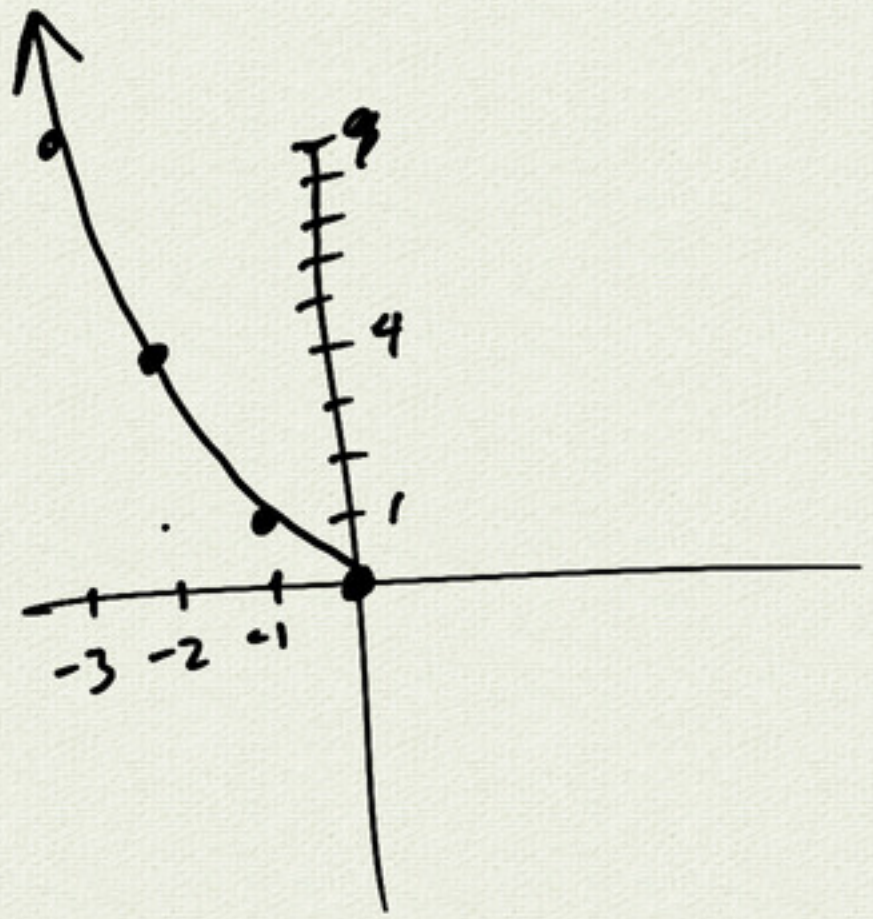
8.7 (13)

$x(t) = -\sqrt{t}$

$y(t) = t$

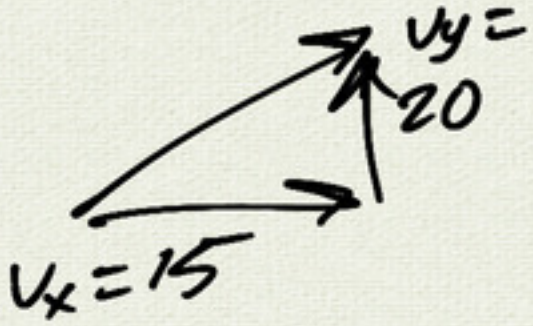
t	$x(t)$	$y(t)$
0	0	0
1	−1	1
4	−2	4
9	−3	9



$x(t) = x_0 + v_x t$

$y(t) = y_0 + v_y t - 16t^2$

(63)



$y_0 = 0$

$y(t) = -16t^2 + 20t$ (v_y)

$x(t) = 15t$

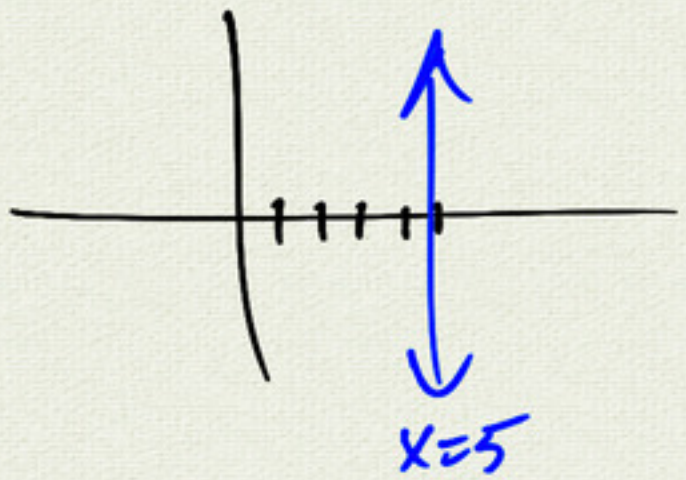
$y(x)$

$x = 15t \Rightarrow t = \frac{x}{15}$

$y\left(\frac{x}{15}\right) = -16\left(\frac{x}{15}\right)^2 + 20\left(\frac{x}{15}\right)$

3.5 Polar Graphs

$x=5$



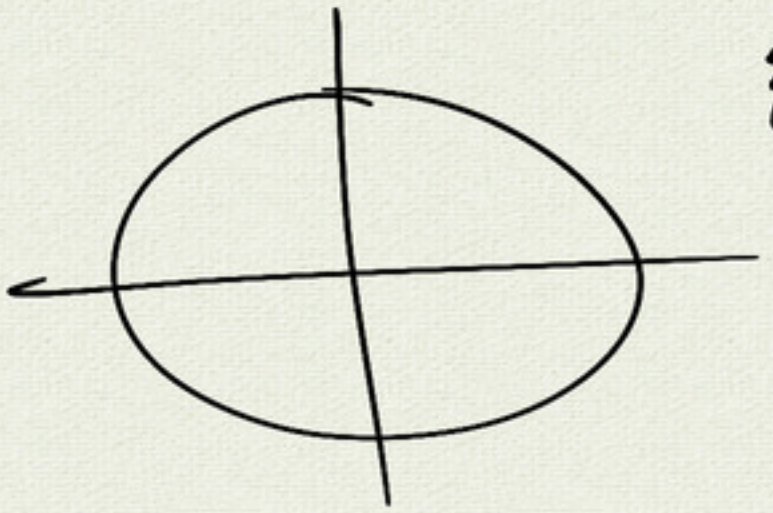
graph

set notation

$\{(x,y) \mid x=5\}$

y can be anything

$r=5$

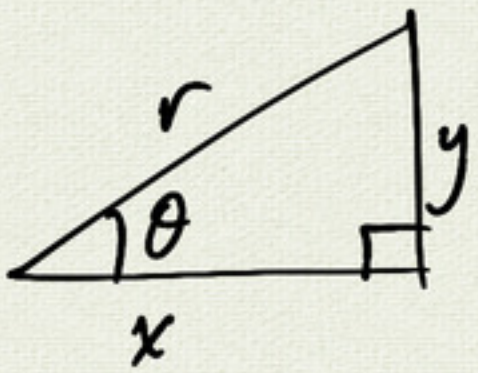


$\{(r,\theta) \mid r=5\}$

θ can be anything

$r^2=5^2$

$x^2+y^2=5^2$

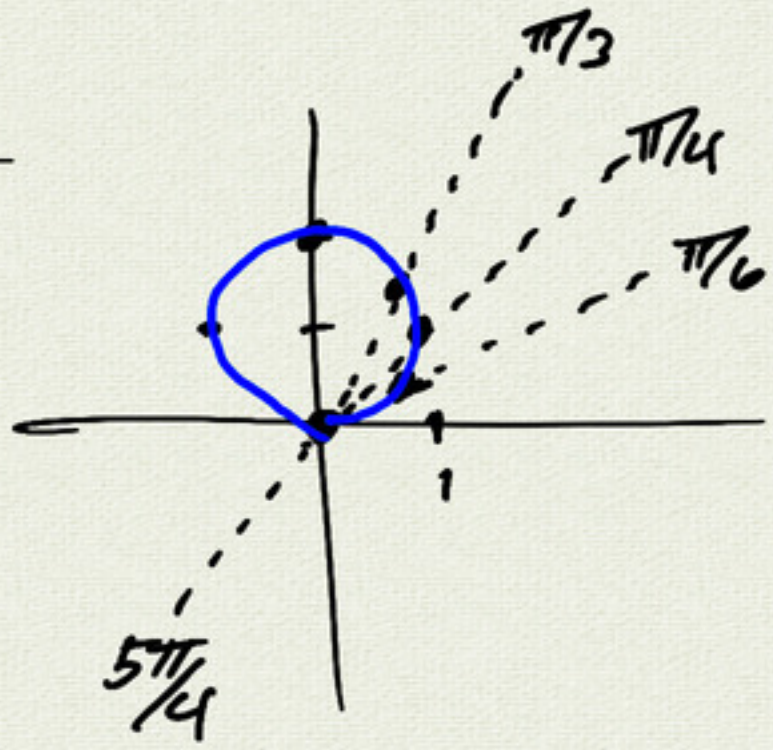


$x=r\cos\theta$ $\quad r^2=x^2+y^2$

$y=r\sin\theta$ $\quad \tan\theta=y/x$

$r = 2\sin\theta$

θ	$\sin\theta$	$r = 2\sin\theta$
0	0	0
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}$
$\pi/2$	1	2
$3\pi/4$	$\sqrt{2}/2$	$\sqrt{2}$
π	0	0
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\sqrt{2}$



$r = 2\sin\theta$

$x = r\cos\theta$
$y = r\sin\theta$

$r^2 = 2r\sin\theta$

$x^2 + y^2 = 2y$

$x^2 + (y^2 - 2y + 1) = 0 + 1$

$x^2 + (y-1)^2 = 1$

circle
center $(0, 1)$
radius 1

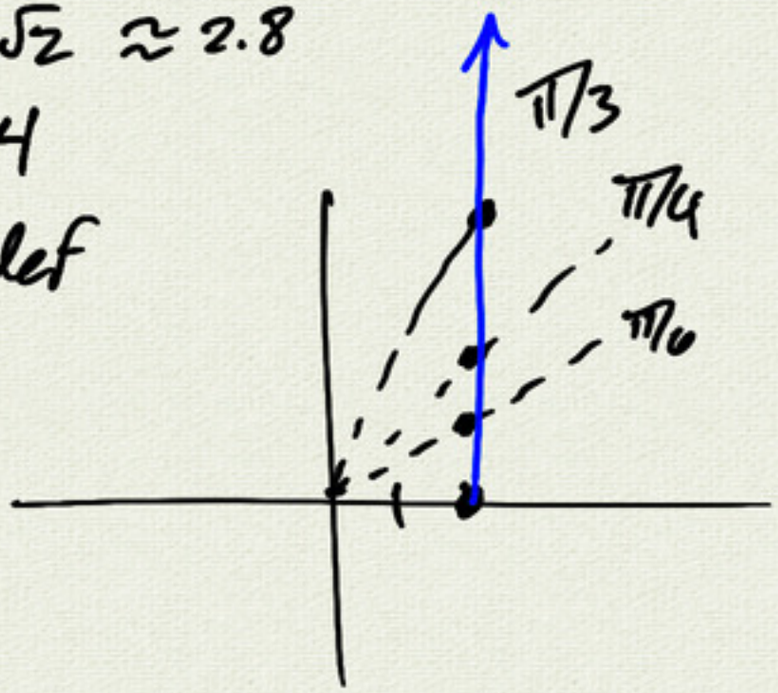
$(y+a)^2 = y^2 + 2ay + a^2$
$(y-a)^2 = y^2 - 2ay + a^2$

$r = 2\sec\theta$

θ	$\cos\theta$	$\sec\theta$	$r = 2\sec\theta$
0	1	1	2
$\pi/6$	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$	$4/\sqrt{3}$ >2
$\pi/4$	$\frac{\sqrt{2}}{2}$	$\sqrt{2}$	$2\sqrt{2}$ ≈ 2.8
$\pi/3$	$1/2$	2	4
$\pi/2$	0	undef	undef

$\sqrt{3} \approx 1.732$

$\sqrt{2} \approx 1.414$



$r = 2\sec\theta$

$r = \frac{2}{\cos\theta}$

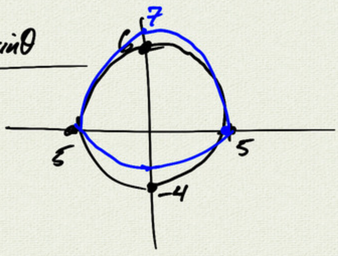
$r\cos\theta = 2$

$x = 2$ vertical line

$r=5$

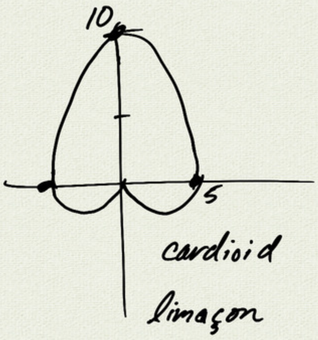
$r=5+\sin\theta$

θ	$\sin\theta$	$r=5+\sin\theta$	$5+2\sin\theta$
0	0	5	5
$\pi/2$	1	6	7
π	0	5	5
$3\pi/2$	-1	4	3
2π	0	5	5



$r=5+5\sin\theta$

θ	$5\sin\theta$	$5+5\sin\theta$
0	0	5
$\pi/2$	5	10
π	0	5
$3\pi/2$	-5	0
2π	0	5

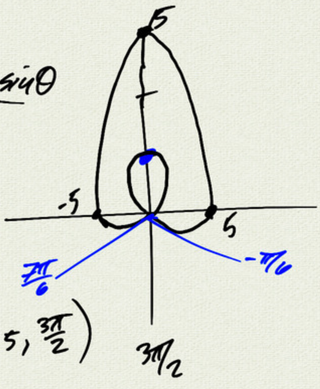


cardioid

limaçon

$r=5+10\sin\theta$

θ	$10\sin\theta$	$r=5+10\sin\theta$
0	0	5
$\pi/2$	10	15
π	0	5
$3\pi/2$	-10	-5
2π	0	5



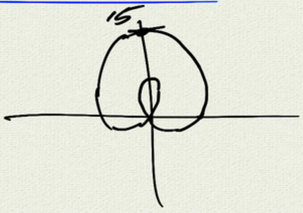
$r=0 \Rightarrow 5+10\sin\theta=0$

$10\sin\theta=-5$

$\sin\theta=-\frac{1}{2}$

$\theta=$

HW: analyze graph of $r=5+10\sin\theta$



(1) graph (on Desmos)

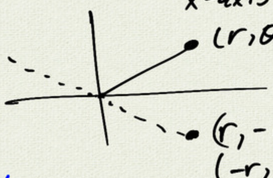
(2) max $|r|$ value (and angles)

$r=5+10\sin\theta \Rightarrow r=15$ at $\theta=\pi/2$

(3) symmetry

check for x, y, origin symmetry

x-axis



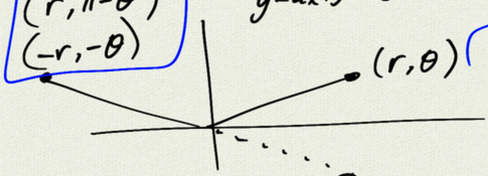
check these

check

$(r,\pi-\theta)$

$(-r,-\theta)$

y-axis test



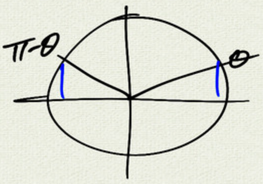
check $(r,\pi-\theta)$:

$r \stackrel{?}{=} 5+10\sin(\pi-\theta)$

$-r \stackrel{?}{=} 5+10\sin(-\theta)$

$-r=5-10\sin\theta$ ✗

$\sin(\pi-\theta)=\sin\theta$? use sum formula to prove



$r \stackrel{?}{=} 5+10\sin(\pi-\theta)$

$=5+10[\underbrace{\sin\pi}_{0}\cos(-\theta)+\underbrace{\cos\pi}_{-1}\sin(-\theta)]$

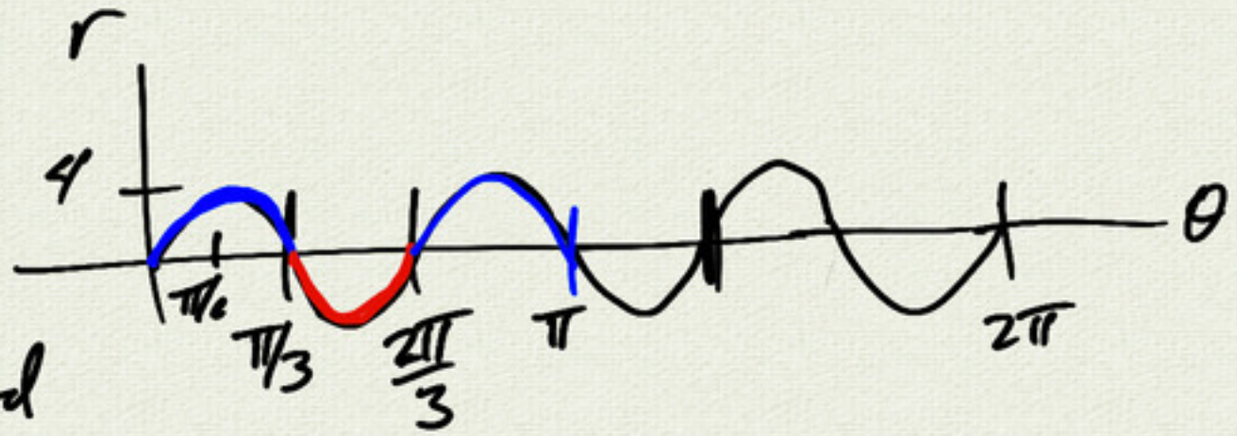
$=5+10\sin\theta$ ✓

example:

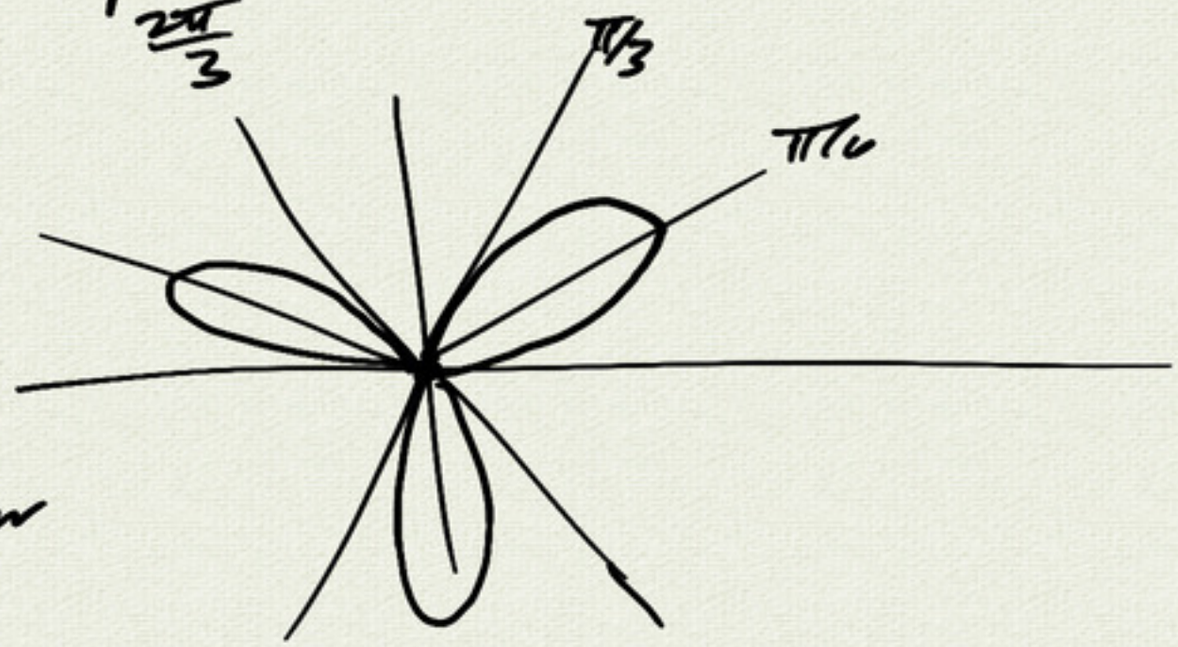
$r = 4\sin 3\theta$

↑ amplitude

← period $\frac{2\pi}{3}$



rose curve



check y-axis symmetry

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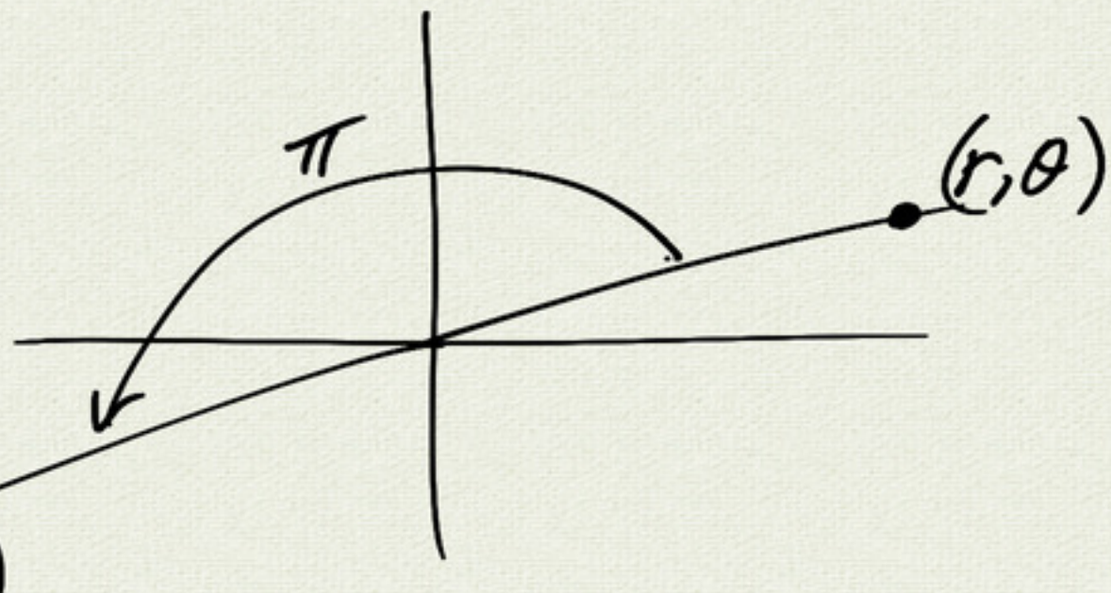
origin symmetry test



check these points $\begin{cases} (-r, \theta) \\ (r, \theta+\pi) \end{cases}$