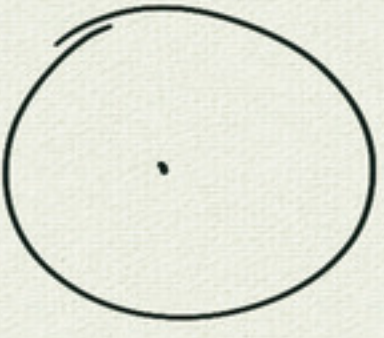
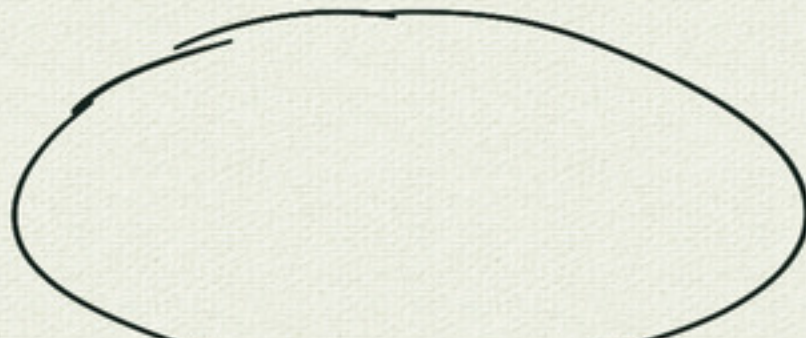


# 5.2 Ellipses



circle

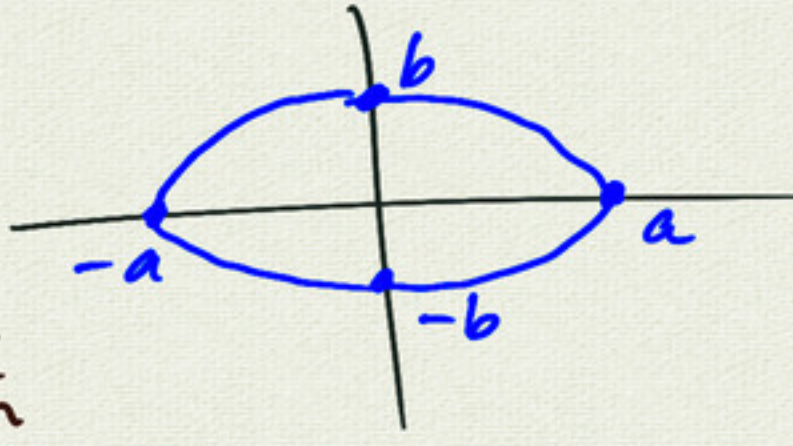
$(x-h)^2 + (y-k)^2 = r^2$   
 center  $(h, k)$   
 radius  $r$   
 $x^2 + y^2 = r^2$   
 (at origin)



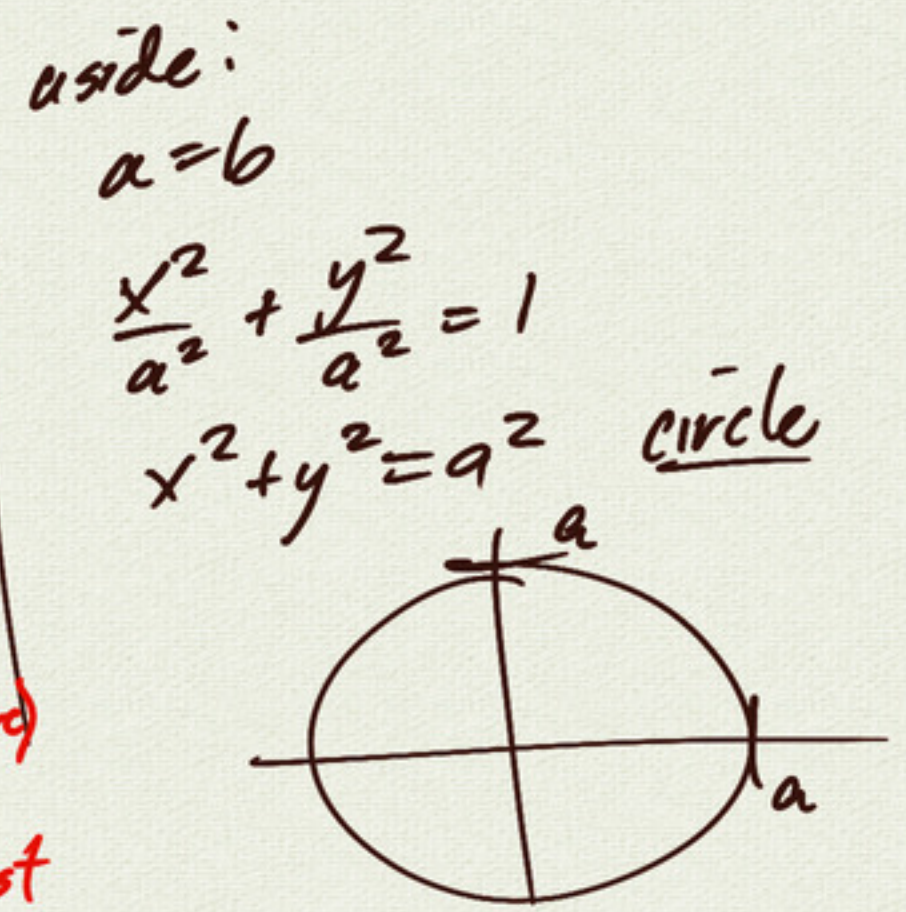
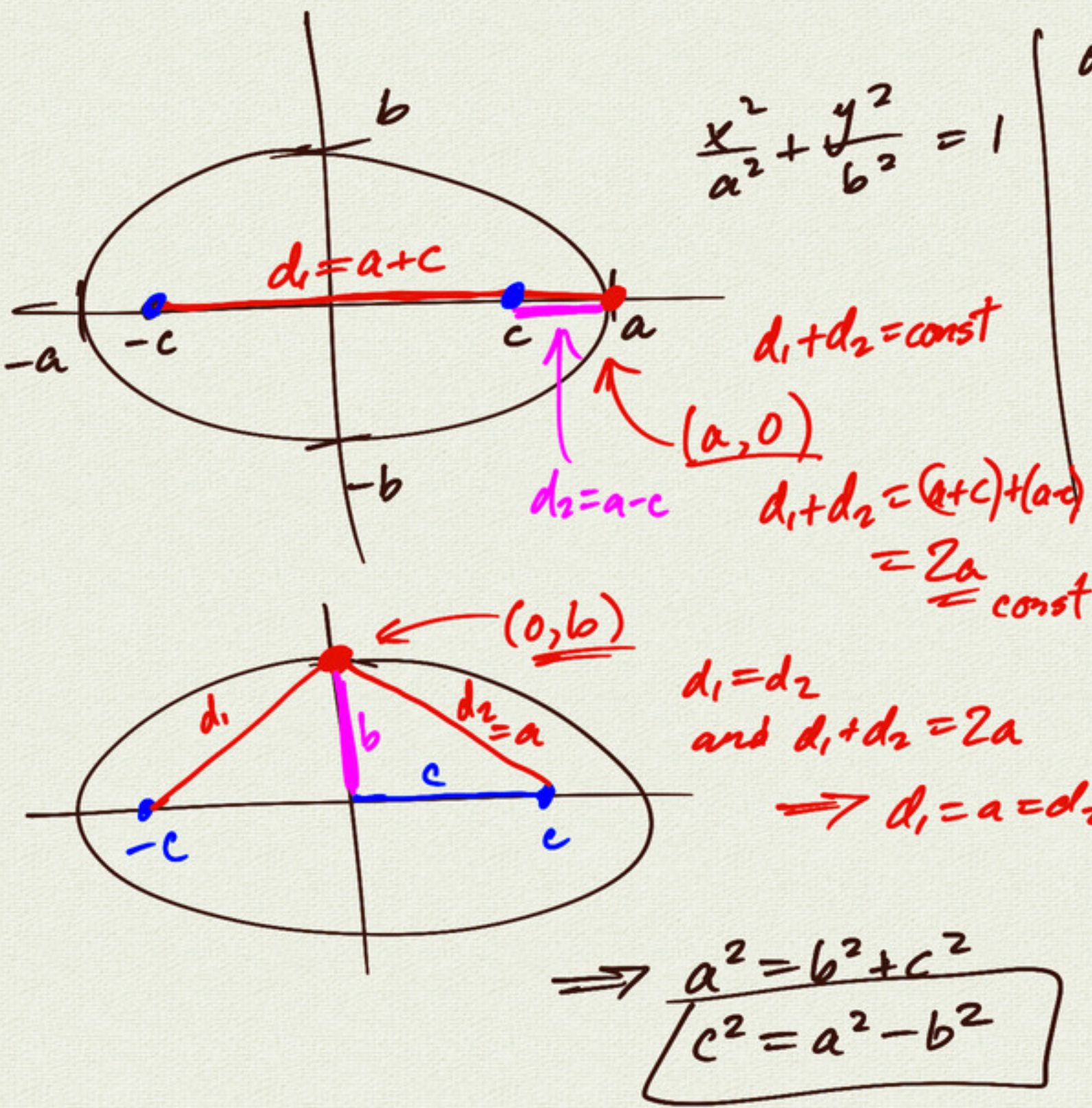
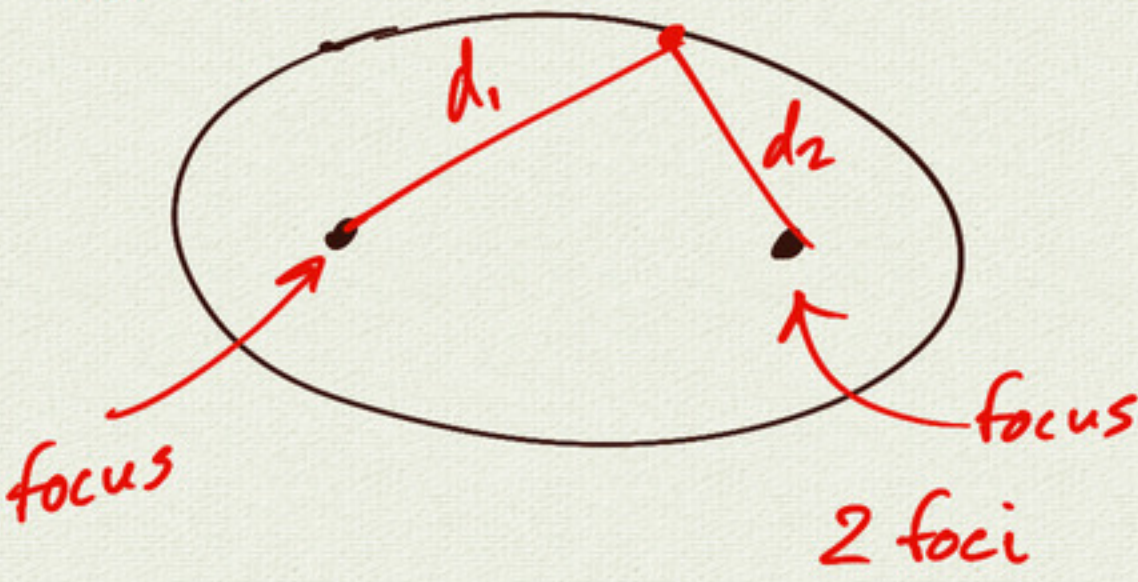
ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\rightarrow y=0: \frac{x^2}{a^2} = 1$   
 $x^2 = a^2$   
 $x = \pm a$   
 $x=0 \rightarrow y = \pm b$



geometric definition  
 $d_1 + d_2 = \text{const}$

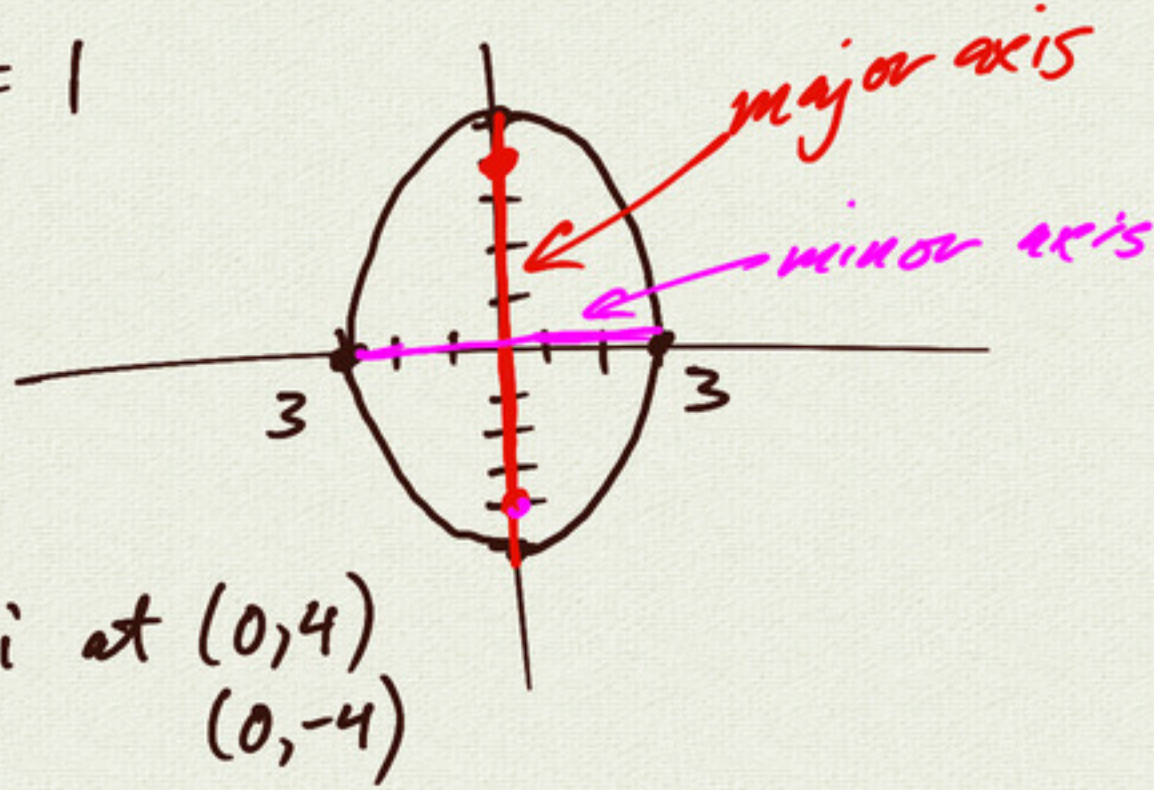


example:

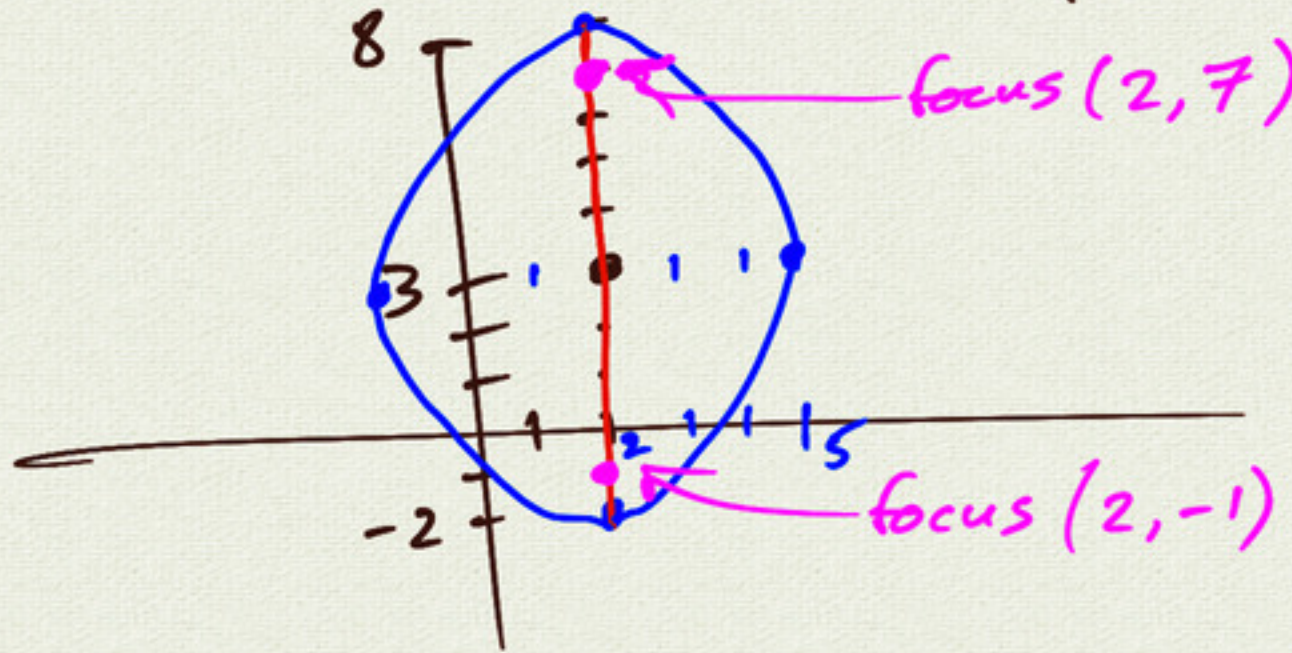
$\frac{x^2}{9} + \frac{y^2}{25} = 1$

$c^2 = a^2 - b^2$   
 $= 25 - 9$   
 $= 16$

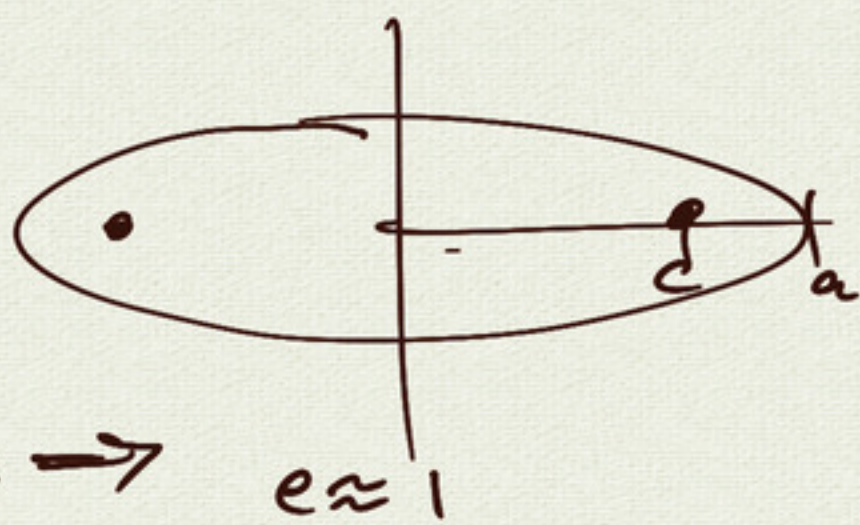
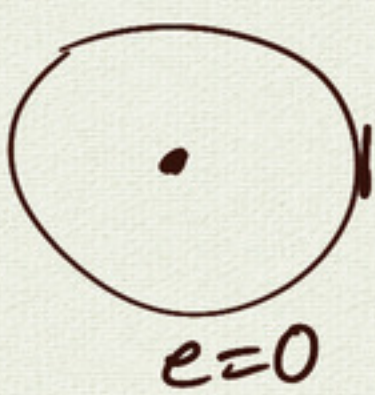
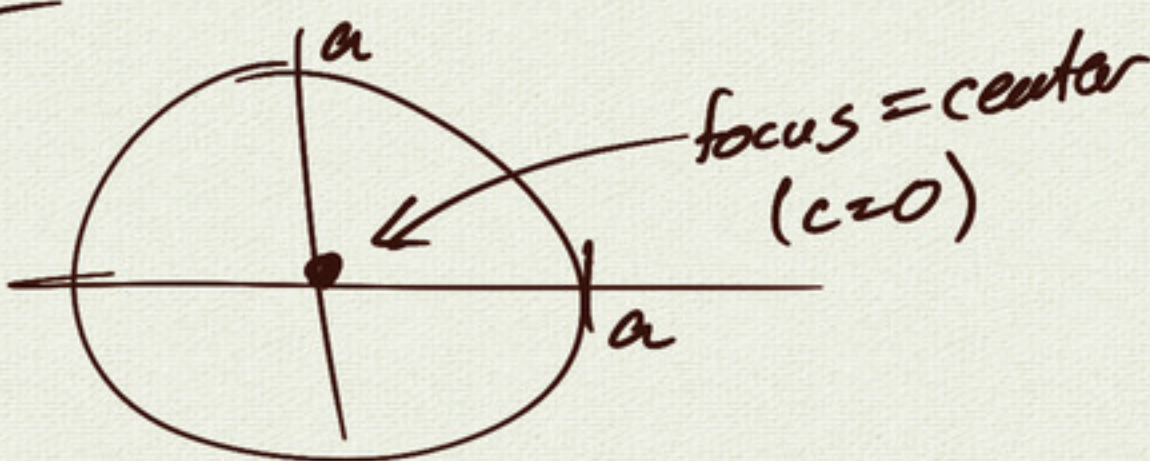
$c=4 \Rightarrow$  foci at  $(0, 4)$   
 $(0, -4)$



translate to center  $(2, 3)$ :  $\frac{(x-2)^2}{9} + \frac{(y-3)^2}{25} = 1$



$a=b \Rightarrow c^2 = a^2 - b^2 = 0$

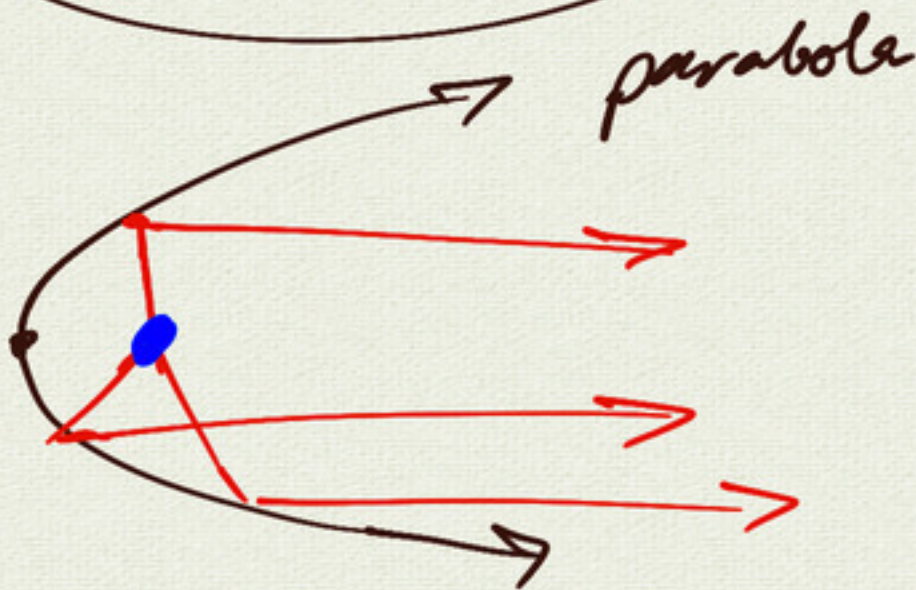
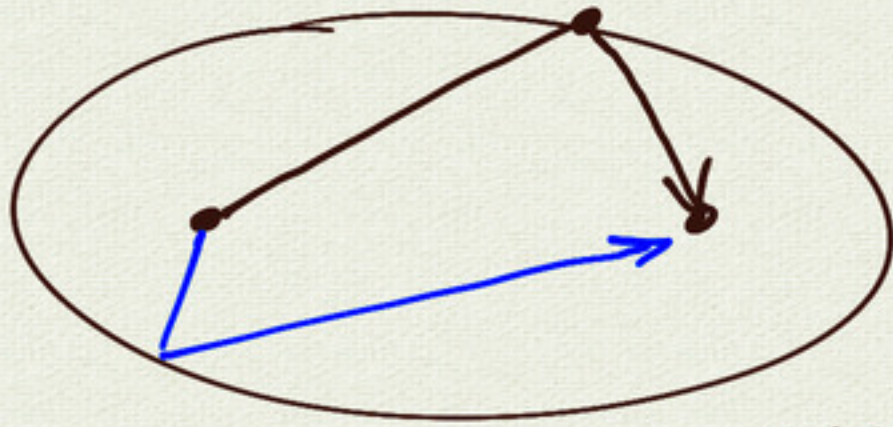


more eccentric  $\rightarrow e \approx 1$

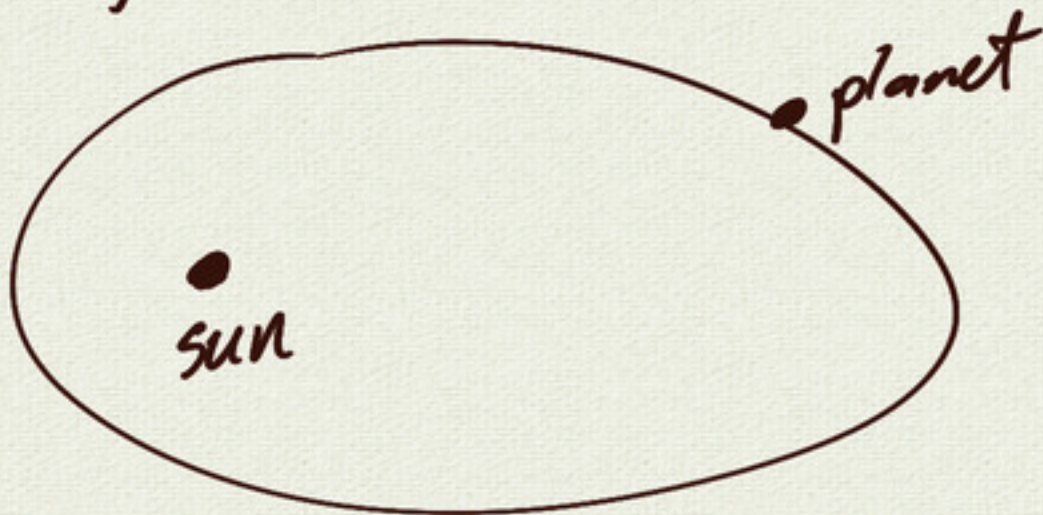
eccentricity  $e = \frac{c}{a}$



reflections



planetary motion



Kepler:  
planets travel  
in elliptical  
orbits



$$h(x) = \frac{x-1}{x^2-x}$$

$$= \frac{x-1}{x(x-1)} = \begin{cases} \frac{1}{x} & \text{if } x \neq 1 \\ \text{undef} & \text{if } x = 1 \end{cases}$$

