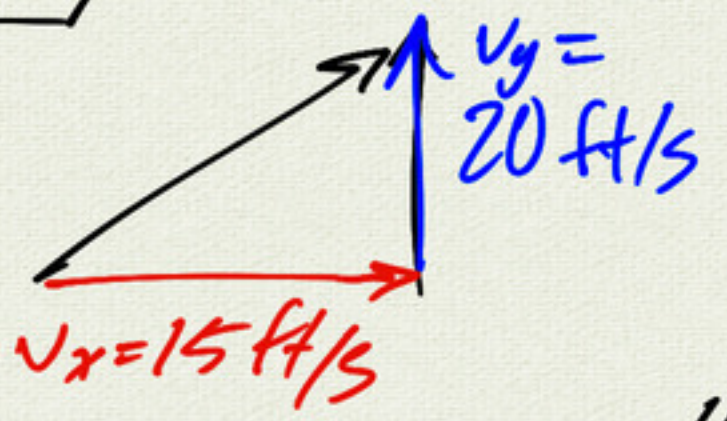
8.7

63

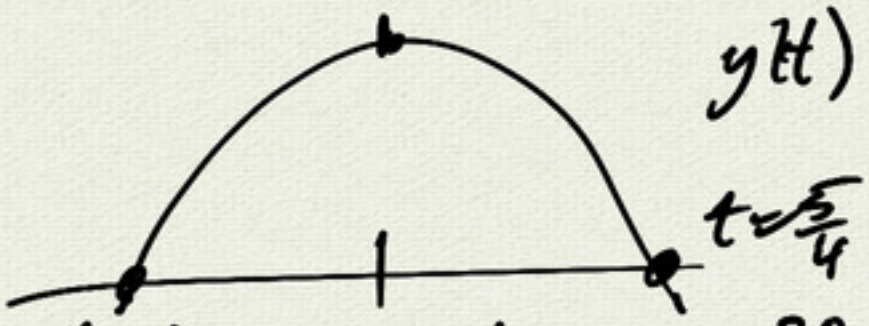


$$x(t) = x_0 + v_x t$$

$$y(t) = y_0 + v_y t - 16t^2$$

$$y(t) = -16t^2 + \underbrace{20t}_{v_y}$$

$$x(t) = 15t$$



$$t = -\frac{b}{2a} = \frac{-20}{-32} = \frac{+5}{+8}$$

$$\text{max height} = y\left(\frac{5}{8}\right) = \frac{25}{4} = 6.25$$

---

hit ground:

$$y(t) = 0 \longrightarrow -16t^2 + 20t = 0$$

$$t(-16t + 20) = 0$$

$$\Rightarrow t = 0 \text{ or } t = \frac{20}{16} = \frac{5}{4}$$

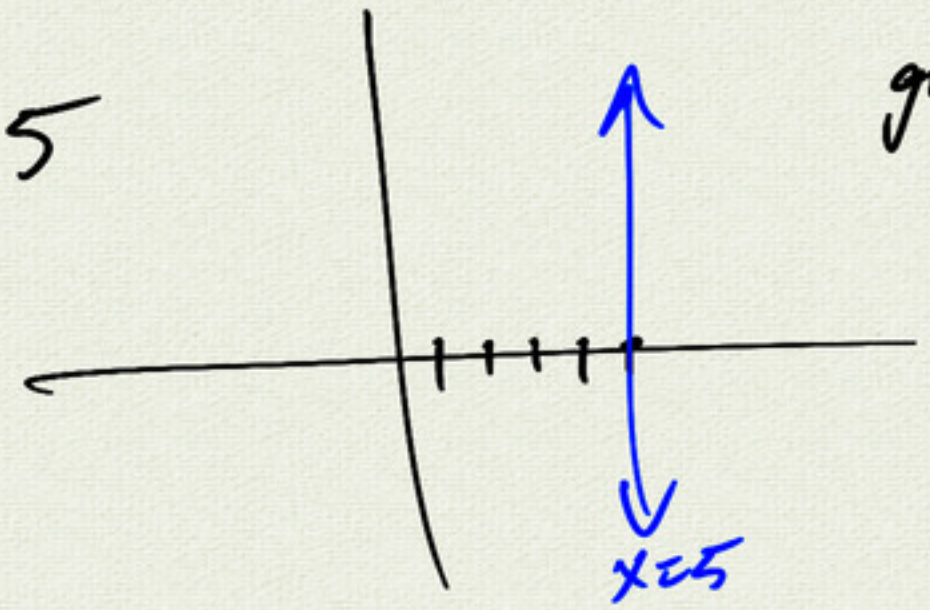
$$\text{max distance} = x\left(\frac{5}{4}\right)$$

$$= 15 \cdot \frac{5}{4}$$

$$= \frac{75}{4}$$

3.5 Polar Graphs

$x=5$



graph = set

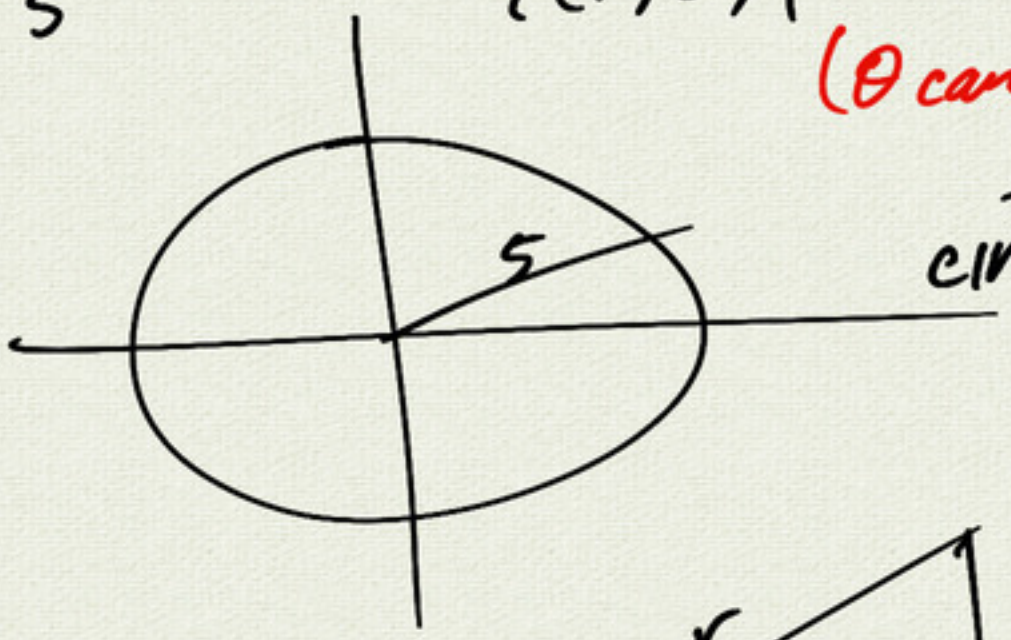
$\{(x,y) \mid x=5\}$

(y can be anything)

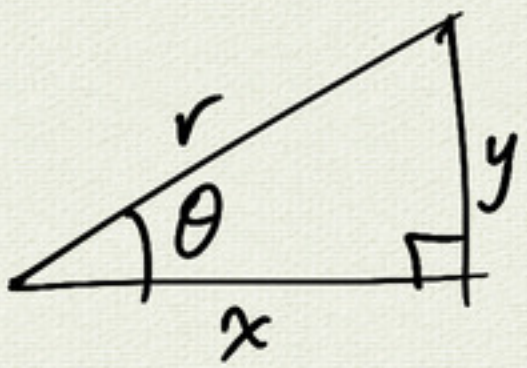
$r=5$

$\{(r,\theta) \mid r=5\}$

(θ can be anything)



circle radius 5



$x = r\cos\theta$ $\quad r^2 = x^2 + y^2$

$y = r\sin\theta$ $\quad \tan\theta = \frac{y}{x}$

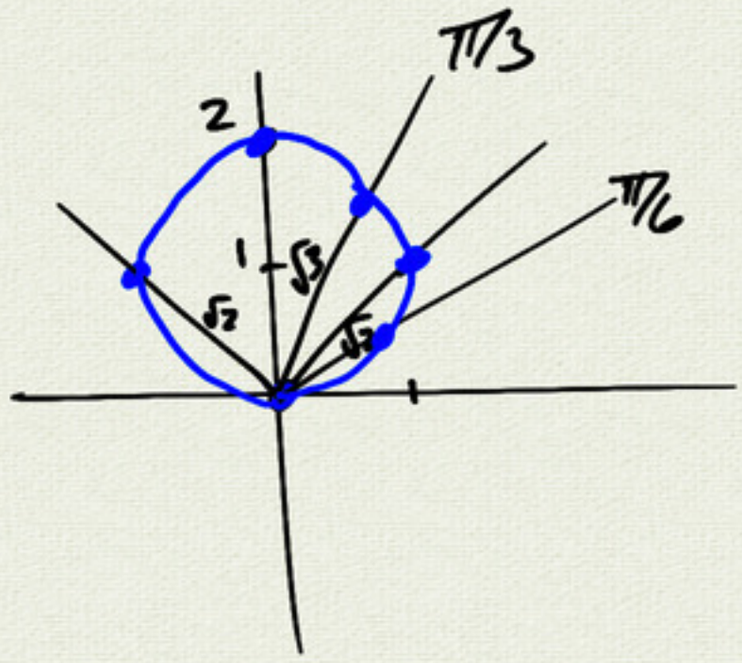
$r = 5$

$r^2 = 5^2$

$x^2 + y^2 = 5^2$ (standard circle equation)

$r = 2\sin\theta$

θ	$\sin\theta$	$r = 2\sin\theta$
0	0	0
$\pi/4$	$\frac{\sqrt{2}}{2}$	$\sqrt{2}$
$\pi/2$	1	2
$3\pi/4$	$\frac{\sqrt{2}}{2}$	$\sqrt{2}$
π	0	0



$r = 2\sin\theta$ $\qquad$ $y = r\sin\theta$

$r^2 = 2r\sin\theta$

$x^2 + y^2 = 2y$

$x^2 + (y^2 - 2y + 1) = 0 + 1$

$x^2 + (y-1)^2 = 1$

radius 1

center $(0, 1)$

$x^2 + y^2 = r^2$

$(x-h)^2 + (y-k)^2 = r^2$

circle $\quad$ center (h, k)

$(y-k)^2$

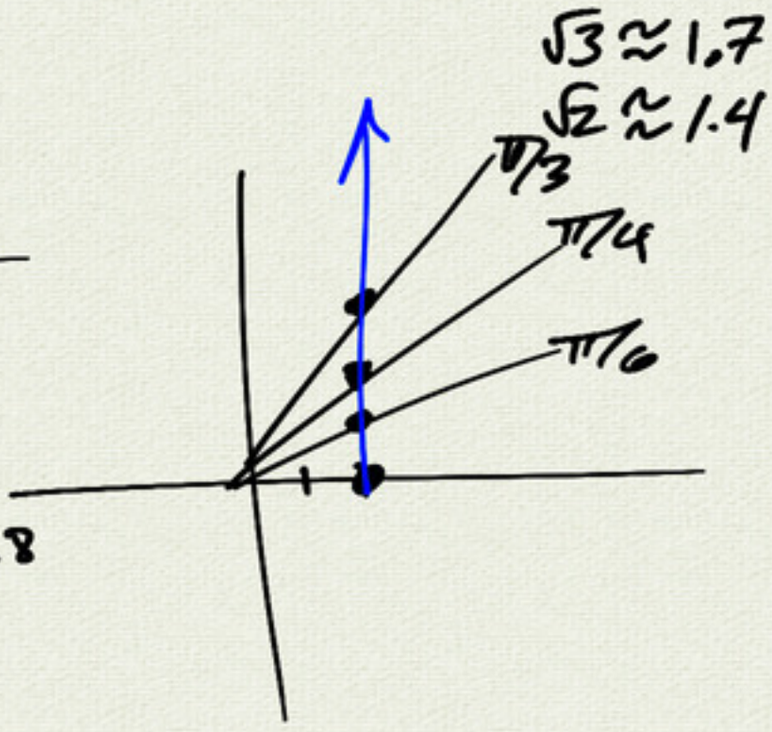
$= y^2 - 2ky + k^2$

---

example

$r = 2\sec\theta$

θ	$\cos\theta$	$\sec\theta$	$r = 2\sec\theta$
0	1	1	2
$\pi/6$	$\sqrt{3}/2$	$2/\sqrt{3}$	$4/\sqrt{3}$? 2
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}$	$2\sqrt{2} \approx 2.8$
$\pi/3$	$1/2$	2	4
$\pi/2$	0	undef.	undef.



$r = 2\sec\theta$

$r = \frac{2}{\cos\theta}$

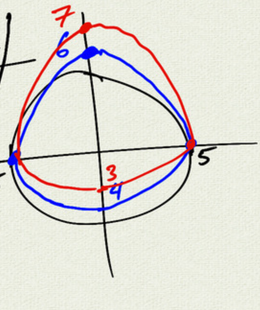
$r\cos\theta = 2$

$x = 2$

$r=5$

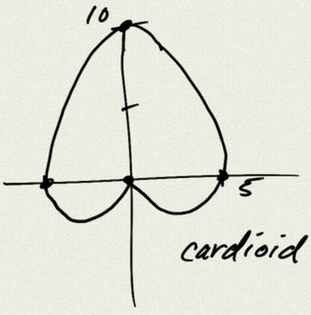
$r=5+\sin\theta$

θ	$\sin\theta$	$r=5+\sin\theta$	$5+2\sin\theta$
0	0	5	5
$\pi/2$	1	6	7
π	0	5	5
$3\pi/2$	-1	4	3
2π	0	5	5



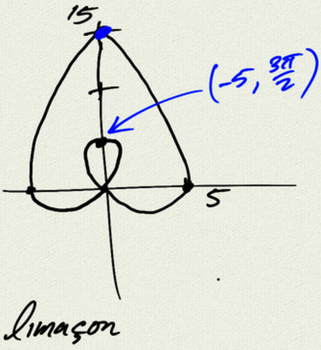
$r=5+5\sin\theta$

θ	$5\sin\theta$	$5+5\sin\theta$
0	0	5
$\pi/2$	5	10
π	0	5
$3\pi/2$	-5	0
2π	0	5



$r=5+10\sin\theta$

θ	$10\sin\theta$	$r=5+10\sin\theta$
0	0	5
$\pi/2$	10	15
π	0	5
$3\pi/2$	-10	-5
2π	0	5



---

analyze $r=5+10\sin\theta$

(1) graph (on Desmos)

(2) max $|r|$ value $\quad r=5+10\sin\theta$

max $|r|=15$ when $\sin\theta=1$

$\theta=\pi/2$

(3) symmetry: x-axis

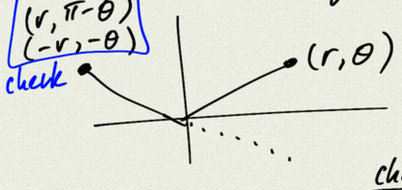
y-axis

origin (rotate 180°)

$(r,\pi-\theta)$

$(-r,-\theta)$

check



check $(-r,-\theta)$:

$-r \stackrel{?}{=} 5+10\sin(-\theta)$

$-r = 5-10\sin\theta$ ✗

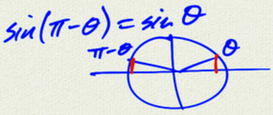
check $(r,\pi-\theta)$:

$r \stackrel{?}{=} 5+10\sin(\pi-\theta)$

$=5+10[\underbrace{\sin\pi}_{0}\cos(-\theta)+\underbrace{\cos\pi}_{-1}\sin(-\theta)]$

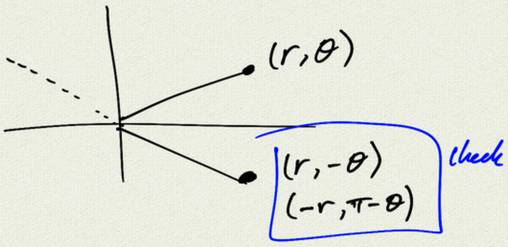
$=5+10\sin\theta$ ✓

$\sin(\pi-\theta)=\sin\theta$



---

x-axis symmetry

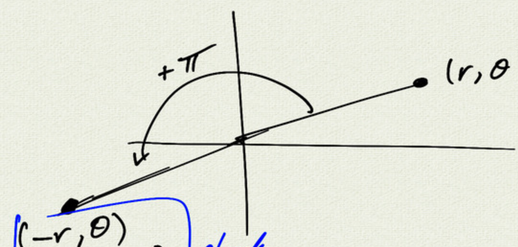


$(r,-\theta)$

$(-r,\pi-\theta)$

check

origin symmetry



$(-r,\theta)$

$(r,\theta+\pi)$

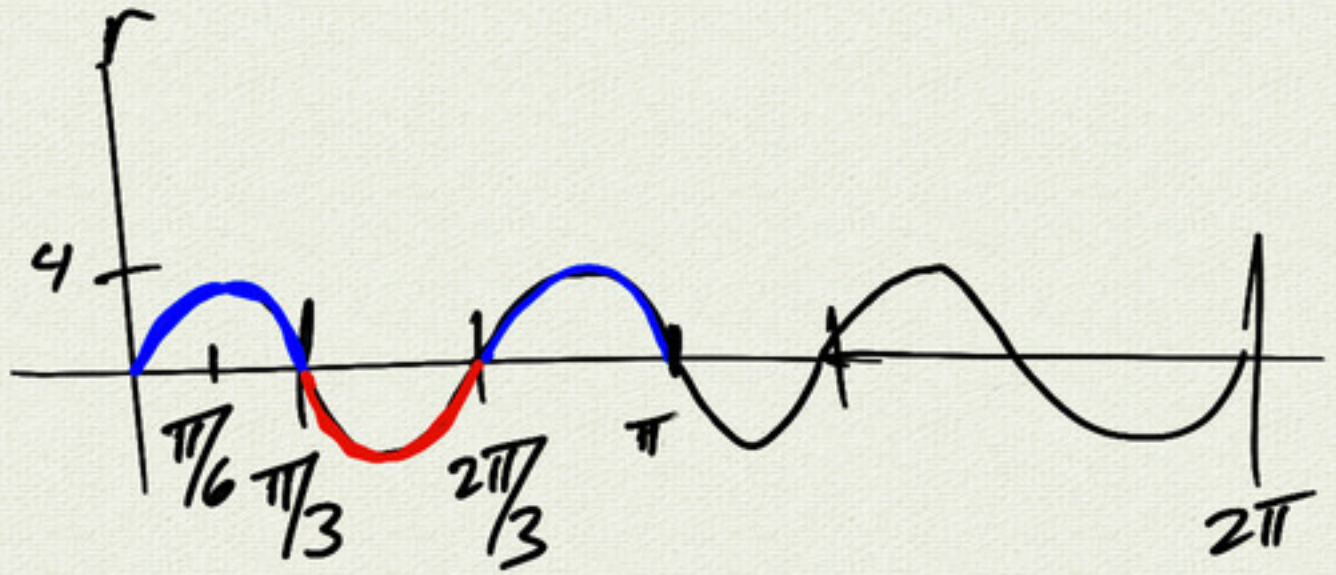
check

example

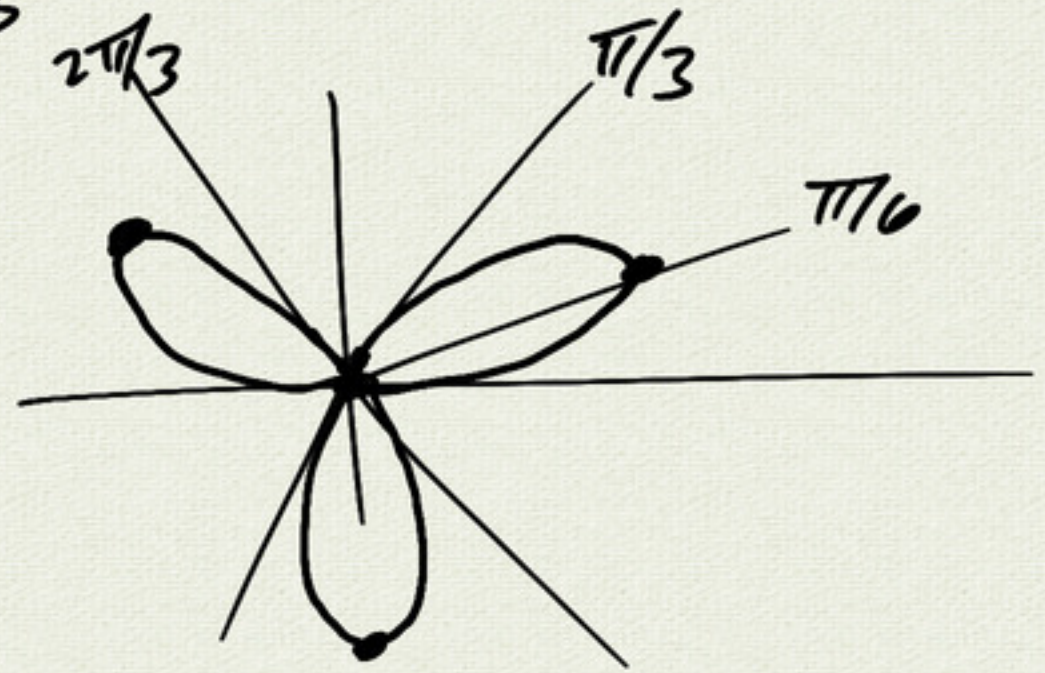
$r = 4\sin 3\theta$

amplitude

period $\frac{2\pi}{3}$



rose curve



$r = 4\sin 3\theta$

max $|r|$ value $= 4$ when $\sin 3\theta = \pm 1$

$3\theta = \frac{\pi}{2} + k\pi$

$\theta = \frac{\pi}{6} \pm k\frac{\pi}{3}$

check y-axis symmetry