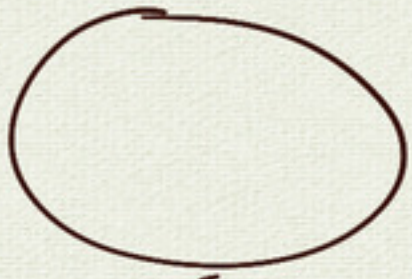
5.2 Ellipses



circle

ellipse

$$x^2+y^2=r^2$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$(x-h)^2+(y-k)^2=r^2$$

↖ center (h,k)

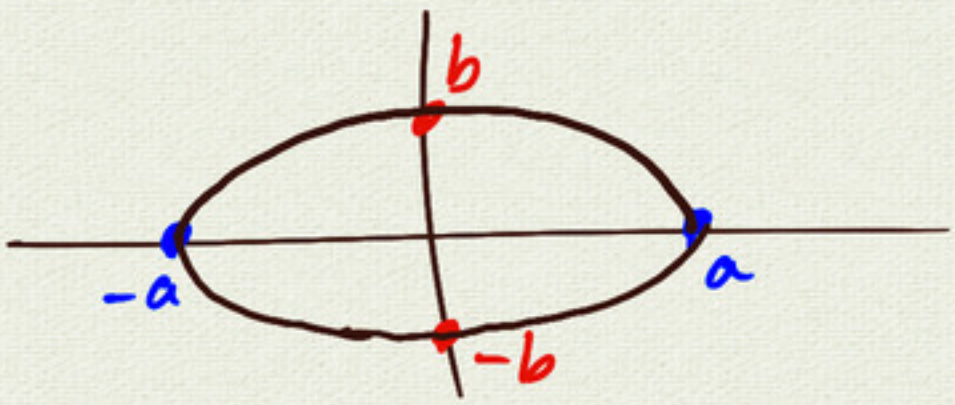
$$\frac{x^2}{a^2}+\left(\frac{y^2}{b^2}\right)=1$$

$y=0 \Rightarrow \frac{x^2}{a^2}=1$

$x^2=a^2$

$x=\pm a$

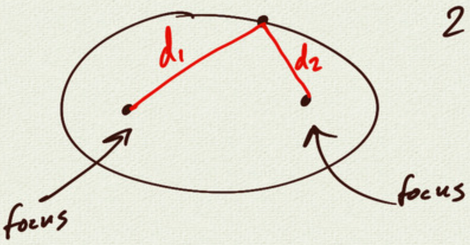
$x=0 \Rightarrow y=\pm b$



center (h,k):

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

geometric definition



2 foci

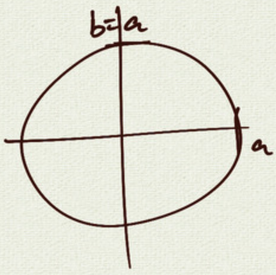
$d_1 + d_2 = \text{const}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

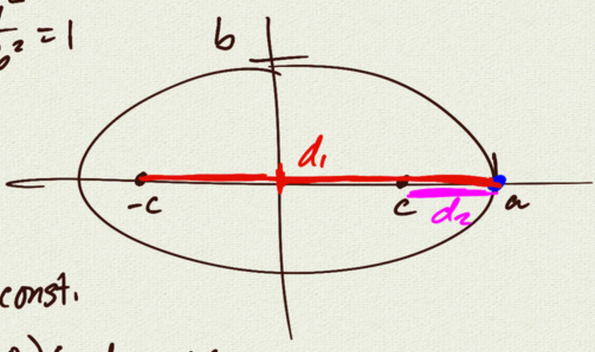
$a = b \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$

$x^2 + y^2 = a^2$

circle



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



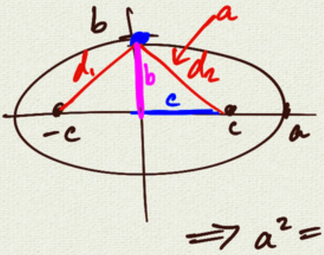
$d_1 + d_2 = \text{const.}$

point $(a, 0)$: $d_1 = a + c$

$d_2 = a - c$

$d_1 + d_2 = (a+c) + (a-c)$

$= 2a \leftarrow$ const.



point $(0, b)$

$d_1 = d_2$

$d_1 + d_2 = 2a$

$\Big] \; d_1 = a = d_2$

$\Rightarrow a^2 = b^2 + c^2$

$c^2 = a^2 - b^2$

---

example: $\frac{x^2}{9} + \frac{y^2}{25} = 1$

foci on y-axis

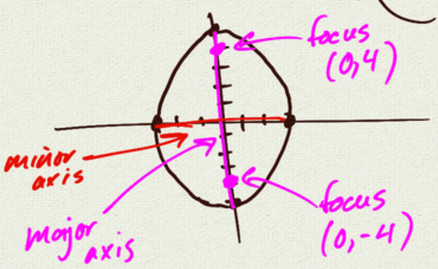
$c^2 = a^2 - b^2$

↑ bigger

$c^2 = 25 - 9$

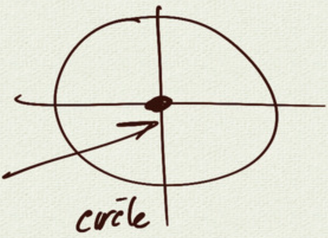
$= 16$

$c = 4$



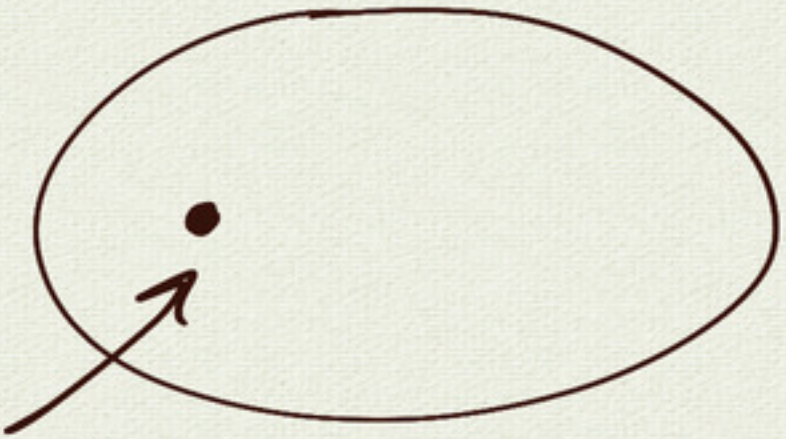
$a = b \Rightarrow a^2 - b^2 = 0$

$c = 0$



eccentricity $e = \frac{c}{a}$

planets travel in ellipses



sun = focus

reflection