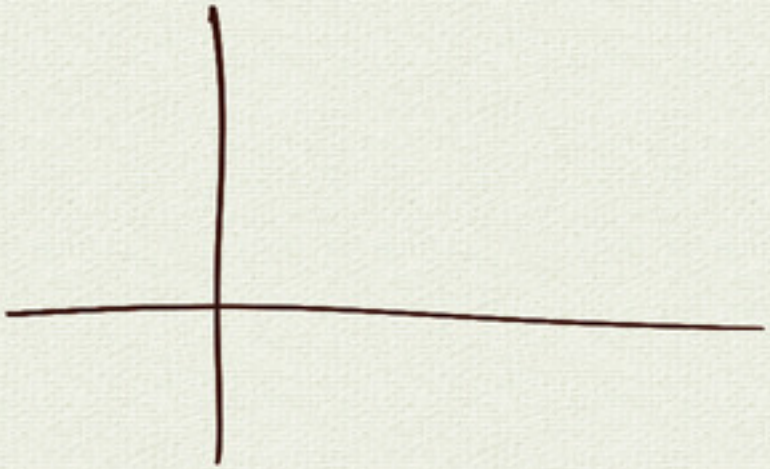
**Bonus:**

(1) Repeat #2 (5 transformations) for $\text{int}(x)$, the greatest integer function

(2) Write a precise definition for what it means for a function to be *increasing* on an interval. (or decreasing)



$$f(x) = \begin{cases} \cos x & x < 0 \\ 0 & x = 0 \\ 1 - x^2 & x > 0 \end{cases}$$

# 4.5 Fundamental Theorem of Algebra

Factor Theorem: $p(x)$ polynomial

$$p(a)=0 \iff x-a \mid p(x)$$

$a$ is a zero / root

$x-a$ is a factor

---

$$p(x)=a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$$

$\deg(p)=n$

$\deg(p)=n \Rightarrow$ at most $n$ linear factors $\Rightarrow$ at most $n$ roots

---

last time: $p(x)=(x-2)(x+2)(x-3)(x^2+1)$

$(x-2)(x+2)(x-3)$: 3 real roots

$(x^2+1)$: irreducible quadratic

factored over $\mathbb{R}$

$x^2+1=0 \Rightarrow x^2=-1$

$x=\pm i$

$p(x)=(x-2)(x+2)(x-3)(x-i)(x+i)$

factored over $\mathbb{C}$ complex numbers

Fundamental Theorem of Algebra:

$\deg(p)=n \rightarrow p$ has $n$ complex roots

# complex numbers

$z = a + bi$

real part — imaginary part

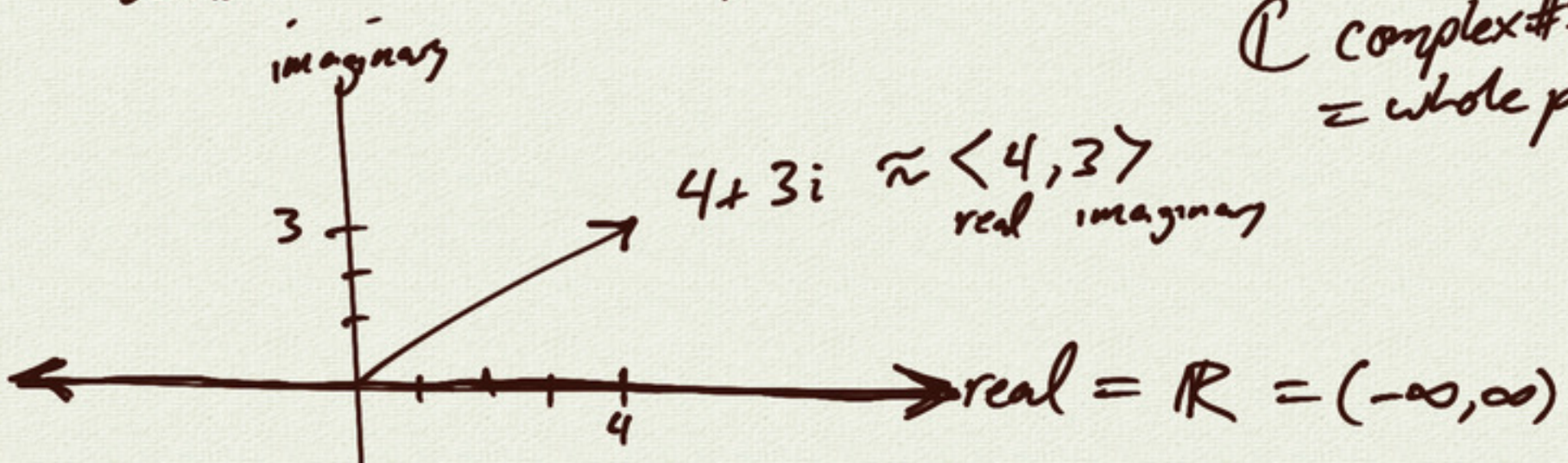
$(3+4i) + (2+5i) = 5+9i$

$2(3+4i) = 6+8i$

] just like vectors

$a+bi \leftrightarrow \langle a, b \rangle$

$\mathbb{C}$ complex #'s = whole plane




multiplication: (FOIL)

$(4+3i)(2+3i) = 8+12i+6i+9i^2$

$9i^2 = -9$

$= -1+18i$

$i = \sqrt{-1}$

$i^2 = -1$

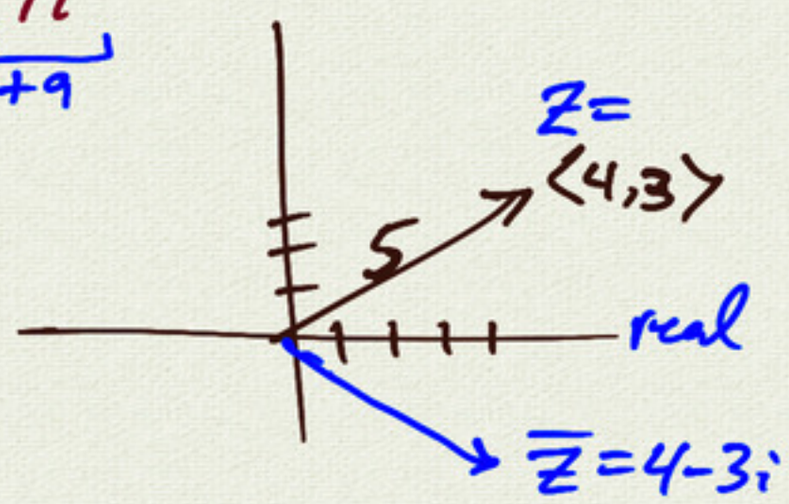
$i^3 = i(i^2) = -i$

$i^4 = 1$

$(4+3i)(4-3i) = 16 - 12i + 12i - 9i^2$

$-9i^2 = +9$

$= 25$

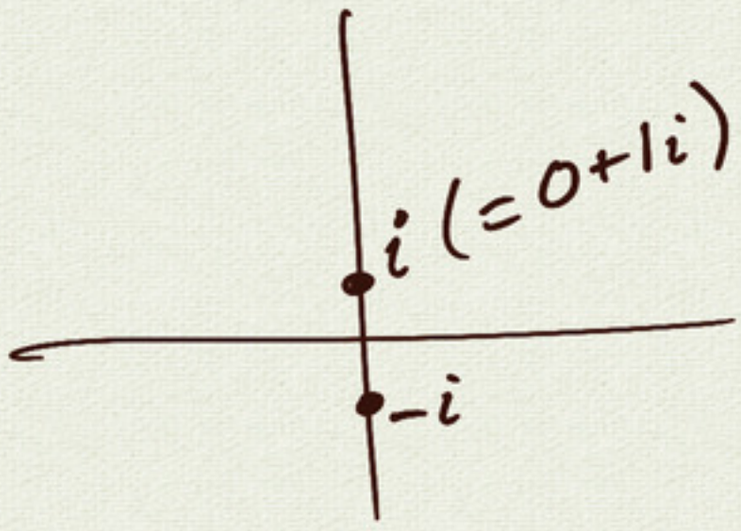



$z = 4+3i$

$\bar{z} = 4-3i$ complex conjugate

$z\bar{z} = |z|^2 \quad (= \sqrt{a^2+b^2})$

$\bar{5} = 5$




$5 = 5+0i$

$\bar{i} = -i$

---

Fundamental Theorem: $p(x)$ polynomial, $\deg(p) = n$

Then $p$ has exactly $n$ complex roots.

And any non-real roots occur in complex conjugate pairs.

**example:**

$p(x) = x^4 + 2x^2 + 8x + 5$

factor completely / find all roots

---

potential roots: $\pm 1, 5$

$p(1) = 0$

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & 2 & 8 & 5 \\ & & -1 & 1 & -3 & -5 \\ \hline & 1 & -1 & 3 & 5 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 3 & 5 \\ & & -1 & 2 & -5 \\ \hline & 1 & -2 & 5 & \boxed{0} \end{array}$$

$\Rightarrow p(x) = (x+1)^2(x^2 - 2x + 5)$

$p(x) = (x+1)^2(x-(1+2i))(x-(1-2i))$

check FOIL

quadratic formula

$x = \frac{2 \pm \sqrt{4-20}}{2}$

$= 1 \pm \sqrt{-4}$

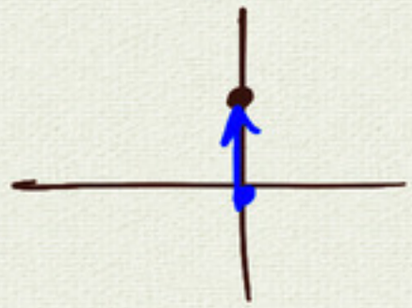
$= 1 \pm 2i$

↖ conjugates

$|i| = i(-i) = -i^2$

$= -(-1)$

$= 1$



little bird: $-1$ is a double root