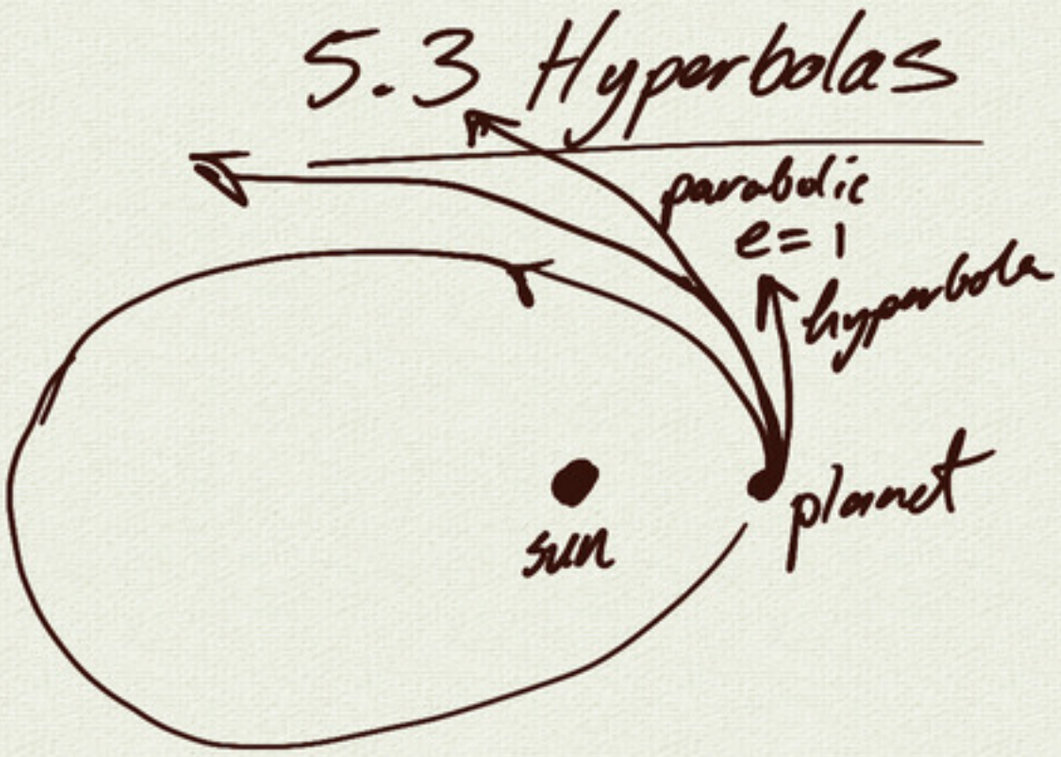
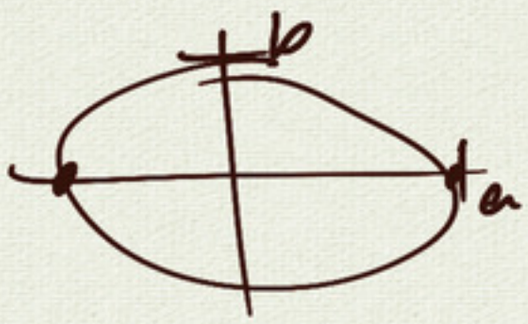
5.3 Hyperbolas



ellipse:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



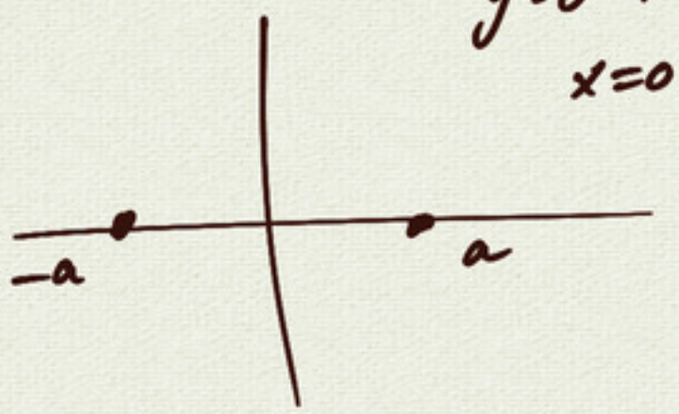
hyperbola:

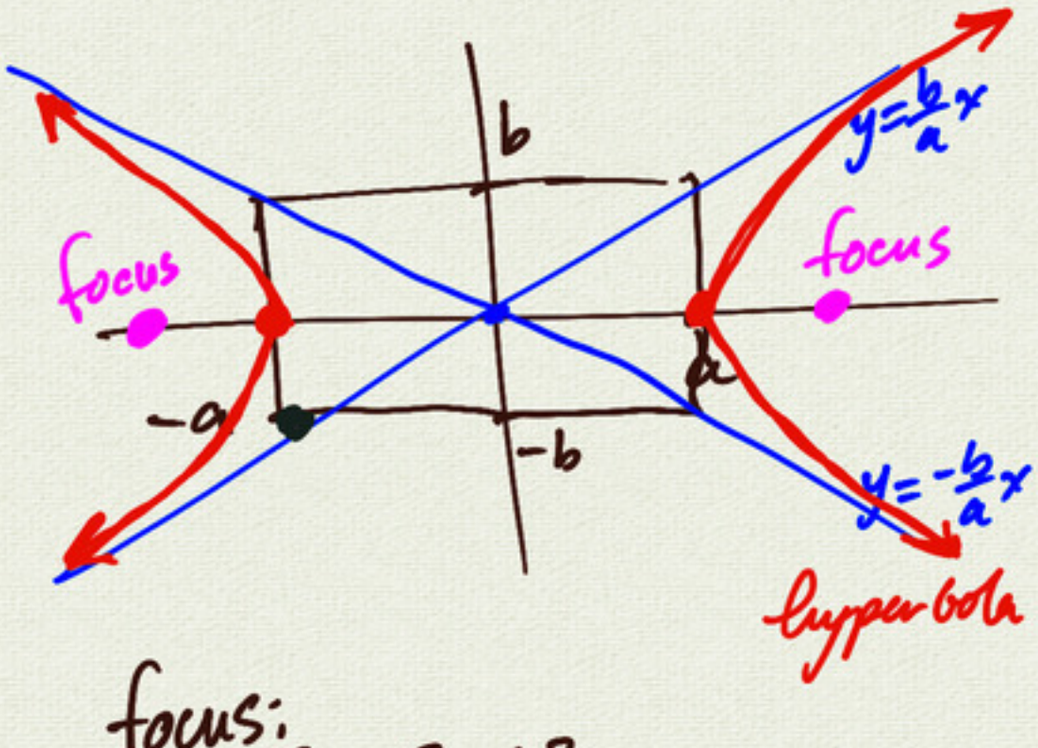
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$y=0 \Rightarrow x=\pm a$

$x=0 \Rightarrow -\frac{y^2}{b^2}=1$

$y^2=-b^2$ ✗





$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{x^2}{a^2}=\frac{y^2}{b^2}+1$$

if x, y really big

$$\frac{x^2}{a^2}\approx\frac{y^2}{b^2}$$

$$y^2\approx\left(\frac{b}{a}\right)^2x^2$$

$$\boxed{y\approx\pm\frac{b}{a}x}$$

focus: $c^2=a^2+b^2$

Example:

$$-9x^2-18x+16y^2-64y-89=0$$

ellipse:
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

hyperbola:
$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

$$-9(x^2+2x+1)+16(y^2-4y+4)=89-9+64$$

$$-9(x+1)^2+16(y-2)^2=144$$

$9\cdot 16=144$

$$-\frac{(x+1)^2}{16}+\frac{(y-2)^2}{9}=1$$

$$\boxed{\frac{(y-2)^2}{9}-\frac{(x+1)^2}{16}=1}$$

center $(-1,2)$

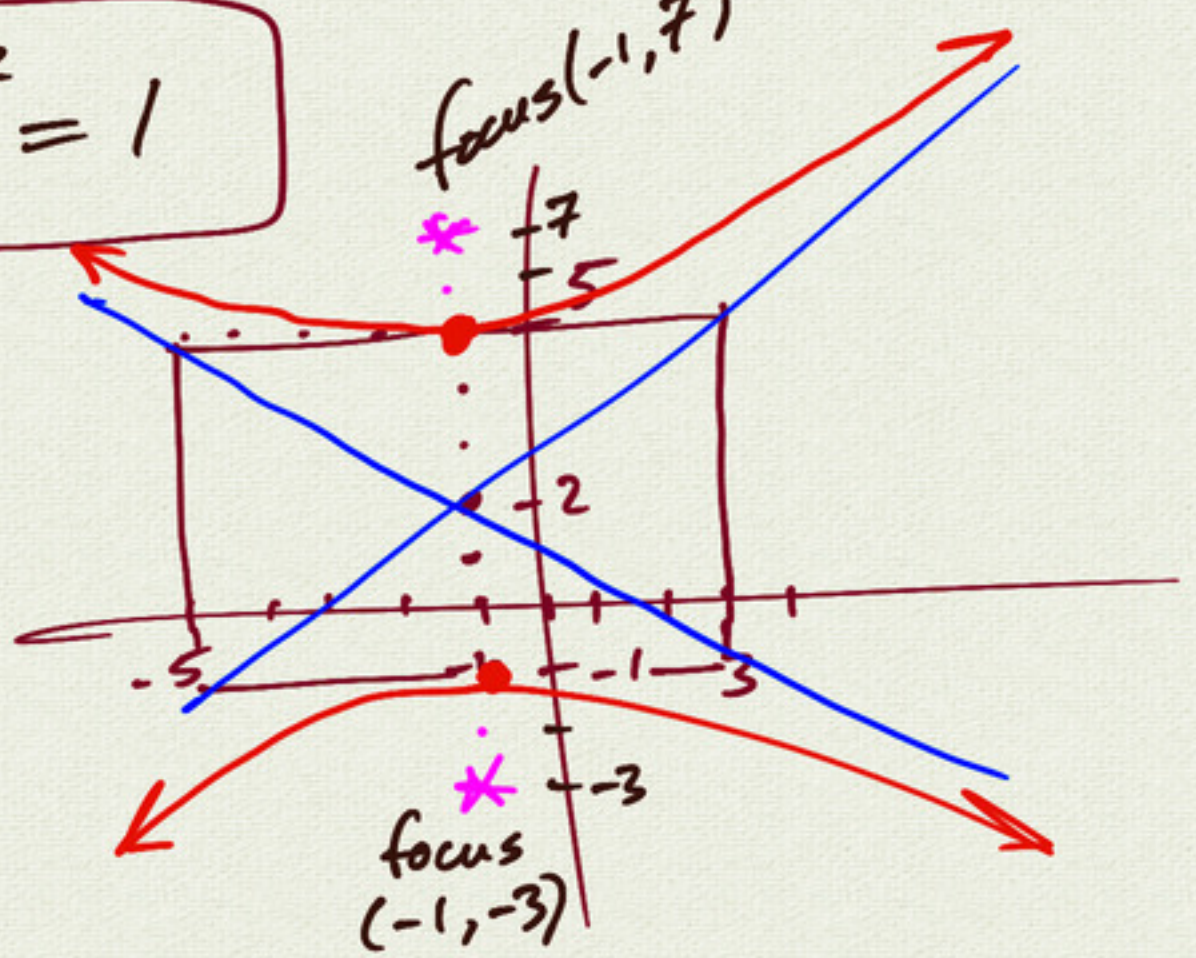
$c^2=a^2+b^2$

$=9+16$

$=25$

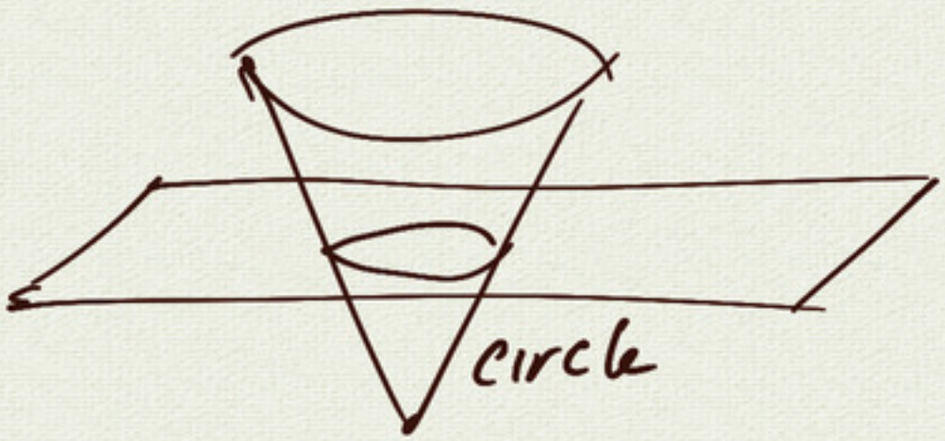
$c=5$

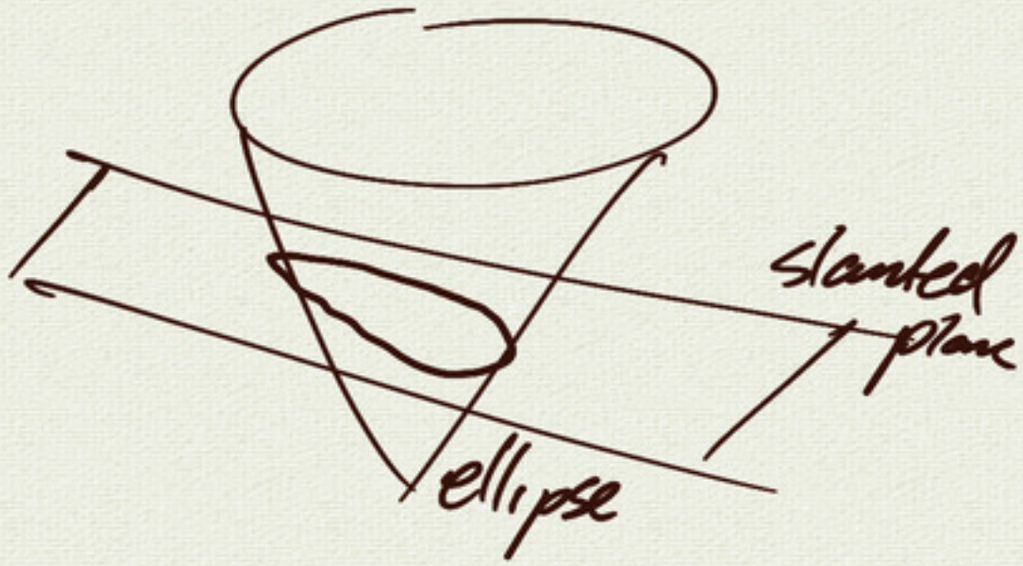
$e=\frac{c}{a}=\frac{5}{3}$

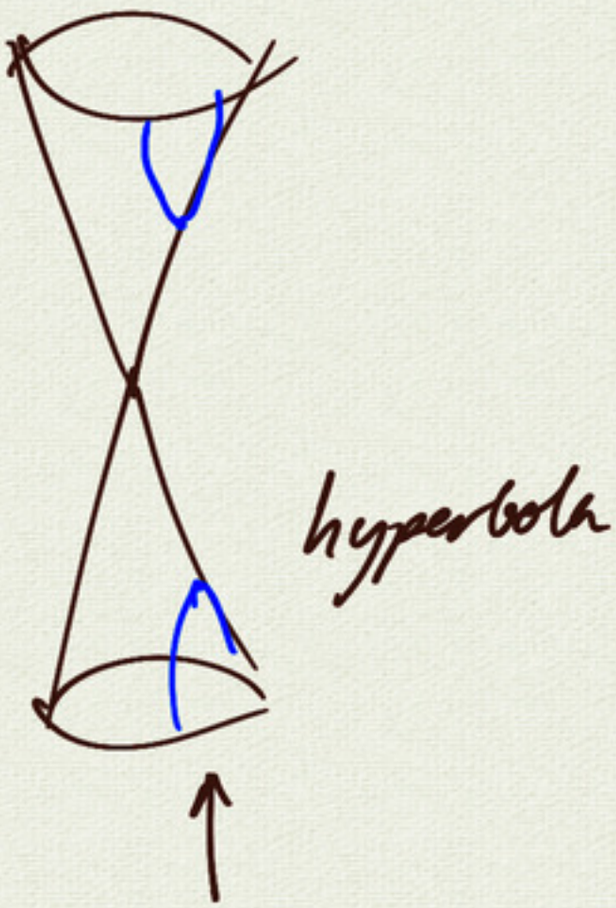


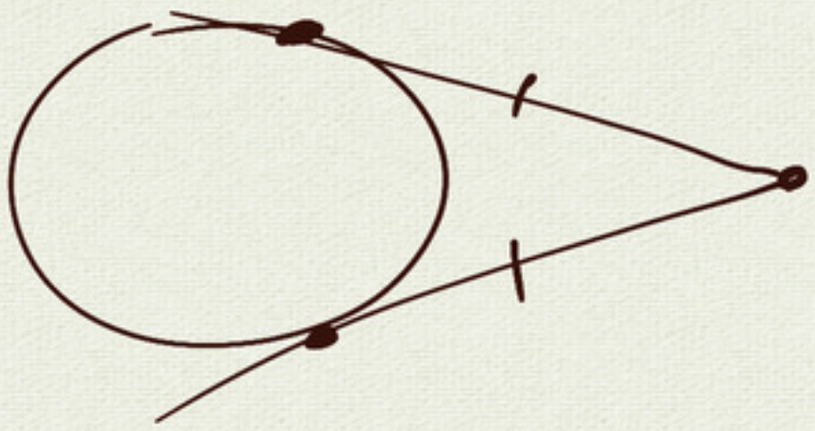
asymptotes: $y-2=\pm\frac{3}{4}(x+1)$

conic sections









Sphere in 3D