3.6 Linear Systems

$$3x + 6y = 15$$
$$y - z = -1$$
$$-2x - 4y + z = -7$$

substitution
elimination

$$\left(\begin{array}{ccc|c} 3 & 6 & 0 & 15 \\ 0 & 1 & -1 & -1 \\ -2 & -4 & 1 & -7 \end{array}\right)$$

goal $\Longrightarrow$ $\left(\begin{array}{ccc|c} 1 & 0 & 0 & A \\ 0 & 1 & 0 & B \\ 0 & 0 & 1 & C \end{array}\right)$ $\Longrightarrow$ $x = A$, $y = B$, $z = C$

$\downarrow \frac{1}{3}R_1$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 1 & -1 & -1 \\ -2 & -4 & 1 & -7 \end{array}\right)$$

$\downarrow 2R_1 + R_3$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

use 1 to clear column

$\downarrow -2R_2 + R_1$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$\downarrow -2R_3 + R_1, \quad R_3 + R_2$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right) \longrightarrow \begin{array}{l} x = 1 \\ y = 2 \\ z = 3 \end{array}$$

$r^2 = a^2 \cos 2\theta$

lemniscate