9.6

47

$$\frac{x-1}{7} + \frac{y-2}{8} + \frac{z-3}{4} = 0$$

$$x + y + z = 6$$

$$\frac{x+2}{3} + 2y + \frac{z-3}{3} = 5$$

$$8(x-1) + 7(y-2) + 14(z-3) = 0$$

$$8x - 8 + 7y - 14 + 14z - 42 = 0$$

$$8x + 7y + 14z = 64$$

3.7 Matrices

$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ 2D array of numbers

$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$ 2×3 matrix

#rows #columns

$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ row 2, column 3

a_{ij} row column

---

basic operations:

① addition $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 3 & 6 \end{pmatrix}$

② scalar multiplication $3\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 9 \\ 6 & 12 \end{pmatrix}$

$\langle x, y \rangle \iff \begin{pmatrix} x \\ y \end{pmatrix}$

vector 2×1 matrix column vector

$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix} = ?$

undefined

addition: matrices must be same size

Matrix multiplication

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 9 & 16 \\ 14 & 22 \end{pmatrix}$$

$1\cdot 3 + 3\cdot 2 = 9$

$\langle 1,3\rangle \cdot \langle 1,5\rangle = 1\cdot 1 + 3\cdot 5 = 16$

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} = ? \qquad \begin{pmatrix} \square & \end{pmatrix}$$

can't do dot product $\langle 1,3\rangle \cdot \langle 1,2,3\rangle$

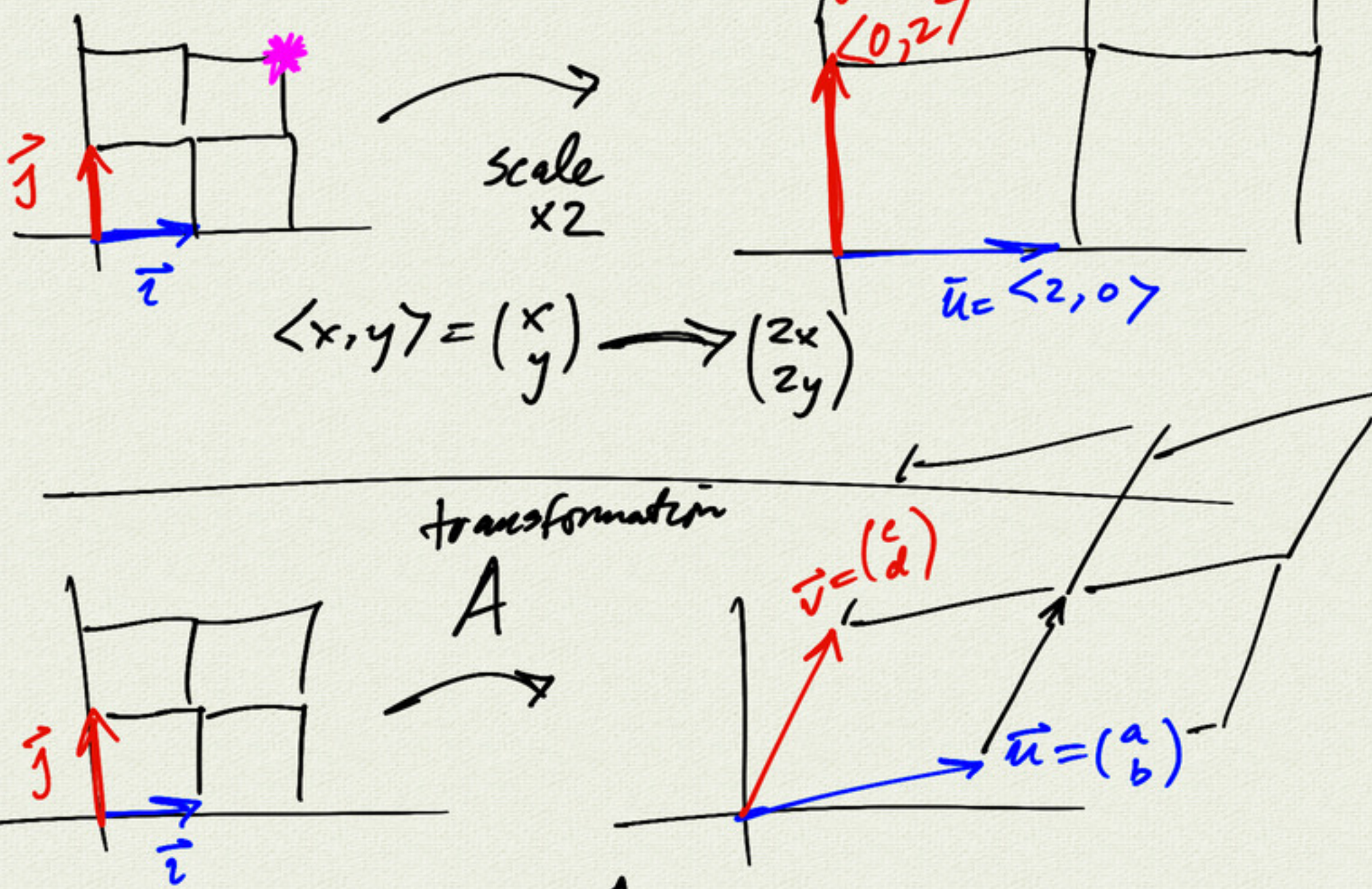
undefined

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix} = \begin{pmatrix} 7 & \cdot & 23 \\ \cdot & \cdot & \cdot \end{pmatrix}$$

2 rows — 3 columns — 2×3

$(2\times 2)\cdot(2\times 3)$

must be same (dot product)



$$\langle x,y\rangle = \begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{A} x\vec{u}+y\vec{v}$$

$x\vec{i}+y\vec{j}$

$$= x\begin{pmatrix} a \\ b \end{pmatrix} + y\begin{pmatrix} c \\ d \end{pmatrix}$$

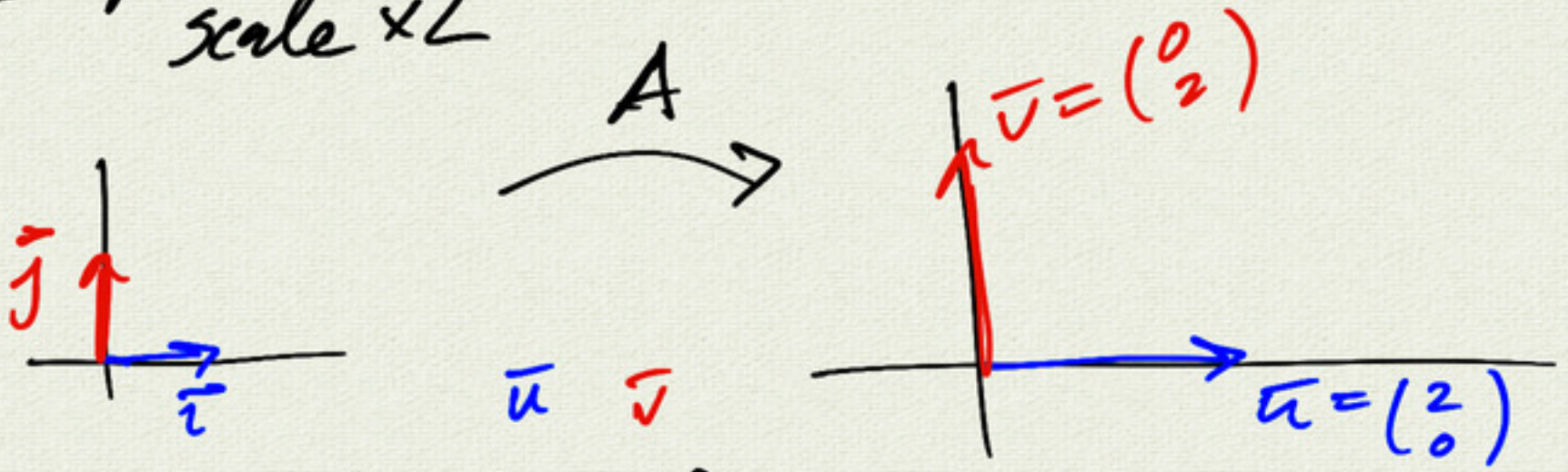
$$= \begin{pmatrix} ax+cy \\ bx+dy \end{pmatrix}$$

$$= \begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

A (linear) transformation $\Longleftrightarrow$ matrix

multiplication $\Longleftrightarrow$ apply the transformation

example: scale ×2



$$A=\begin{pmatrix}a & c\\ b & d\end{pmatrix}$$

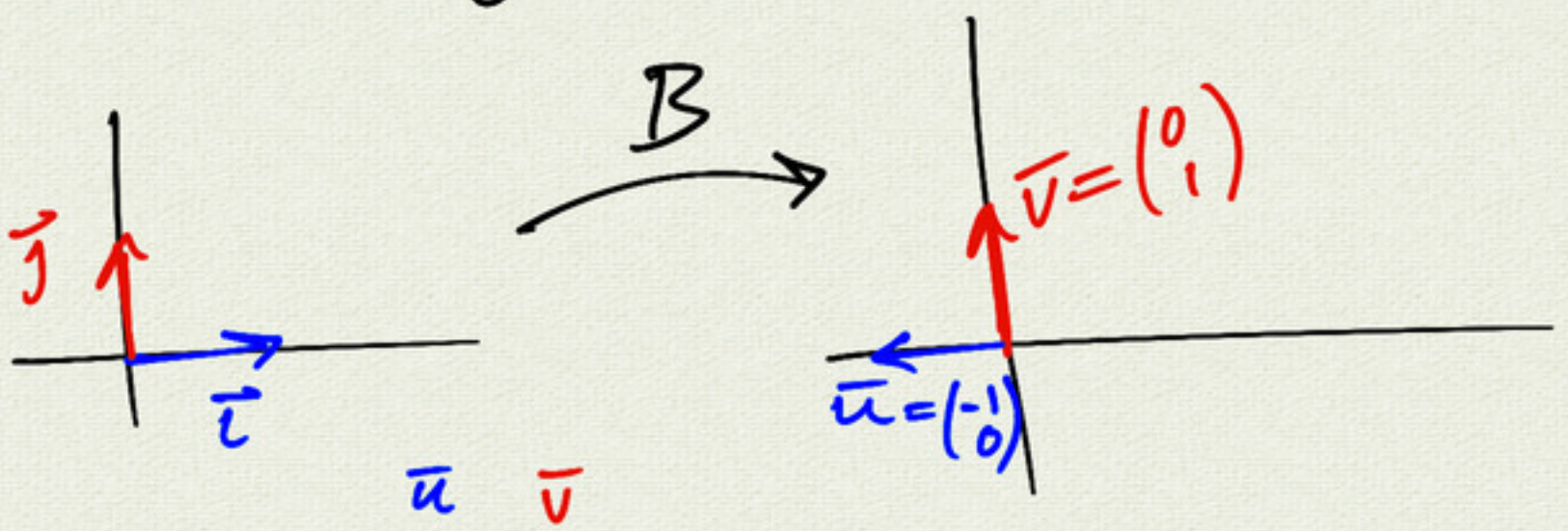
$$=\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$$

apply A to $\begin{pmatrix}x\\y\end{pmatrix}$:

$$A\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2x\\2y\end{pmatrix}$$

---

example 2 reflect across y-axis

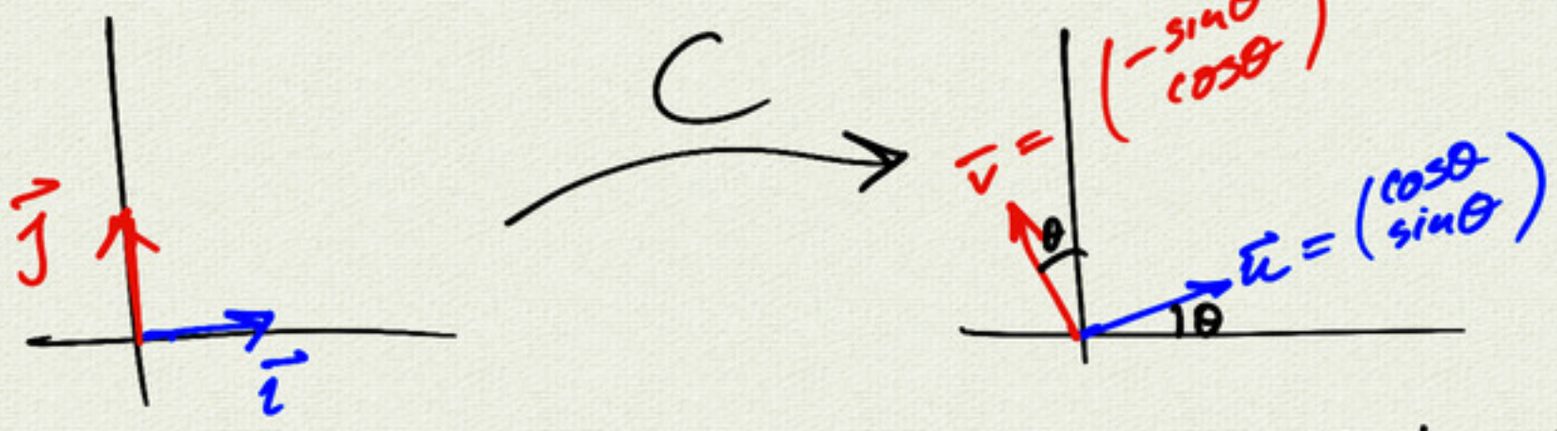


$$B=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$$

apply B to $\begin{pmatrix}x\\y\end{pmatrix}$:

$$B\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-x\\y\end{pmatrix}$$

example 3 rotation by θ



$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

check:

$$\vec{u}\cdot\vec{v} = \begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}\cdot\begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}$$

$$= -\sin\theta\cos\theta + \sin\theta\cos\theta$$

$$= 0 \ \checkmark$$

$\vec{u}\cdot\vec{v} = 0$ orthogonal

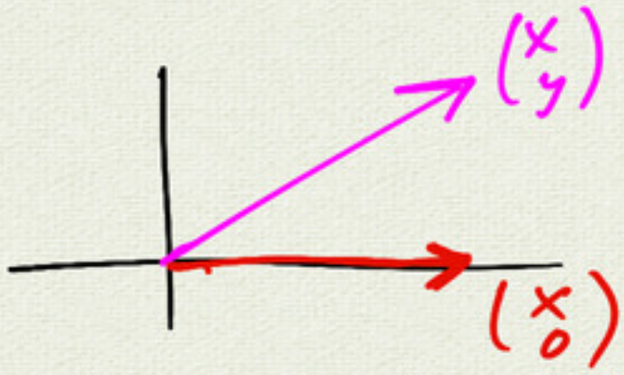
$\Leftrightarrow$ $|\vec{u}| = 0$ or $|\vec{v}| = 0$

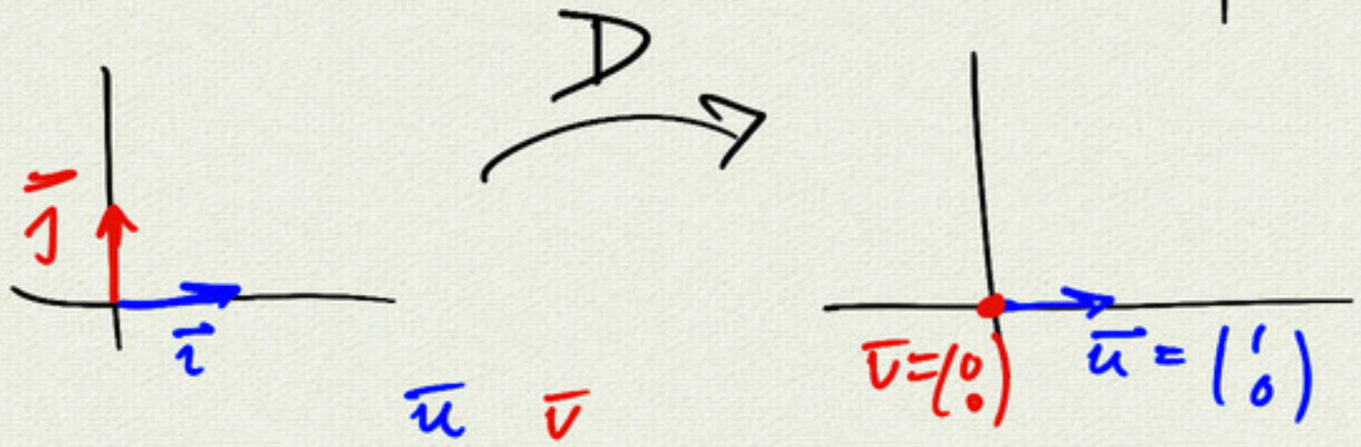
or $\cos\theta = 0$

$\theta = \pi/2$ (perpendicular)

---

example 4 projection onto x-axis





$$D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

apply D to $\begin{pmatrix} x \\ y \end{pmatrix}$:

$$D\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$$

---

matrix $\Longleftrightarrow$ linear transformation

multiplication $\Longleftrightarrow$ apply the transformation