6.1 Combinatorics ← counting

discrete math vs. continuous

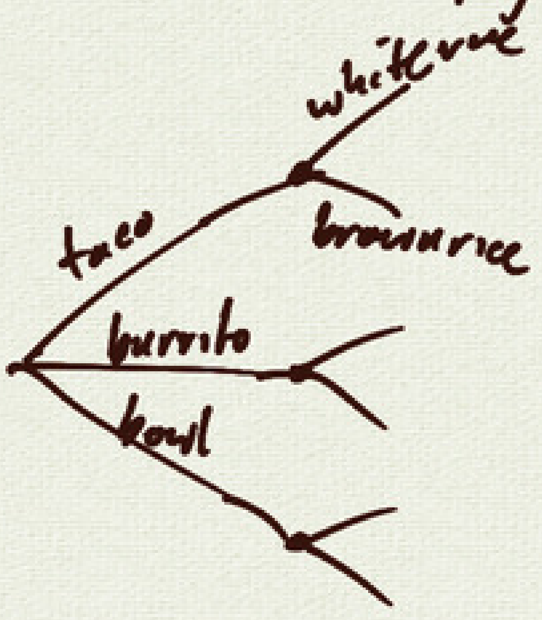
example: Chipotle

$$\begin{pmatrix}\text{taco}\\\text{burrito}\\\text{bowl}\end{pmatrix}\begin{pmatrix}\text{brown rice}\\\text{white rice}\end{pmatrix}\begin{pmatrix}\text{beef}\\\text{chicken}\\\text{tofu}\end{pmatrix}\begin{pmatrix}\text{mild}\\\text{medium}\\\text{hot}\end{pmatrix}\begin{pmatrix}\text{cheese}\\\text{no cheese}\end{pmatrix}\begin{pmatrix}\text{lettuce}\\\text{no lettuce}\end{pmatrix}$$

$$3 \cdot 2 \cdot 3 \cdot 3 \cdot 2 \cdot 2 = 3^3 \cdot 2^3 = 216$$

independent choices: multiply

visualize:



independent choice = branch

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pizza: 10 possible toppings

how many pizzas (different)?

$\begin{pmatrix}\text{cheese}\\\text{no}\end{pmatrix}\begin{pmatrix}\text{tomato}\\\text{no}\end{pmatrix}$ _ _ _ _ _ _ _ _

$$\underbrace{2 \cdot 2 \cdot \dots \cdot 2}_{10 \text{ choices}} = 2^{10} = 1024$$

0/1 0/1 0/1 _ _ _ _ _ _ 0/1

binary sequence:

00000 00000	nothing
11111 11111	supreme
11000 00000	cheese + tomato

pizzas w/ 10 toppings ⟷ # binary sequences length 10 (2^{10})

set $\{A, B, C, D\}$

set of toppings {cheese, tomato, ... } (10 items)

how many subsets?

subsets of a set of size 10 $= 2^{10}$

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pizzas with n toppings $= 2^n$ = # of binary sequences of length n

= # of subsets of a set size n

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5 toppings ⟶ 2^5 pizzas $= 32$ (= # subsets of a set w/ 5 elements)

(= # binary sequences length 5)

3 symbols: A, B, C

$\underline{3} \cdot \underline{2} \cdot \underline{1}$ = 6 possible orderings (permutations)

ABC BAC CAB
ACB BCA CBA

4 symbols: $4 \cdot 3 \cdot 2 \cdot 1 = 4!$ "4 factorial"

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$6! = 6(5!) = 720$

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example: class of 12 students
pick P, VP, Treas.
how many ways to do this?

$\underline{12} \cdot \underline{11} \cdot \underline{10} = 1320$

notation:

${}_{12}P_3$ (order matters)

#items #picked

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formula: $${}_nP_r = \underbrace{\underline{n}\,\underline{(n-1)}\,\underline{(n-2)} \cdots \underline{(n-r+1)}}_{r} = \frac{n(n-1)(n-2)\cdots(n-r+1)(n-r)\cdots(1)}{(n-r)\cdots(1)}$$

$$\boxed{{}_nP_r = \frac{n!}{(n-r)!}}$$

don't use this

Combinations (order doesn't matter)

example: $\{A, B, C, D\}$ 4 symbols

how many ways to choose 2 symbols from the 4?

AB	BC	CD
AC	BD	
AD		

6 total

2 spots: $\dfrac{\underline{4} \cdot \underline{3}}{2!} = 6$

$2!$ ← # of ways to reorder the spots

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example: # ways to pick 3 items from 6

$$\frac{\underline{6} \cdot \underline{5} \cdot \underline{4}}{3!} = 20$$

3 spots (pick 3 items)

# of ways to reorder 3 spots

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example: # of ways to pick 4 items from 8 total

$$= \frac{\underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5}}{4!}$$

← 4 spots

$$= \frac{8 \cdot 7 \cdot \cancel{6}^{2} \cdot 5}{4 \cdot \cancel{3} \cdot 2 \cdot 1} = 70$$

notation: ${}_8C_4 = \binom{8}{4}$ "8 choose 4"

${}_nC_r = \binom{n}{r}$ "n choose r"

= # ways to pick r items from n total

practice:

$\binom{3}{1} = \frac{3}{}$

${}_3C_1$

$\binom{3}{2} = \frac{3 \cdot 2}{2} = 3$

formula:

${}_nC_r = \frac{n(n-1)\dots(n-r+1)}{r!} \cdot \frac{(n-r)\dots(1)}{(n-r)\dots(1)}$

$\boxed{{}_nC_r = \frac{n!}{r!(n-r)!}}$ don't use!

$\binom{3}{0} = 1$

$\binom{3}{3} = 1$

$\Longrightarrow 0! = 1$

practice:

$\binom{4}{2} = \frac{4 \cdot 3 \cdot}{2!} = 6$

$\binom{4}{1} = 4 = \binom{4}{3}$

$\binom{4}{0} = 1 = \binom{4}{4}$

$\binom{n}{1} = n = \binom{n}{n-1}$