4.4 More Polynomials

$p(x) = (2x-3)(5x-7)$

$2x-3=0$

$2x=3$

$x=\frac{3}{2}$

roots: $\frac{3}{\boxed{2}}, \frac{7}{\boxed{5}}$

factors

$p(x) = (2x-3)(5x-7)$

$= \boxed{10}x^2 - 29x + 21$

factors

Rational Roots Theorem:

$p(x)$ polynomial $\quad (a_i \in \mathbb{Z})$

$p(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$

Suppose $p\left(\frac{a}{b}\right) = 0$

Then: $a \mid a_0$ and $b \mid a_n$

"a divides a_0"

Factor Theorem

$p(a) = 0 \iff x-a \mid p(x)$

a is a root / zero — "$x-a$ is a factor"

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example 1 $\boxed{p(x) = x^4 - 5x^2 + 4}$ Factor completely / Find all roots

potential rational roots: $\pm\frac{1,2,4}{1}$

$p(1) = 1 - 5 + 4 = 0$ ✓

divide by $x-1$

roots a	factors $x-a$
1	$x-1$

$$\begin{array}{c|ccccc} 1 & 1 & 0 & -5 & 0 & 4 \\ & & 1 & 1 & -4 & -4 \\ \hline & 1 & 1 & -4 & -4 & \boxed{0} \end{array}$$

should be zero

$p_2(x) = x^3 + x^2 - 4x - 4$ potential rational roots: $\pm 1, 2, 4$

$p_2(1) = 1 + 1 - 4 - 4 \neq 0$
$p_2(-1) = -1 + 1 + 4 - 4 = 0$

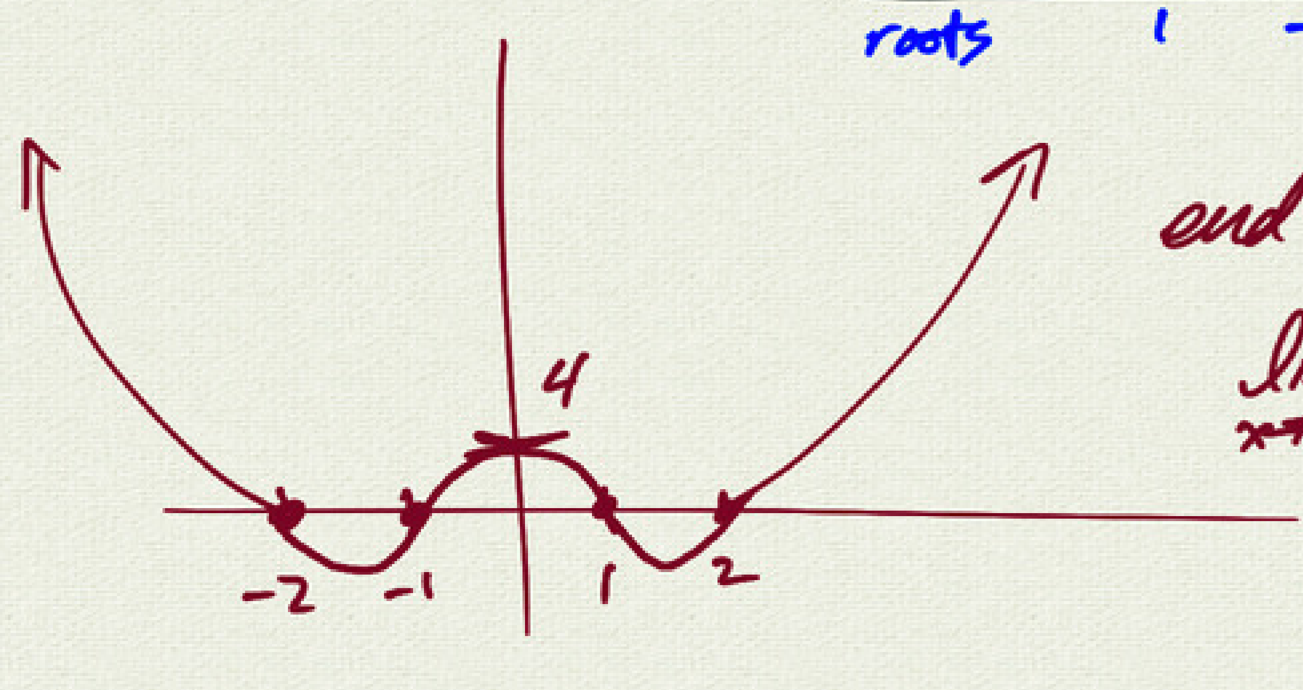
$$\begin{array}{c|cccc} -1 & 1 & 1 & -4 & -4 \\ & & -1 & 0 & 4 \\ \hline & 1 & 0 & -4 & \boxed{0} \end{array}$$

$x^2 - 4$

$\Longrightarrow p(x) = (x-1)(x+1)(x^2-4)$

$\boxed{p(x) = (x-1)(x+1)(x-2)(x+2)}$

roots 1 -1 2 -2



end behavior:

$\lim_{x\to\infty} p(x) = \infty$

$\lim_{x\to-\infty} p(x) = \infty$

example 2

$$p(x) = x^5 - 3x^4 - 3x^3 + 9x^2 - 4x + 12$$

potential rational roots: $\pm 1, 2, 3, 4, 6, 12$

$p(1) \neq 0$
$p(-1) \neq 0$
$p(2) = 0$

$$\begin{array}{r|rrrrrr} 2 & 1 & -3 & -3 & 9 & -4 & 12 \\ & & 2 & -2 & -10 & -2 & -12 \\ \hline & 1 & -1 & -5 & -1 & -6 & \boxed{0} \end{array}$$

$p_2(x) = x^4 - x^3 - 5x^2 - x - 6$ potential roots: $\pm 1, 2, 3, 6$

little bird $\Rightarrow$ $p(-2) = 0$
$p(3) = 0$

$$\begin{array}{r|rrrrr} -2 & 1 & -1 & -5 & -1 & -6 \\ & & -2 & 6 & -2 & 6 \\ \hline & 1 & -3 & 1 & -3 & \boxed{0} \end{array}$$

$x^3 - 3x^2 + x - 3$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 1 & -3 \\ & & 3 & 0 & 3 \\ \hline & 1 & 0 & 1 & \boxed{0} \end{array}$$

$x^2 + 1$

irreducible quadratic

$$\Rightarrow \boxed{p(x) = (x-2)(x+2)(x-3)(x^2+1)}$$

complete factorization over $\mathbb{R}$

$(x+1)(x-1) = x^2 - 1$

$x^2 + 1 = 0$
$x^2 = -1$
$x = \pm i$

quadratic formula:
$$x = \frac{-0 \pm \sqrt{0^2 - 4}}{2}$$
$$= \pm \frac{\sqrt{-4}}{2}$$
$$= \pm i$$

3 real roots