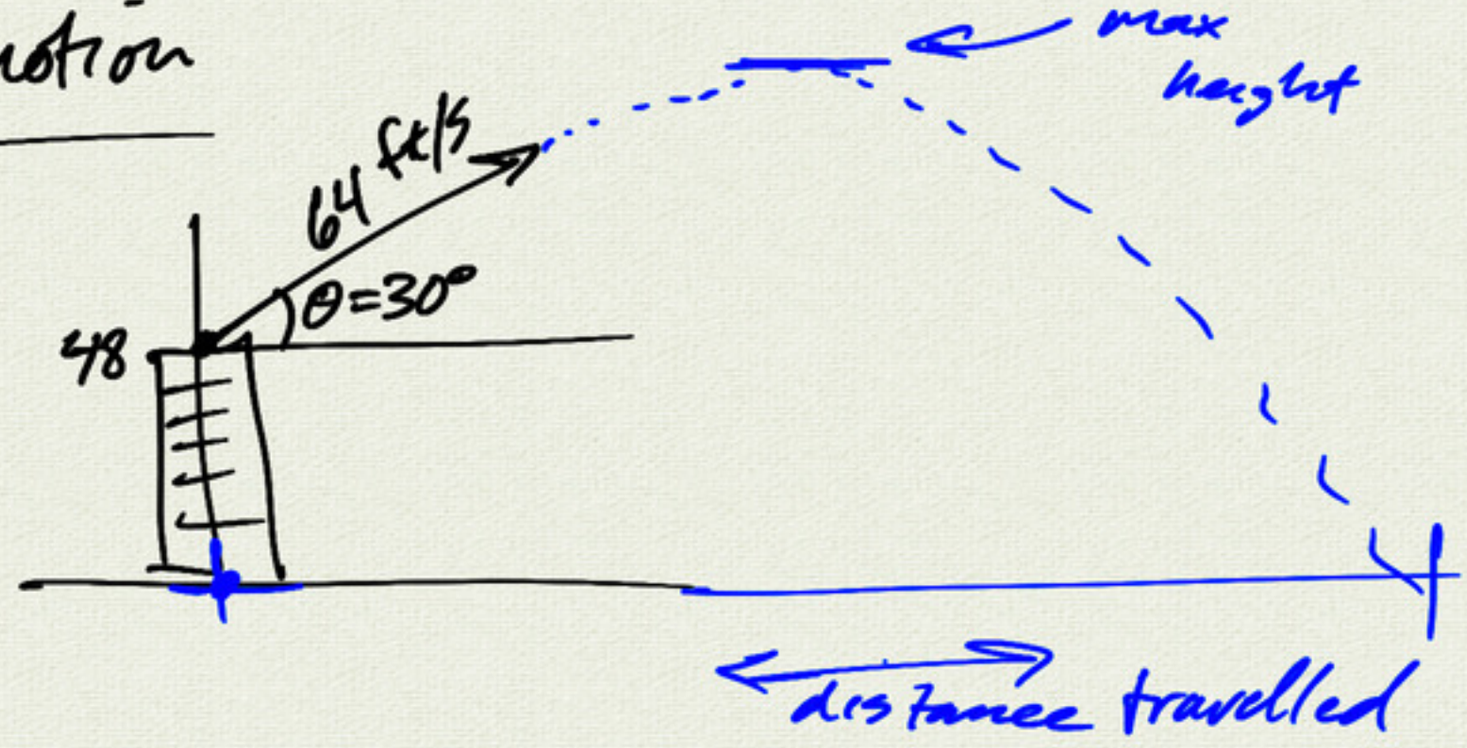
3.3 Parametric Equations

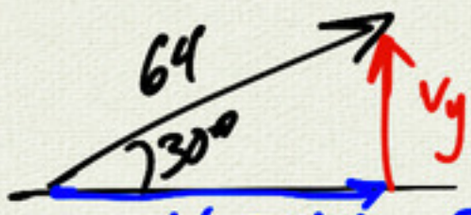
$x(t)$
$y(t)$
↖ parameter ("time")



projectile motion

Example:





$x(t) = x_0 + v_x t$ ($r \cdot t = d$)
↖ initial position

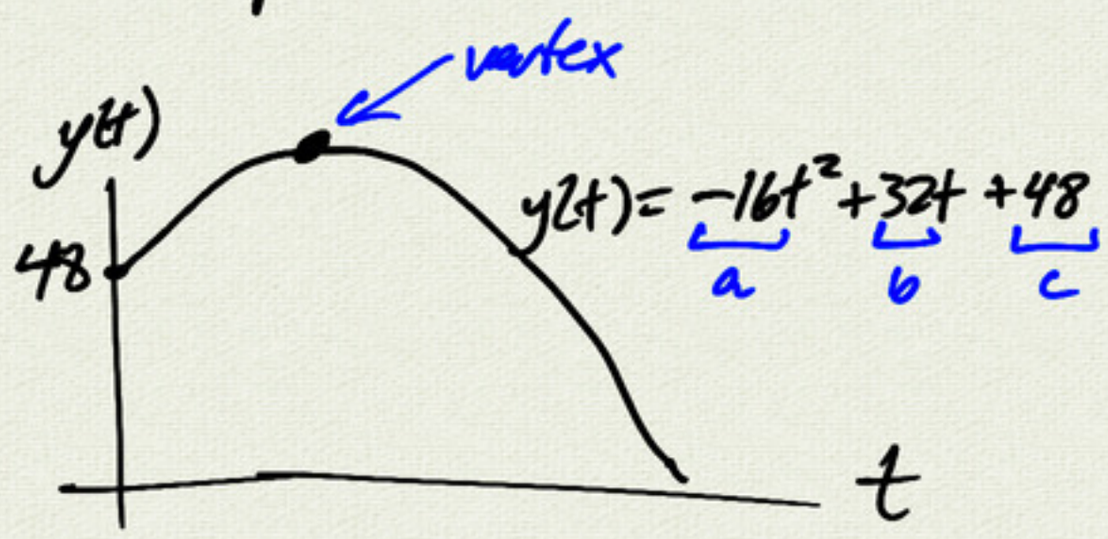
$y(t) = y_0 + v_y t - 16t^2$
↖ initial y position; ↖ initial y speed

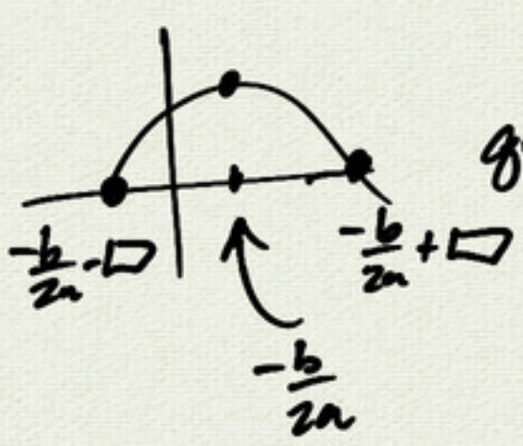
units: feet, seconds

$$\begin{cases} x(t) = (32\sqrt{3})\,t \\ y(t) = 48 + 32t - 16t^2 \end{cases}$$

(1) what is max height?

vertex at $t = -\frac{b}{2a}$

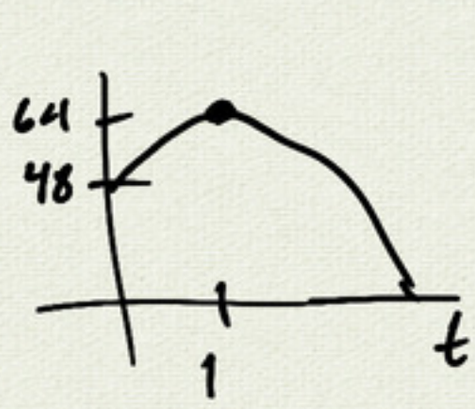




quadratic formula
zeros at $t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$= -\frac{b}{2a} \pm \square$

vertex at $t = -\frac{b}{2a} = -\frac{32}{2(-16)} = 1$

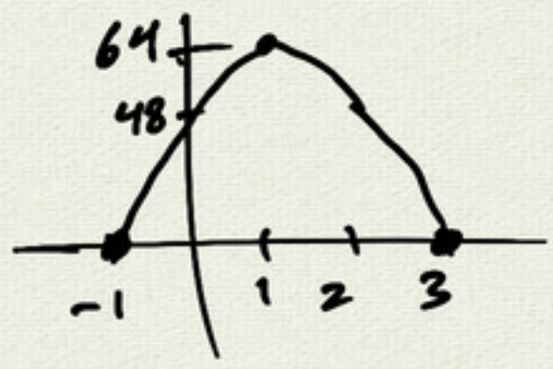
max height $y(1) = -16(1)^2 + 32(1) + 48$
$= 64$



(2) how far (x distance)?

when do we hit ground? $y = 0$

$y(t) = -16t^2 + 32t + 48 = 0$
$-16(t^2 - 2t - 3) = 0$
$-16(t-3)(t+1) = 0$
$\rightarrow t = 3, -1$

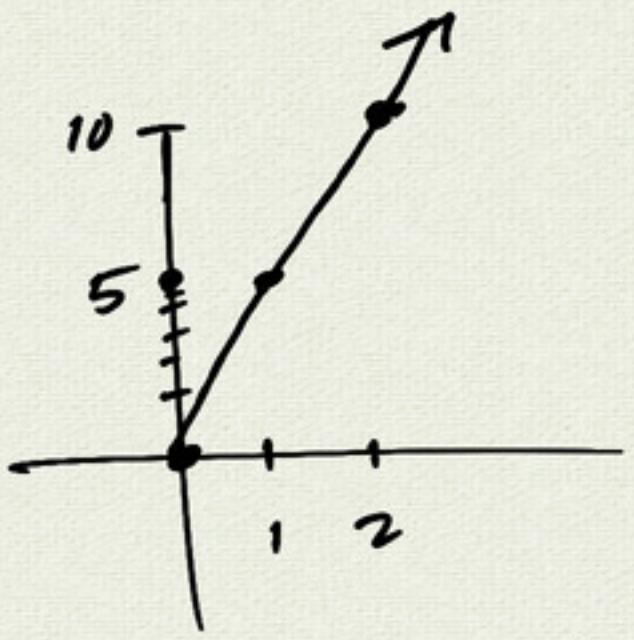


distance travelled
$= x(3) = (32\sqrt{3})3$
$= 96\sqrt{3}$

Some simpler parametric equations:

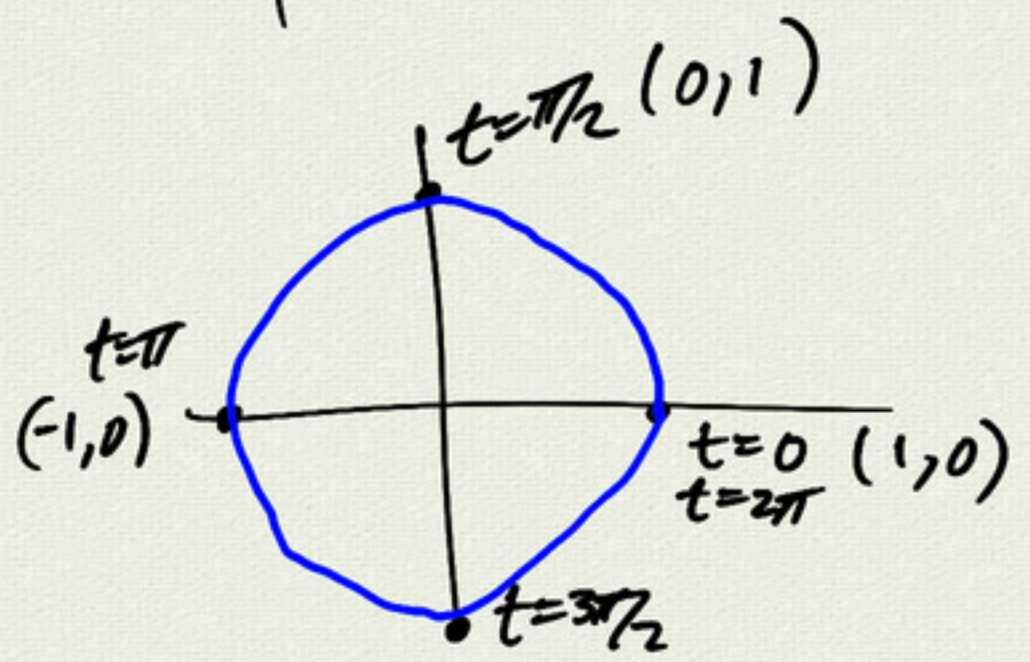
$x(t) = t$
$y(t) = 5t$

t	$x(t)$	$y(t)$
0	0	0
1	1	5
2	2	10



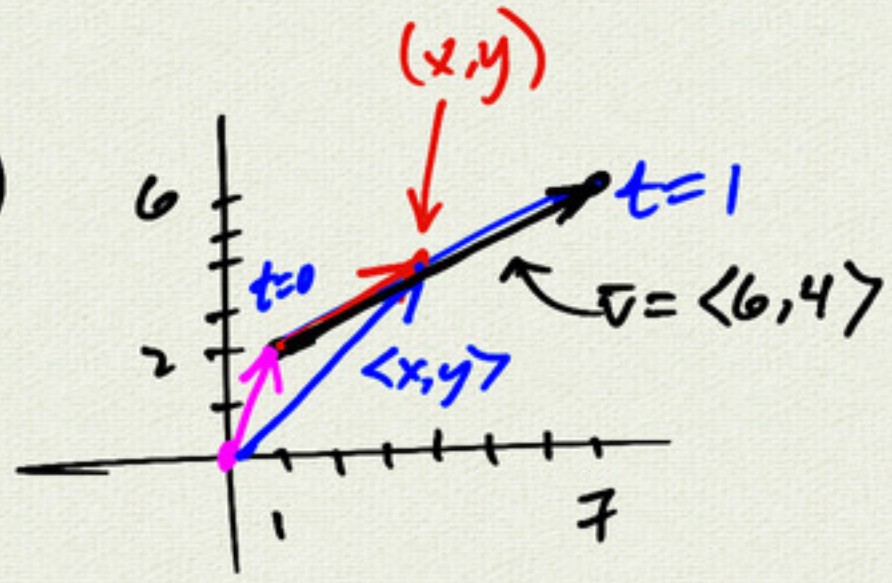
$x(t) = \cos t$
$y(t) = \sin t$

t	$x(t)$	$y(t)$
0	1	0
$\pi/2$	0	1
π	-1	0
$3\pi/2$	0	-1
2π	1	0



Example: parametrize line (segment) from $(1,2)$ to $(7,6)$

$t=0$ $\qquad$ $t=1$



$\langle x, y \rangle = \langle 1, 2 \rangle + t\langle 6, 4 \rangle$

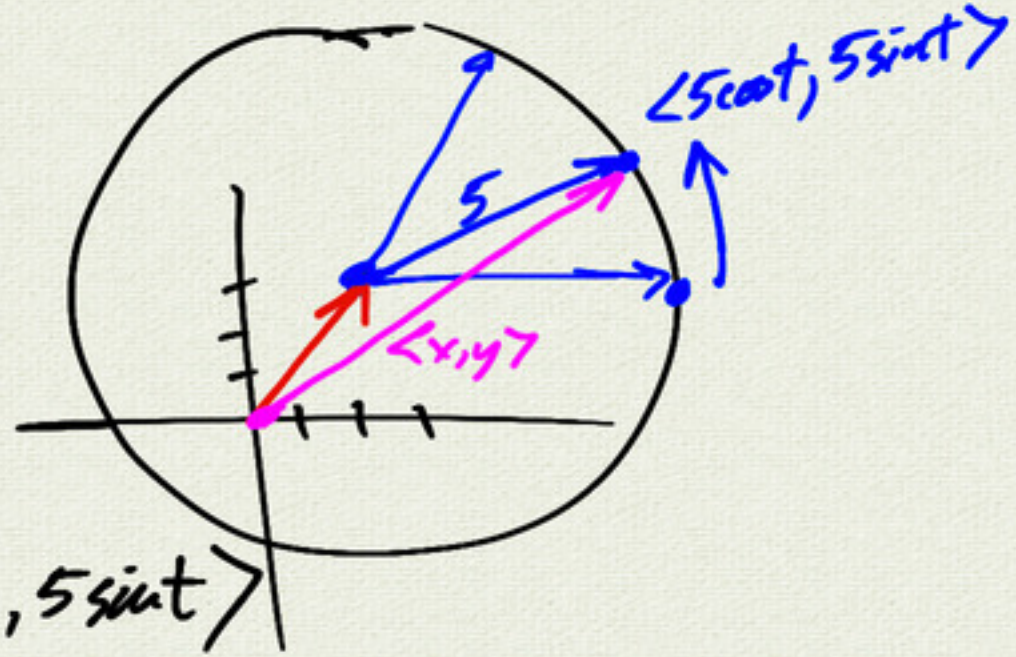
$x(t) = 1 + t \cdot 6$
$y(t) = 2 + t \cdot 4$

$$\begin{cases} x(t) = 6t + 1 \\ y(t) = 4t + 2 \end{cases}$$

Check: $x(0) = 1$ ✓
$y(0) = 2$

$x(1) = 7$ ✓
$y(1) = 6$

Example: parametrize a circle
radius 5
center (2, 3)

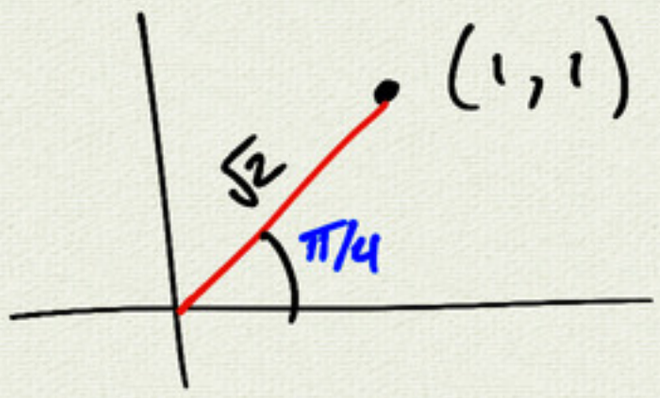


$$\langle x, y \rangle = \langle 2, 3 \rangle + \langle 5\cos t, 5\sin t \rangle$$

$$x(t) = 2 + 5\cos t$$

$$y(t) = 3 + 5\sin t$$

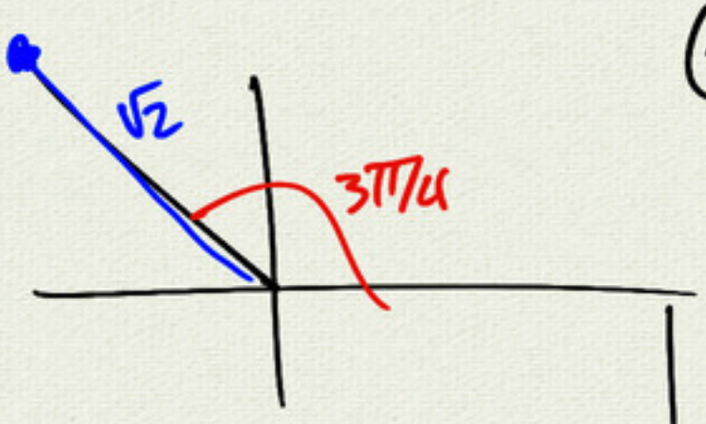
3.4 Polar coordinates

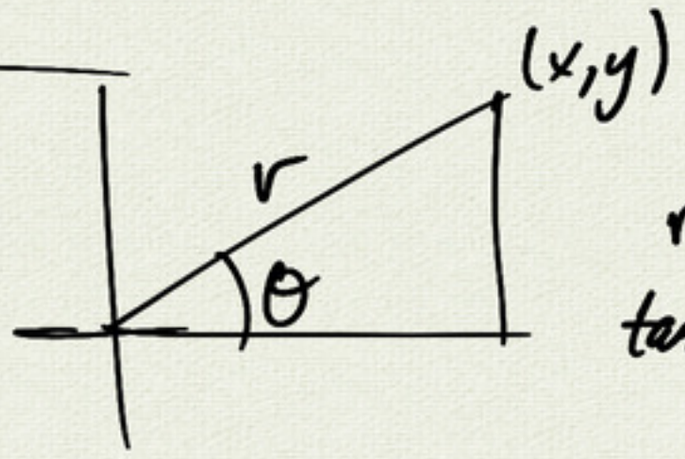


$(1,1)$ rectangular coordinates

$\left(\sqrt{2}, \frac{\pi}{4}\right)$ polar coordinates

$\left(\sqrt{2}, \frac{3\pi}{4}\right)$





$r^2 = x^2 + y^2$ | $x = r\cos\theta$

$\tan\theta = \frac{y}{x}$ | $y = r\sin\theta$

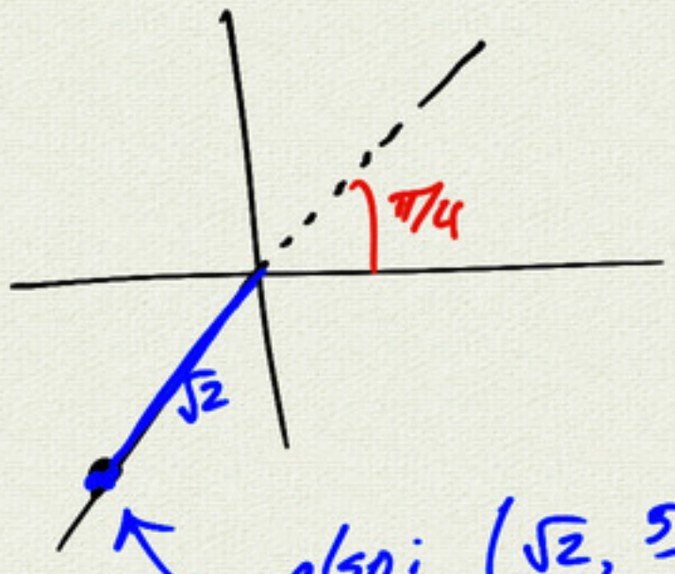
$\left(\sqrt{2}, \frac{3\pi}{4}\right) \rightarrow$ rectangular

$x = \sqrt{2}\cos\frac{3\pi}{4} = \sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = -1$

$y = \sqrt{2}\sin\frac{3\pi}{4} = 1$

$\left(-\sqrt{2}, \frac{\pi}{4}\right)$

walk backwards



also: $\left(\sqrt{2}, \frac{5\pi}{4}\right)$

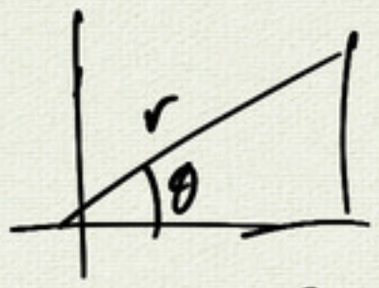
$\left(\sqrt{2}, \frac{5\pi}{4} + 2\pi\right)$

$\left(-\sqrt{2}, \frac{\pi}{4} + 2\pi\right)$

---

example: let $(x,y) = (2\sqrt{3}, 2)$

find all polar coordinates



find r, θ

$r^2 = x^2 + y^2$

$= (2\sqrt{3})^2 + 2^2$

$= 12 + 4$

$= 16$

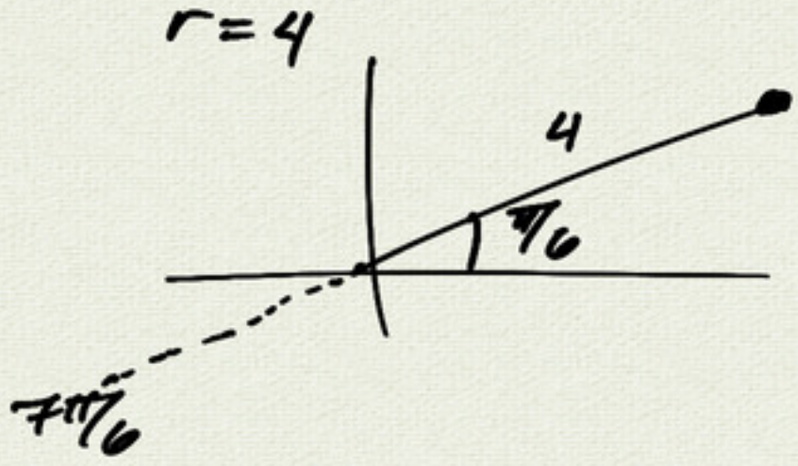
$r = 4$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\theta = \pi/6$

$\left(4, \frac{\pi}{6}\right)$



all polar coordinates

$\left(4, \frac{\pi}{6} + 2\pi k\right)$

$\left(-4, \frac{7\pi}{6} + 2\pi k\right)$