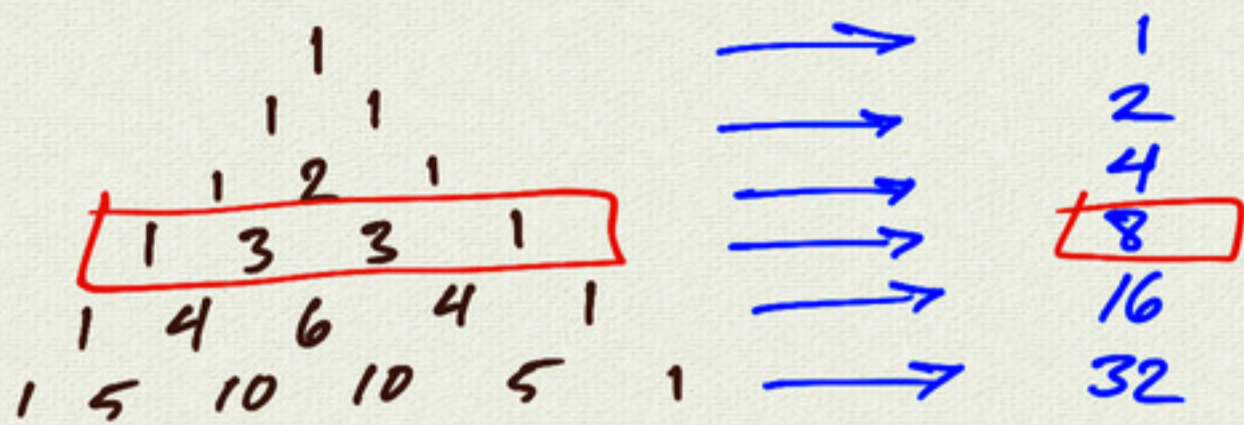


6.5 Sequences

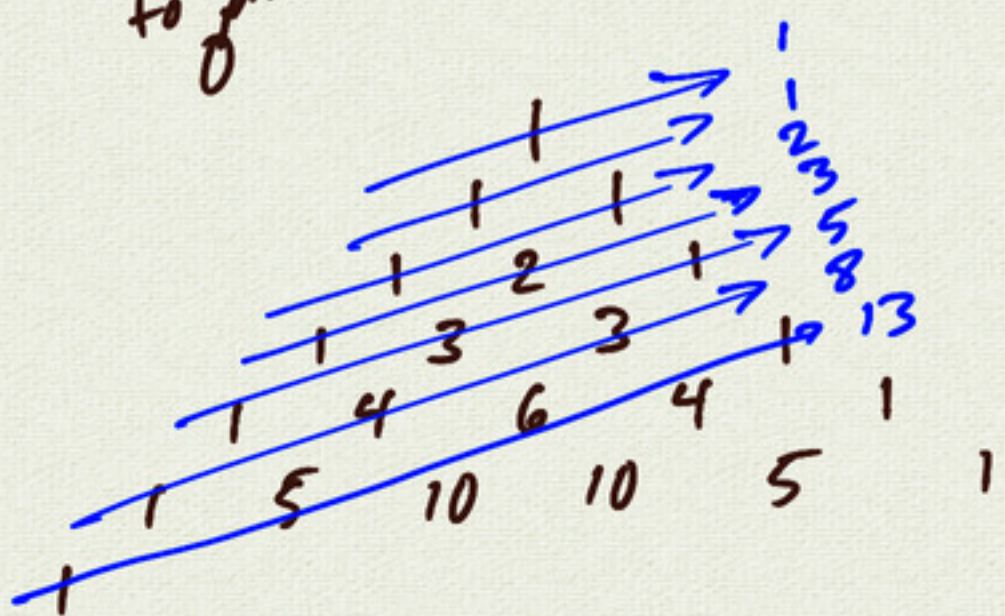


pizzas w/ 3 possible toppings = $2^3 = 8$
 = # subsets of 3 toppings
 {A, B, C}

- \emptyset
- {A}
- {B}
- {C}
- {A, B}
- {B, C}
- {A, C}
- {A, B, C}

$$\binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3} = 2^3$$

ways to pile



Fibonacci sequence
 1, 1, 2, 3, 5, 8, 13, 21, ...

sequences: (infinite list of numbers)

$0, 1, 2, 3, 4, 5, 6, \dots \rightarrow \infty$ (or limit does not exist)

$0, 0, 0, 0, \dots \rightarrow 0$

$1, -1, 1, -1, 1, -1, \dots \rightarrow$ no limit

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots \rightarrow 0$

notation:

$a_1, a_2, a_3, a_4, \dots$

$\{a_n\}_{n=1}^{\infty}$

$\{a_n\}$

explicit formula:

$$a_n = \left(\frac{1}{2}\right)^n$$

index

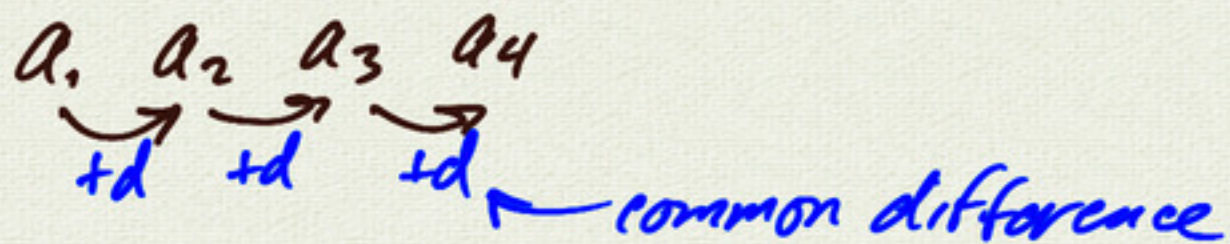
value

a_1	a_2	a_3	a_4
$\left(\frac{1}{2}\right)^1$	$\left(\frac{1}{2}\right)^2$	$\left(\frac{1}{2}\right)^3$	$\left(\frac{1}{2}\right)^4$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

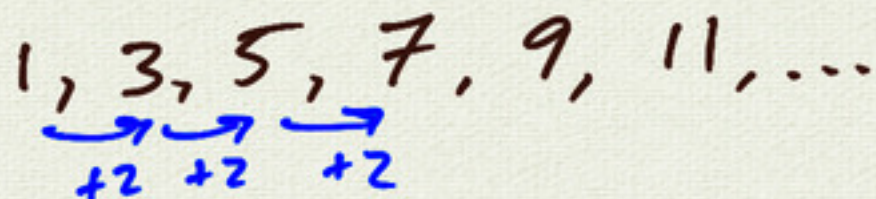
$\{a_n\}$

$a_k = \left(\frac{1}{2}\right)^k$ means same thing

arithmetic sequences



example:



recursive definition:

$$\begin{aligned} a_1 &= 1 \\ a_{n+1} &= a_n + 2 \end{aligned}$$

explicit definition

$$a_n = a_1 + d(n-1)$$

$$a_n = 1 + 2(n-1)$$

$$a_n = 1 + 2n - 2$$

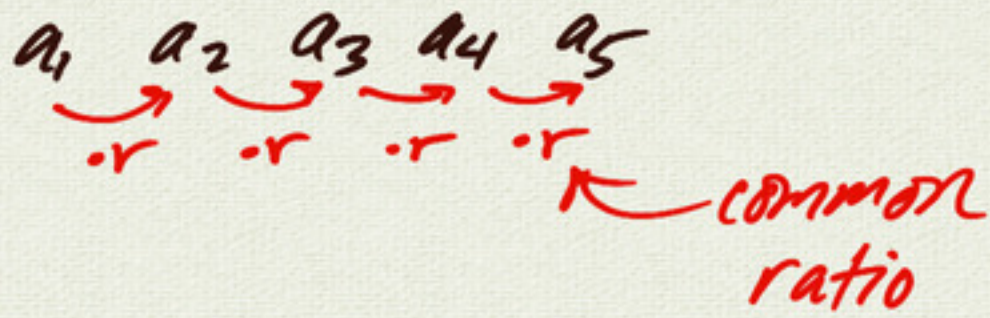
$$\boxed{a_n = -1 + 2n}$$

(general)

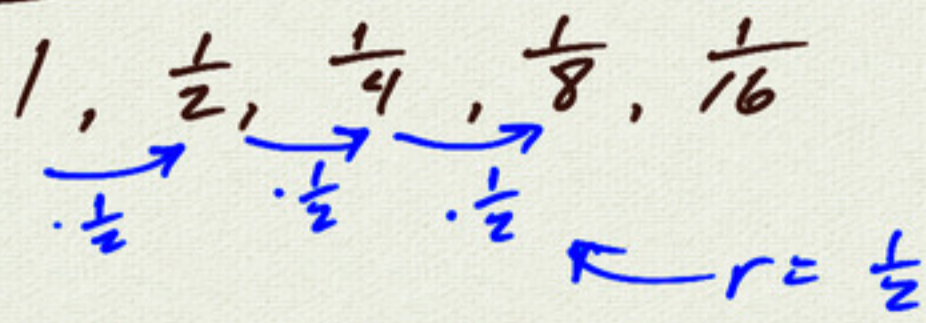
(this case)

index n	value $a_n = -1 + 2n$
1	1
2	3
3	5
4	7

geometric sequence



example:



recursive definition:

$$\begin{cases} a_1 = 1 \\ a_{n+1} = a_n \cdot \frac{1}{2} \end{cases}$$

explicit definition:

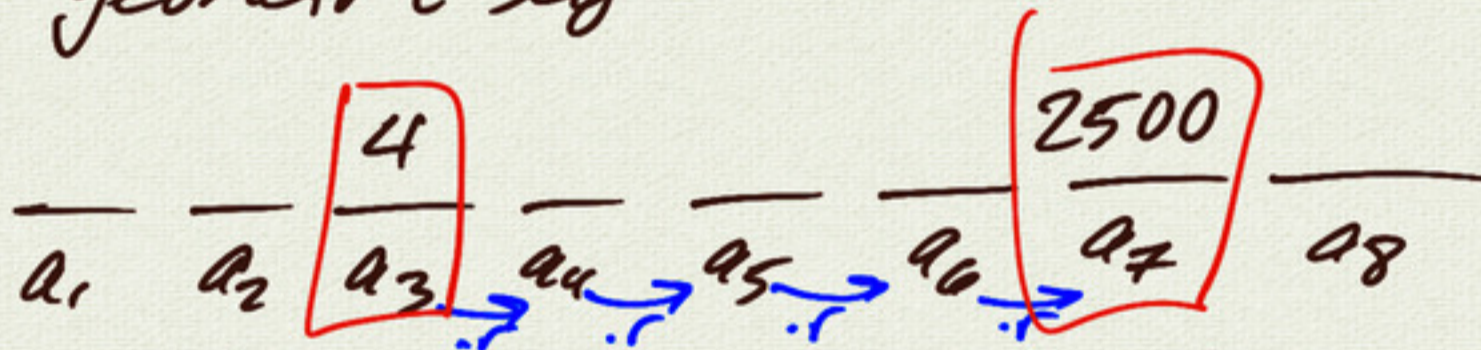
$$a_n = a_1 \cdot r^{n-1} \quad (\text{general})$$

$$= 1 \cdot \left(\frac{1}{2}\right)^{n-1}$$

$$a_n = \left(\frac{1}{2}\right)^{n-1}$$

n	a_n
1	$\left(\frac{1}{2}\right)^0 = 1$
2	$\left(\frac{1}{2}\right)^1 = \frac{1}{2}$
3	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

example: geometric sequence



Find recursive + explicit formulas.

explicit formula: $a_n = a_1 \cdot r^{n-1}$

$$a_7 = a_1 \cdot r^6$$

$$a_3 = a_1 \cdot r^2$$

divide

$$\frac{a_7}{a_3} = r^4$$

$$\frac{2500}{4} = r^4$$

$$625 = r^4$$

$$r = 5 \Rightarrow \text{check}$$

$\frac{4}{25}$	$\frac{4}{5}$	4	20	100	500	2500
a_1		a_3				a_7
		$\xrightarrow{\cdot 5}$				

recursive definition:

$$\begin{cases} a_1 = \frac{4}{25} \\ a_{n+1} = a_n \cdot 5 \end{cases}$$

explicit definition:

$$a_n = a_1 \cdot r^{n-1}$$

$$a_n = \left(\frac{4}{25}\right) \cdot 5^{n-1}$$

check:

$$a_3 = 4$$

$$a_7 = 2500$$

Fibonacci: 1, 1, 2, 3, 5, 8, 13, 21

recursive: $a_n = a_{n-1} + a_{n-2}$

$$a_1 = a_2 = 1$$