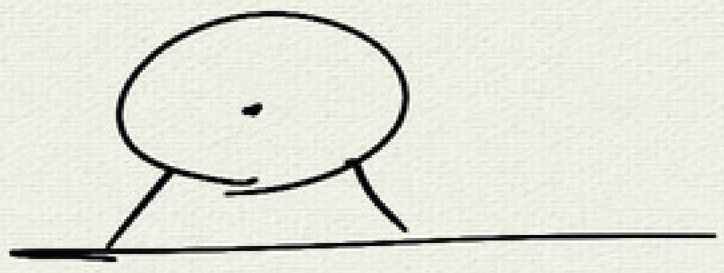
3.3 Parametric Equations

$x(t)$
$y(t)$ ← parameter ("time")



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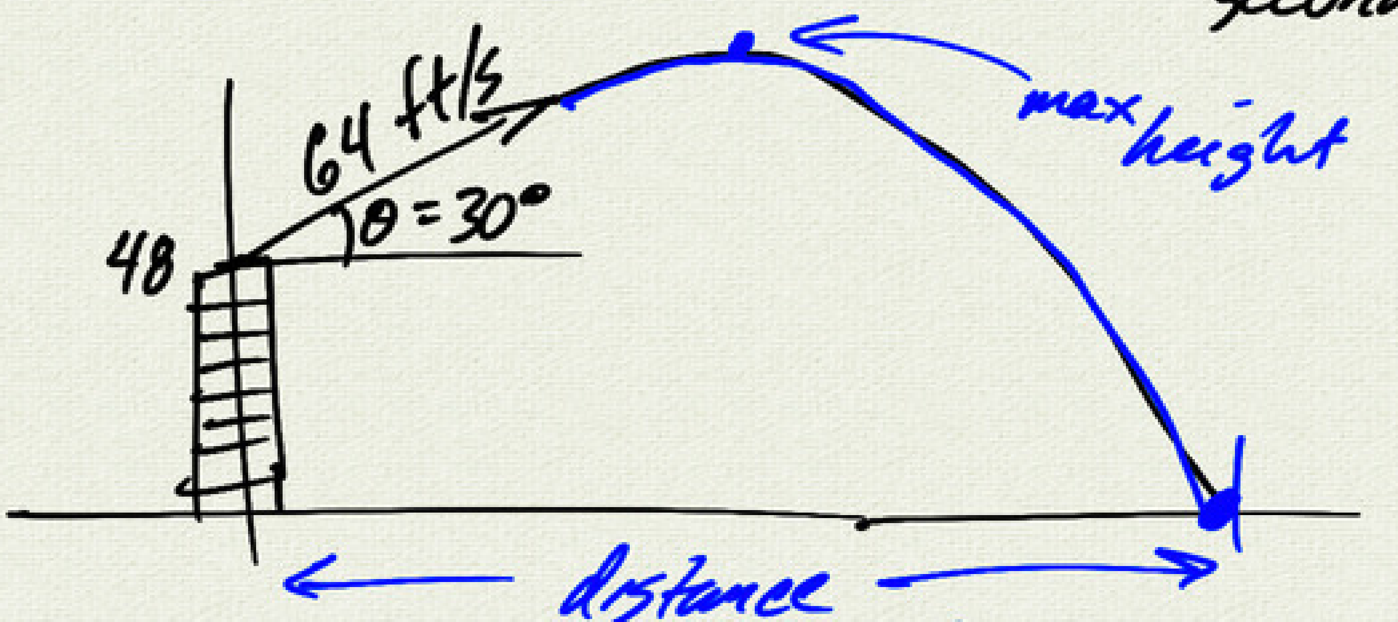
projectile motion

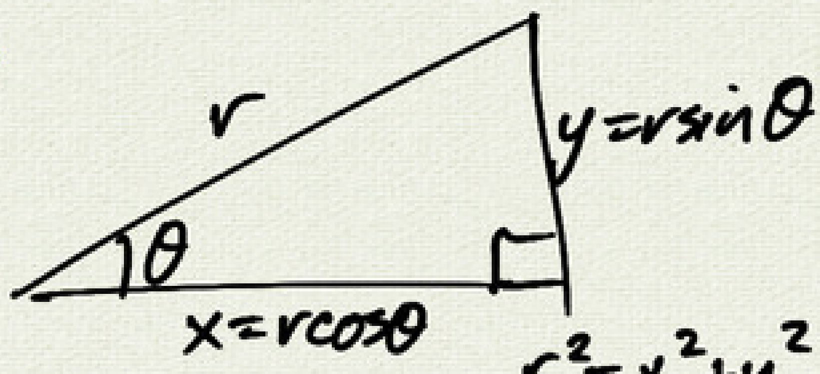
unit: feet, seconds

example:



$v_y = 64\sin 30° = 32$
initial y-velocity

$v_x = 64\cos 30° = 64\left(\frac{\sqrt{3}}{2}\right) = 32\sqrt{3}$
initial x velocity



$r^2 = x^2 + y^2$
$\tan\theta = y/x$

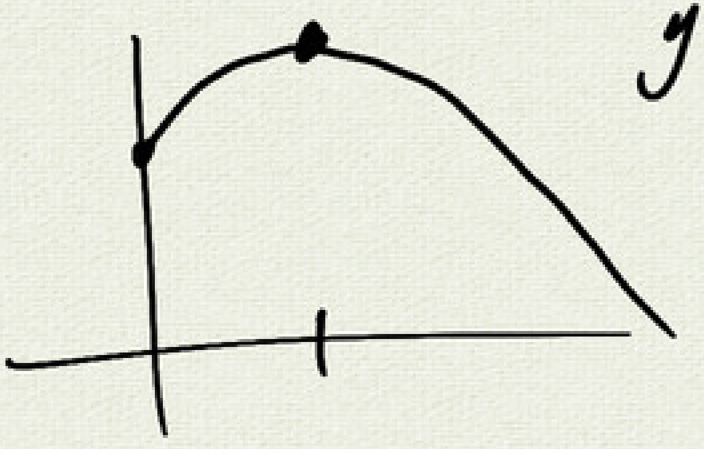
equations:

$x(t) = \underbrace{x_0}_{\text{initial position}} + \underbrace{v_x t}_{r \cdot t = d}$

$y(t) = y_0 + v_y t \underbrace{- 16t^2}_{\text{acceleration due to gravity}}$

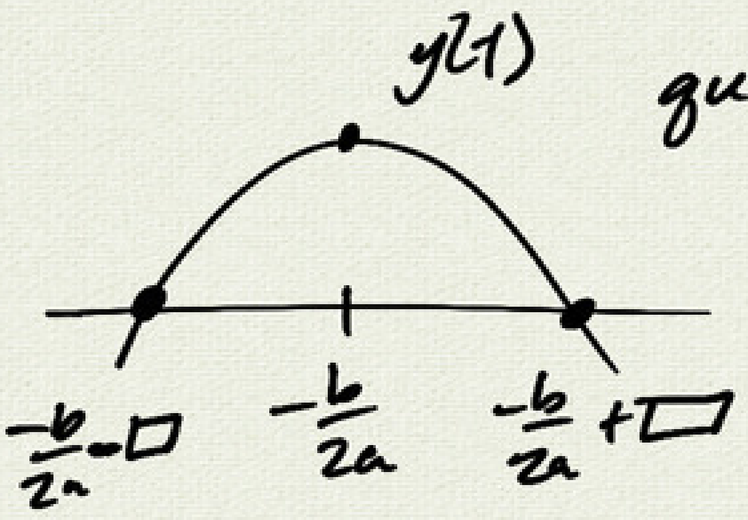
$$\begin{cases} x(t) = (32\sqrt{3})t \\ y(t) = 48 + 32t - 16t^2 \end{cases}$$

(1) max height?
find vertex



$y(t) = \underbrace{-16}_{a}t^2 + \underbrace{32}_{b}t + \underbrace{48}_{c}$

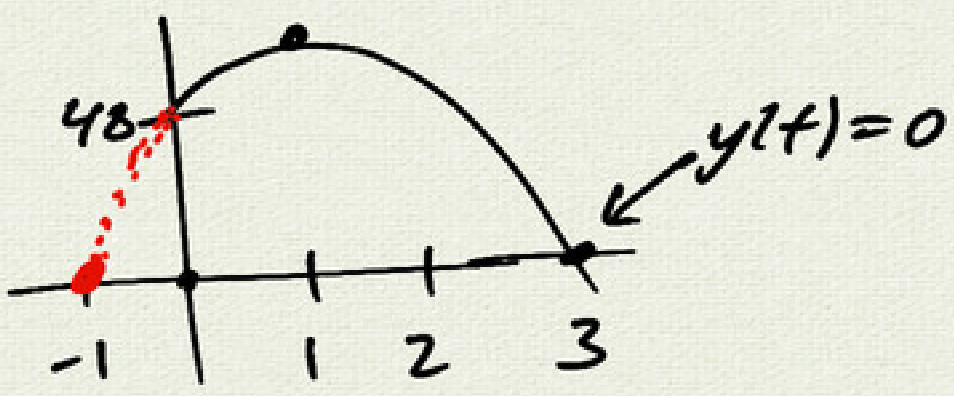
$t = -\frac{b}{2a}$
$= \frac{-32}{2(-16)}$
$= 1$



quadratic formula
$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$= \frac{-b}{2a} \pm \square$

max height $y(1) = -16(1)^2 + 32(1) + 48$
$= 64$

(2) distance (x-direction) on impact



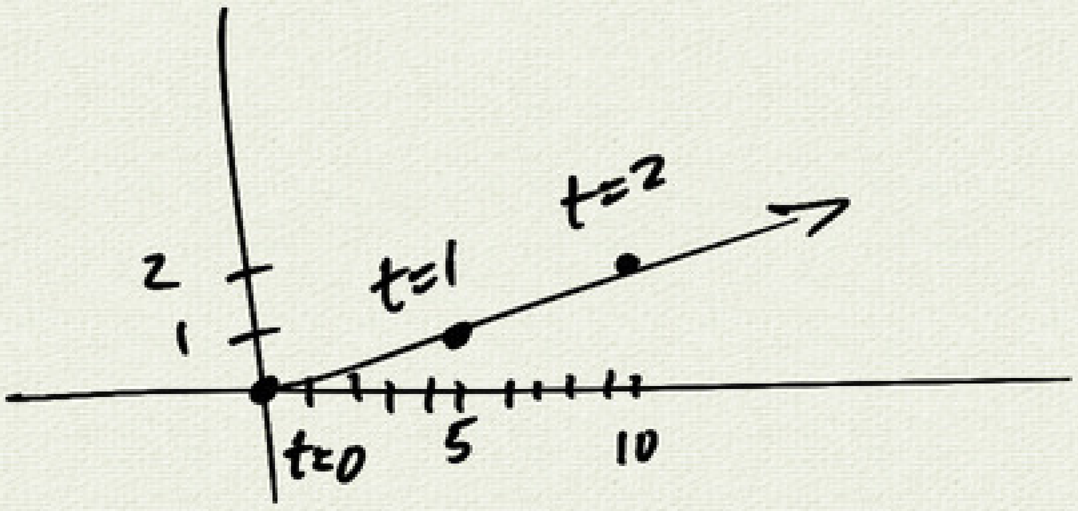
$y(t) = -16t^2 + 32t + 48 = 0$
$-16(t^2 - 2t - 3) = 0$
$-16(t-3)(t+1) = 0$
$\Rightarrow t = 3, -1$

x distance $x(3) = (32\sqrt{3}) \cdot 3$
$= 96\sqrt{3}$

parametric equations

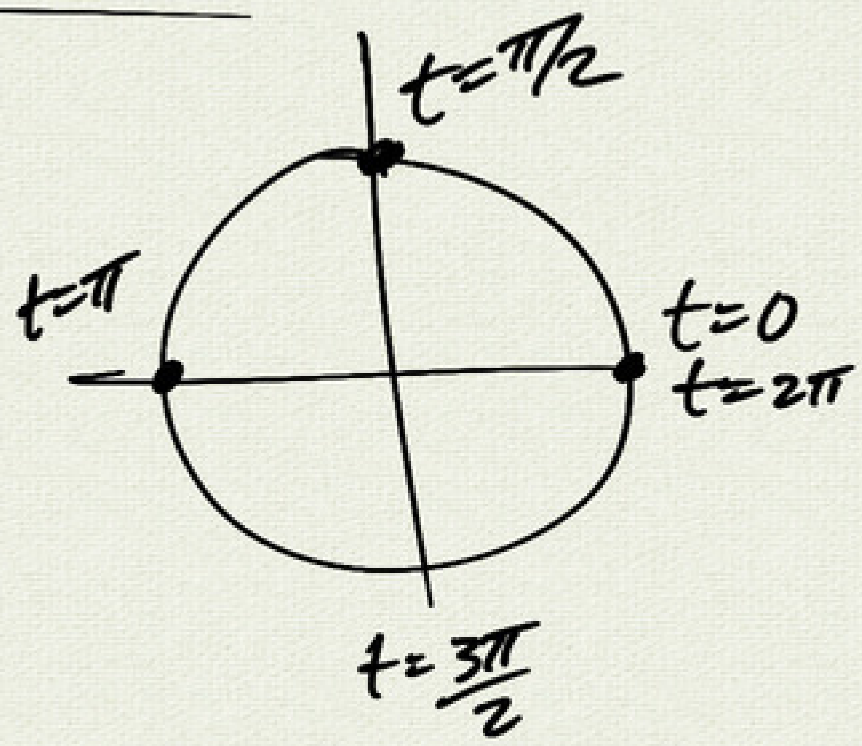
$x(t) = 5t$
$y(t) = t$

t	$x(t)$	$y(t)$
0	0	0
1	5	1
2	10	2



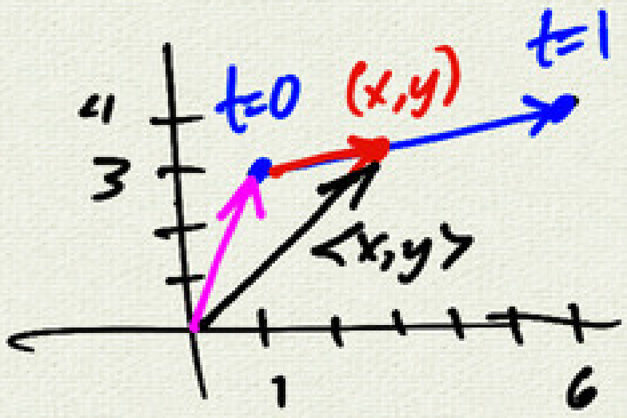
$x(t) = \cos t$
$y(t) = \sin t$

t	$x(t)$	$y(t)$
0	1	0
$\pi/2$	0	1
π	-1	0
$3\pi/2$	0	-1
2π	1	0



<u>example</u> parametrize line (segment)

from $(1,3)$ to $(6,4)$



$\langle x,y \rangle = \langle 1,3 \rangle + t\langle 5,1 \rangle$

$\bar{v} = \langle 5,1 \rangle$

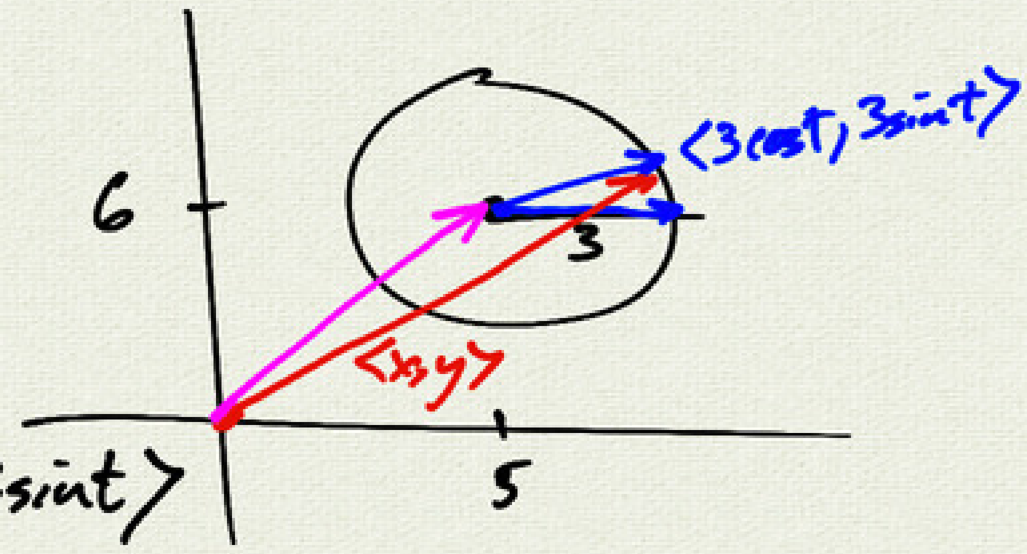
$x = 1 + t \cdot 5$
$y = 3 + t \cdot 1$

$x(t) = 5t + 1$
$y(t) = t + 3$

check: $x(0) = 1$
$y(0) = 3$

$x(1) = 6$
$y(1) = 4$ ✓

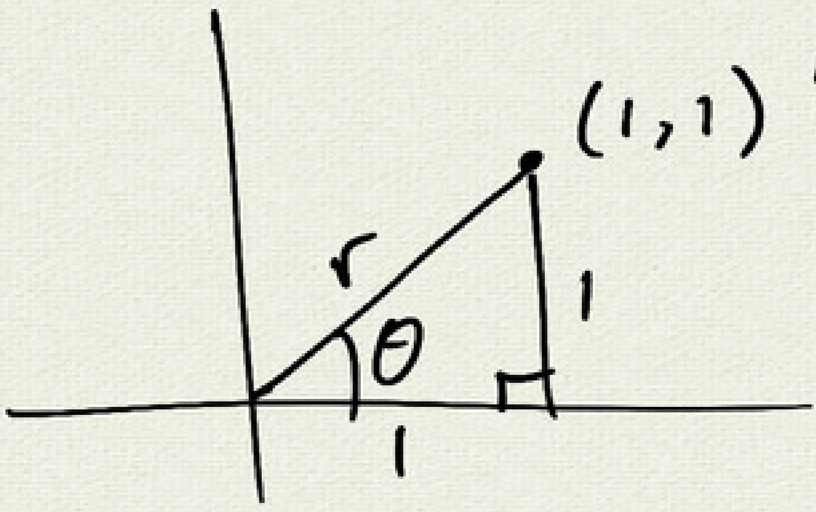
example
parametrize circle
radius 3
center (5,6)



$$\langle x,y\rangle = \langle 5,6\rangle + \langle 3\cos t, 3\sin t\rangle$$

$$x(t) = 5 + 3\cos t$$
$$y(t) = 6 + 3\sin t$$

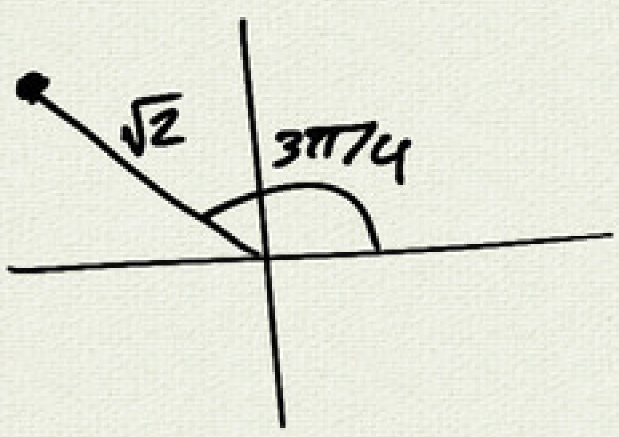
3.4 Polar Coordinates



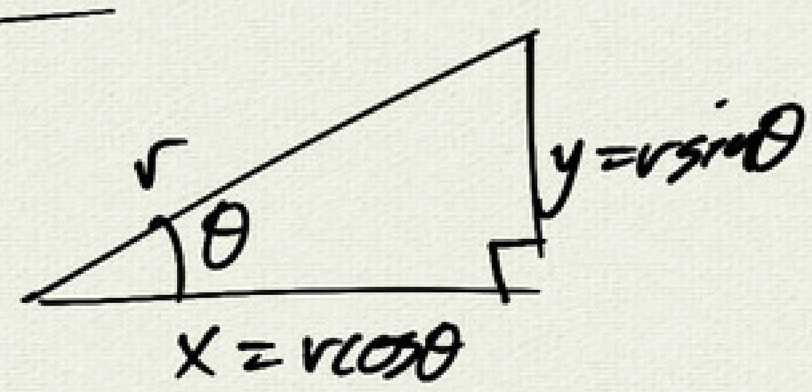
rectangular coordinates (x,y)

polar coordinates (r,θ)

$\left(\sqrt{2}, \frac{\pi}{4}\right)$



$\left(\sqrt{2}, \frac{3\pi}{4}\right)$



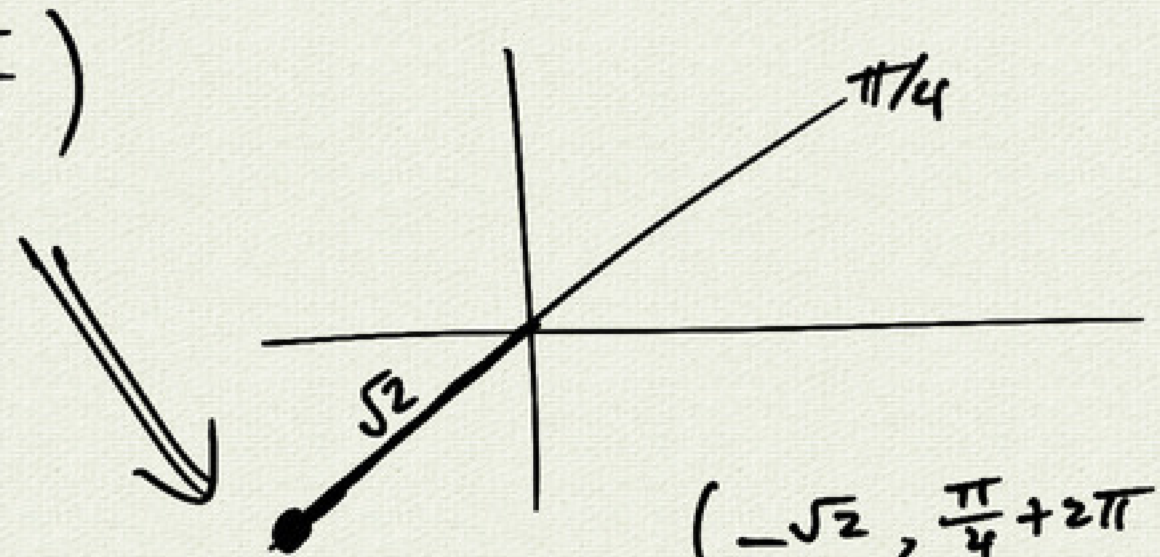
$$x = \sqrt{2}\cos\frac{3\pi}{4} = \sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = -1$$

$$y = \sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = 1$$

rectangular $(-1, 1)$

$\left(-\sqrt{2}, \frac{\pi}{4}\right)$

walk backwards



$\left(-\sqrt{2}, \frac{\pi}{4} + 2\pi\right)$

$\left(\sqrt{2}, \frac{5\pi}{4}\right)$

$\left(\sqrt{2}, \frac{5\pi}{4} + 2\pi\right)$

all polar coordinates

$$\boxed{\begin{array}{c}\left(\sqrt{2}, \frac{5\pi}{4} + 2\pi k\right) \\ \left(-\sqrt{2}, \frac{\pi}{4} + 2\pi k\right)\end{array}}$$