6.4 More Probability

example: Kaila's free throws

success probability: $p = .9$

10 free throws

What is $P(8 \text{ success}_{\text{exactly}})$?

1 1 1 1 1 1 1 1 0 0 ⟸ single sequence

$(.9)(.9)(.9)\ldots(.9)(.1)(.1)$ $\quad (.9)^8(.1)^2$

0 0 1 1 1 1 1 1 1 1 ⟸ same probability $(.9)^8(.1)^2$

possible sequences with 8 success, 2 failure

$= \binom{10}{2} = \binom{10}{8}$

$\Rightarrow$ $P(\text{exactly 8 successes}) = \binom{10}{2}(.9)^8(.1)^2$

$P(\text{exactly 7 successes}) = \binom{10}{3}(.9)^7(.1)^3$

"binomial distribution"

binomial theorem: $1 = (.9 + .1)^{10} =$

$(.9)^{10} + \binom{10}{1}(.9)^9(.1)^1 + \binom{10}{2}(.9)^8(.1)^2$

$P(10_{\text{success}}) + P(9 \text{ succ}) + P(8 \text{ succ}) \ldots$

example: M&Ms in a bag: 20 red
10 green

pull 8 from the bag

$P(5 \text{ red} + 3 \text{ green}) = ?$

$$\frac{\binom{20}{5}\binom{10}{3}}{\binom{30}{8}}$$

$\binom{20}{5}$: #ways to pick 5 red from 20

$\binom{10}{3}$: #ways to pick 3 green from 10

$\binom{30}{8}$: # ways to choose 8 from 30

$$P(8 \text{ red}) = \frac{\binom{20}{8}\binom{10}{0}}{\binom{30}{8}}$$

$$P(\text{at least 6 red}) = P(6_{\text{red}}) + P(7_{\text{red}}) + P(8 \text{ red})$$

$$= \frac{\binom{20}{6}\binom{10}{2} + \binom{20}{7}\binom{10}{1} + \binom{20}{8}\binom{10}{0}}{\binom{30}{8}}$$

$$P(\text{at least 1 red}) = 1 - P(0 \text{ reds})$$

$$= 1 - \frac{\binom{20}{0}\binom{10}{8}}{\binom{30}{8}}$$

"hypergeometric distribution"