6.2 Binomial Theorem

$$\binom{8}{2} = \frac{8\cdot 7}{2} = 28$$

Pascal's Triangle

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

$$(x+y)^2 = 1x^2 + 2xy + 1y^2$$

$$\begin{aligned}
(x+y)^3 &= (x+y)(x^2+2xy+y^2) \\
&= x^3 + 2x^2y + xy^2 \\
&\qquad\quad x^2y + 2xy^2 + y^3 \\
&= x^3 + 3x^2y + 3xy^2 + y^3
\end{aligned}$$

$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

$$\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1 \\
\end{array}
\qquad
\begin{array}{ccccccc}
 & & & \binom{0}{0} & & & \\
 & & \binom{1}{0} & & \binom{1}{1} & & \\
 & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\
\binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3}
\end{array}$$

$$1 \quad 4 \quad 6 \quad 4 \quad 1$$

$$\binom{2}{1}$$

$$x^2 + 2xy + y^2$$

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \dots + \boxed{\binom{n}{k}x^{n-k}y^k} + \dots + \binom{n}{n}x^0y^n$$

WTF? why this formula?

Binomial Theorem

example:

1
1 1
1 2 1
1 3 3 1
$\boxed{1\ 4\ 6\ 4\ 1}$

expand $(2a-b)^4$

$$= (2a)^4(-b)^0 + 4(2a)^3(-b)^1 + 6(2a)^2(-b)^2 + 4(2a)^1(-b)^3 + 1(2a)^0(-b)^4$$

$$= 16a^4 - 32a^3b + 24a^2b^2 - 8ab^3 + b^4$$

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example:

find the a^4 term in $(2a-b)^6$

a^4b^2

$$\binom{6}{4} \to \binom{6}{2}(2a)^4(-b)^2$$

$$= 15 \cdot 16\, a^4 b^2$$

$$= 240a^4b^2$$

$\binom{6}{4} = \binom{6}{2}$

$\binom{6}{2} = \frac{6 \cdot 5}{2} = 15$

$$(x+y)^6 = (x+y)(x+y)(x+y)(x+y)(x+y)(x+y)$$

$$= x^6 + \binom{6}{1}x^5y^1 + \binom{6}{2}x^4y^2 + \ldots$$

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

$$
\begin{array}{ccccccc}
 & & & \binom{0}{0} & & & \\
 & & \binom{1}{0} & & \binom{1}{1} & & \\
 & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\
\binom{3}{0} & & \binom{3}{1} & & \boxed{\binom{3}{2}} & & \binom{3}{3}
\end{array}
$$

why does this work?

$$
\begin{array}{ccc}
\binom{n-1}{k-1} & & \binom{n-1}{k} \\
 & \boxed{\binom{n}{k}} &
\end{array}
$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

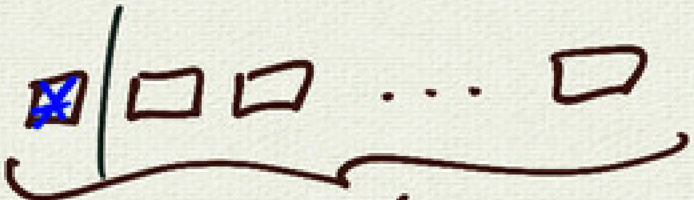
ways to choose k things from n items

ways to choose k-1 from n-1

choose marked

ways choose k from n-1

don't choose marked



n items

$\binom{n}{k}$ binomial coefficient $\left(= {}_nC_k \right)$