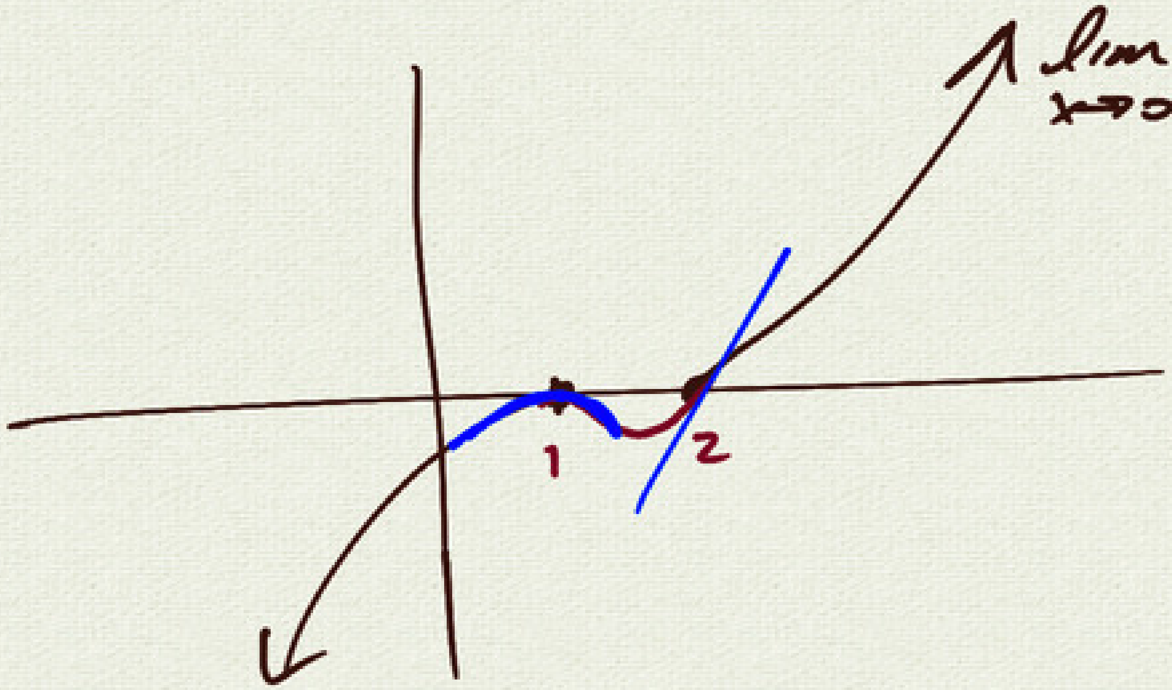
$x = \frac{3}{2}$

$+$ $-$

$$f(x) = \boxed{(x-1)^2}(x-2)$$

multiplicity 2



$\lim_{x \to \infty} f(x) = \infty$

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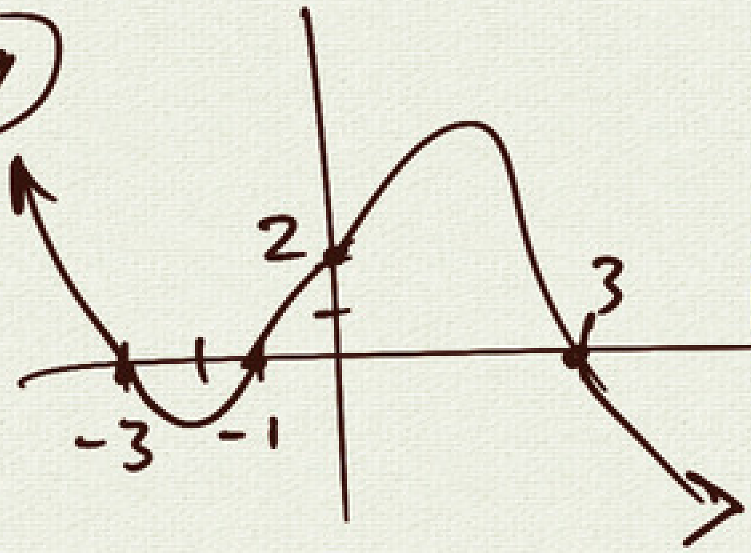


$$\boxed{\begin{array}{l} \text{zeros: } -3,\ -1,\ +3 \\ \updownarrow \\ \text{factors } x+3,\ x+1,\ x-3 \end{array}}$$

| a

| x−a

$$p(x) = a(x+3)(x+1)(x-3)$$
$$= a(x^2-9)(x+1)$$
$$= a(x^3+x^2-9x-9)$$

vertical scale

$+2 = -a9$

$a = -2/9$

$$p(x) = -\frac{2}{9}(x+1)(x-3)(x+3)$$

Factor Thm: $p(x)$ polynomial

$$p(a)=0 \iff x-a \mid p(x)$$

a is a root / zero $\qquad$ $x-a$ is a factor

4.4 More Polynomials

observation:

$$p(x) = (2x-3)(5x-7)$$

$\Rightarrow$ zeros at $\frac{3}{\boxed{2}}, \frac{7}{\boxed{5}}$

$2x-3=0$

$\Rightarrow x = \frac{3}{2}$

$$p(x) = (2x-3)(5x-7)$$
$$= 10x^2 - 29x + 21$$

Rational roots theorem: $\qquad (a_i \in \mathbb{Z})$

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$$

if $\frac{a}{b}$ is a root $\left(p\left(\frac{a}{b}\right)=0\right)$ $\qquad a, b \in \mathbb{Z}$

then $a \mid a_0$ and $b \mid a_n$

example:

$p(x) = 1x^4 - 5x^2 + 4$ factors

find all roots / factor $p(x)$ completely

potential rational roots: $\pm \frac{4,2,1}{\boxed{1}}$

$x^4 + 0x^3 - 5x^2 + 0x + 4$

$p(1) = 1^4 - 5\cdot 1^2 + 4 = 0$ ✓

divide: $x-1$

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -5 & 0 & 4 \\ & & 1 & 1 & -4 & -4 \\ \hline & 1 & 1 & -4 & -4 & \boxed{0} \end{array}$$

$p_2(x) = x^3 + x^2 - 4x - 4$

potential rational roots: $\pm 1, 2, 4$

$p_2(1) = 1 + 1 - 4 - 4 \neq 0$

$p_2(-1) = -1 + 1 + 4 - 4 = 0$ ✓

divide: $x+1$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -4 & -4 \\ & & -1 & 0 & 4 \\ \hline & 1 & 0 & -4 & \boxed{0} \end{array}$$

$x^2 - 4$ $\longrightarrow$ $p(x) = (x-1)(x+1)(x^2-4)$

$$\boxed{p(x) = (x-1)(x+1)(x-2)(x+2)}$$

complete factorization

zeros: $1, -1, 2, -2$

example 2

$p(x) = \boxed{x^5} - 3x^4 - 3x^3 + 9x^2 - 4x + 12$ factors

potential rational roots: $\pm 1, 2, 3, 4, 6, 12$

$p(1) \neq 0$
$p(-1) \neq 0$
$p(2) = 0$ (little bird)

divide by $x-2$:

$$\begin{array}{r|rrrrrr} 2 & 1 & -3 & -3 & 9 & -4 & 12 \\ & & 2 & -2 & -10 & -2 & -12 \\ \hline & 1 & -1 & -5 & -1 & -6 & \boxed{0} \end{array}$$

$p_2(x) = x^4 - x^3 - 5x^2 - x - 6$ | try: $\pm \cancel{1}, 2, 3, 6$

$\Longrightarrow p(x) = (x-2)p_2(x)$

[little bird $\Longrightarrow$ $p_2(-2) = 0$, $p_2(3) = 0$]

$$\begin{array}{r|rrrrr} -2 & 1 & -1 & -5 & -1 & -6 \\ & & -2 & 6 & -2 & 6 \\ \hline & 1 & -3 & 1 & -3 & \boxed{0} \end{array}$$

$x^3 - 3x^2 + x - 3$ $\Longrightarrow p(x) = (x-2)(x+2)(x^3-3x^2+x-3)$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 1 & -3 \\ & & 3 & 0 & 3 \\ \hline & 1 & 0 & 1 & \boxed{0} \end{array}$$

$x^2 + 1$

$$\Longrightarrow p(x) = \underbrace{(x-2)(x+2)(x-3)}_{\text{linear factors}}\underbrace{(x^2+1)}_{\text{irreducible quadratic}}$$

complete factorization over $\mathbb{R}$

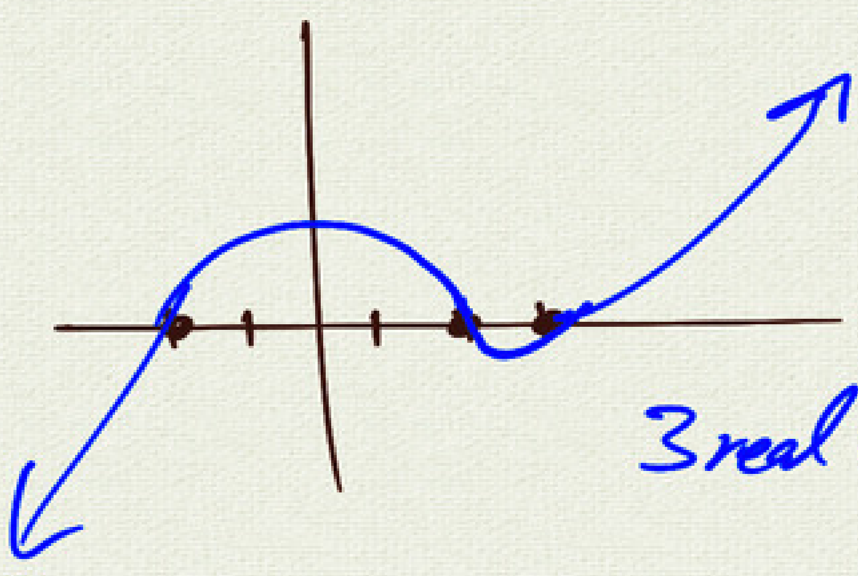
$x^2 + 1 = 0 \Rightarrow x^2 = -1$
$x = \pm i$

quadratic formula:

$$x = \frac{-0 \pm \sqrt{0^2 - 4}}{2}$$

$$= \frac{\pm\sqrt{-4}}{2} = \pm\sqrt{-1}$$

$$= \boxed{\pm i}$$



3 real roots