6.5 Sequences

```
            1                →   1
          1   1              →   2
        1   2   1            →   4
      1   3   3   1          →   8
    1   4   6   4   1        →   16
  1   5   10  10  5   1      →   32
```

\# pizzas w/ 3 possible toppings $= 2^3 = 8$

= # subsets of 3 toppings

$\{A, B, C\}$

$\emptyset$	$\{A\}$	$\{A, B\}$	$\{A, B, C\}$
	$\{B\}$	$\{B, C\}$	
	$\{C\}$	$\{A, C\}$	

$$\binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3} = 2^3$$

\# ways to pick 0

```
                    1
                  1   1        1
                1   2   1      2
              1   3   3   1    3
            1   4   6   4   1  5
          1   5  10  10   5   1  8
        1                        13
```

Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21, ...

$+$

sequences: (infinite list of numbers)

$0, 1, 2, 3, 4, 5, 6, \ldots$ $\to \infty$ (or limit does not exist)

$0, 0, 0, 0, \ldots$ $\to 0$

$1, -1, 1, -1, 1, -1, \ldots$ $\to$ no limit

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$ $\to 0$

notation:

$a_1, a_2, a_3, a_4 \ldots$

$\{a_n\}_{n=1}^{\infty}$

$\{a_n\}$

explicit formula:

$$a_n = \left(\frac{1}{2}\right)^n$$

index value

a_1	a_2	a_3	a_4
$(1/2)^1$	$\left(\frac{1}{2}\right)^2$	$\left(\frac{1}{2}\right)^3$	$\left(\frac{1}{2}\right)^4$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

$\{a_k\}$ $a_k = \left(\frac{1}{2}\right)^k$ means same thing

arithmetic sequences

$$a_1 \xrightarrow{+d} a_2 \xrightarrow{+d} a_3 \xrightarrow{+d} a_4$$

common difference

example:

$$1 \xrightarrow{+2} 3 \xrightarrow{+2} 5 \xrightarrow{+2} 7,\ 9,\ 11, \ldots$$

recursive definition: $\boxed{\begin{array}{l} a_1 = 1 \\ a_{n+1} = a_n + 2 \end{array}}$

explicit definition

$a_n = a_1 + d(n-1)$ (general)

$a_n = 1 + 2(n-1)$ (this case)

$a_n = 1 + 2n - 2$

$\boxed{a_n = -1 + 2n}$

index n	value $a_n = -1+2n$
1	1
2	3
3	5
4	7

geometric sequence

$$a_1 \xrightarrow{\cdot r} a_2 \xrightarrow{\cdot r} a_3 \xrightarrow{\cdot r} a_4 \xrightarrow{\cdot r} a_5$$

$\cdot r$ ← common ratio

example:

$$1 \xrightarrow{\cdot \frac{1}{2}} \frac{1}{2} \xrightarrow{\cdot \frac{1}{2}} \frac{1}{4} \xrightarrow{\cdot \frac{1}{2}} \frac{1}{8}, \frac{1}{16}$$

← $r = \frac{1}{2}$

recursive definition: $\boxed{\begin{array}{l} a_1 = 1 \\ a_{n+1} = a_n \cdot \frac{1}{2} \end{array}}$

explicit definition:

$$a_n = a_1 \cdot r^{n-1} \quad \text{(general)}$$

$$= 1 \cdot \left(\tfrac{1}{2}\right)^{n-1}$$

$$\boxed{a_n = \left(\tfrac{1}{2}\right)^{n-1}}$$

n	a_n
1	$\left(\frac{1}{2}\right)^0 = 1$
2	$\left(\frac{1}{2}\right)^1 = \frac{1}{2}$
3	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

example:

geometric sequence

$$\underline{\quad}_{a_1} \quad \underline{\quad}_{a_2} \quad \boxed{\underline{4}_{a_3}} \xrightarrow{\cdot r} \underline{\quad}_{a_4} \xrightarrow{\cdot r} \underline{\quad}_{a_5} \xrightarrow{\cdot r} \underline{\quad}_{a_6} \xrightarrow{\cdot r} \boxed{\underline{2500}_{a_7}} \quad \underline{\quad}_{a_8}$$

Find recursive + explicit formulas.

explicit formula: $a_n = a_1 r^{n-1}$

$$a_7 = a_1 r^6$$

$$a_3 = a_1 r^2 \quad \text{divide}$$

$$\frac{a_7}{a_3} = r^4$$

$$\frac{2500}{4} = r^4$$

$$625 = r^4$$

$$r = 5 \Rightarrow \text{check}$$

$$\underline{4/25}_{a_1} \quad \underline{4/5} \quad \underline{4}_{a_3} \xrightarrow{\cdot 5} \underline{20} \quad \underline{100} \quad \underline{500} \quad \underline{2500}_{a_7}$$

✓

recursive definition: $\boxed{\begin{array}{l} a_1 = 4/25 \\ a_{n+1} = a_n \cdot 5 \end{array}}$

explicit definition:

$$a_n = a_1 \cdot r^{n-1}$$

$$\boxed{a_n = \left(\tfrac{4}{25}\right) \cdot 5^{n-1}}$$

check:
$a_3 = 4$
$a_7 = 2500$

Fibonacci: 1, 1, 2, 3, 5, 8, 13, 21

recursive: $a_n = a_{n-1} + a_{n-2}$

$a_1 = a_2 = 1$