6.3 Probability

coin flipping: $P(H) = \frac{1}{2}$ $P(T) = \frac{1}{2}$ $\{H, T\}$ outcomes

probability

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flip 2 coins: $P(HH) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

independent events → multiply

$P(HT) = \frac{1}{4}$

$P(TH) = \frac{1}{4}$

$P(\text{exactly one head}) = \frac{1}{2}$

outcomes:

$\{HH, HT, TH, TT\}$

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

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flip 10 coins: $P(\text{all heads}) = \frac{1}{2^{10}} = \underbrace{\left(\frac{1}{2}\right) \cdots \left(\frac{1}{2}\right)}_{10}$ 10 independent choices

$= \frac{1}{1024}$ ← one good option

← 2^{10} possible outcomes

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deck of cards: 4 suits: clubs, spades, hearts, diamonds

13 values: A, 2–10, J, Q, K

1 random card from full deck:

$P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$

$P(10) = \frac{4}{52} = \frac{1}{13}$

dice 1 die $\{1,2,3,4,5,6\}$

roll 2 dice, add

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(7) = \frac{6}{36} = \frac{1}{6}$$

$$P(<5) = \frac{6}{36} = \frac{1}{6}$$

← # good options

← total # outcomes

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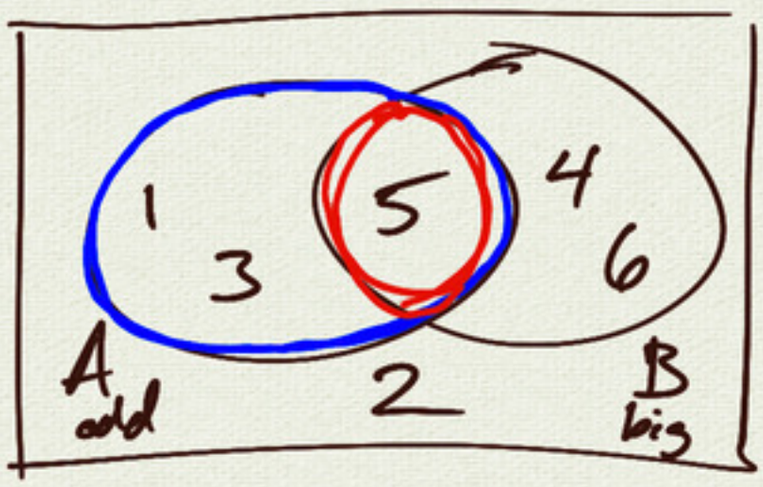
event = subset of outcomes

roll 1 die

A = odd #s = $\{1,3,5\}$ $\quad P(A) = \frac{3}{6} = \frac{1}{2}$

B = big #s = $\{4,5,6\}$ $\quad P(B) = \frac{1}{2}$



$P(A \cap B) = \frac{1}{6}$

↖ A and B

$P(A \cup B) = \frac{5}{6}$

↖ A or B

A, B are independent if $\boxed{P(A \cap B) = P(A) \cdot P(B)}$

for our example A = odd, B = big, are A, B independent?

$$P(A \cap B) = \frac{1}{6} \stackrel{?}{=} P(A) \cdot P(B) = \underbrace{\tfrac{1}{2} \cdot \tfrac{1}{2}}_{1/4}$$

NO: A, B are not independent

conditional probability:

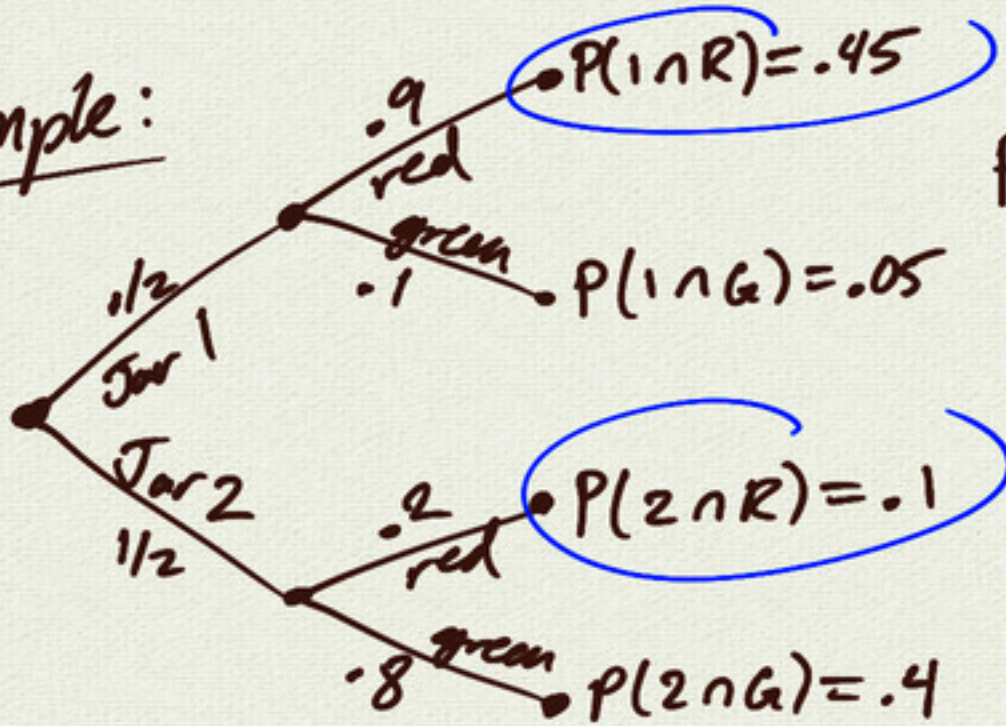
$$\boxed{P(B|A) = \frac{P(A \cap B)}{P(A)}}$$

probability of B given A (assume A is true)

our example:

$$P(B|A) = \frac{(1/6)}{(1/2)} = \frac{1}{3}$$

example:



$P(1|R) = ?$

↖ I know it's red

$P(R) = .45 + .1$
$= .55$

$P(1|R) = \frac{P(1 \cap R)}{P(R)}$

$= \frac{.45}{.55} = \frac{9}{11}$