(24) 4 coins $P(H) = .75 = \frac{3}{4}$

$P\left(\begin{matrix}\text{exactly 2} \\ \text{heads}\end{matrix} \text{ or at least 2 tails}\right)$

2 tails ~~2 heads~~
3 tails 1 head
4 tails 0 heads

$= P\left(\begin{matrix}\text{0 heads} \\ \text{1 head} \\ \text{or} \\ \text{2 heads}\end{matrix}\right) = \underset{\text{success}}{P(0)} + \underset{\text{success}}{P(1)} + \underset{\text{success}}{P(2)}$

$\binom{4}{0}(.75)^0(.25)^4 + \binom{4}{1}(.75)^1(.25)^3 + \binom{4}{2}(.75)^2(.25)^2$

1 4 6

$= \left(\frac{1}{4}\right)^4 + 4\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^3 + 6\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2$

$= \frac{1}{256} + \frac{12}{256} + \frac{54}{256}$

$= \frac{67}{256}$

6.5 Sequences

$$\begin{array}{ccccccccccccc}
 & & & & & 1 & & & & & & \longrightarrow & 1 \\
 & & & & 1 & & 1 & & & & & \longrightarrow & 2 \\
 & & & 1 & & 2 & & 1 & & & & \longrightarrow & 4 \\
 & & \boxed{1} & & \boxed{3} & & \boxed{3} & & \boxed{1} & & & \longrightarrow & 8 \\
 & 1 & & 4 & & 6 & & 4 & & 1 & & & 16 \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 & & 32
\end{array}$$

---

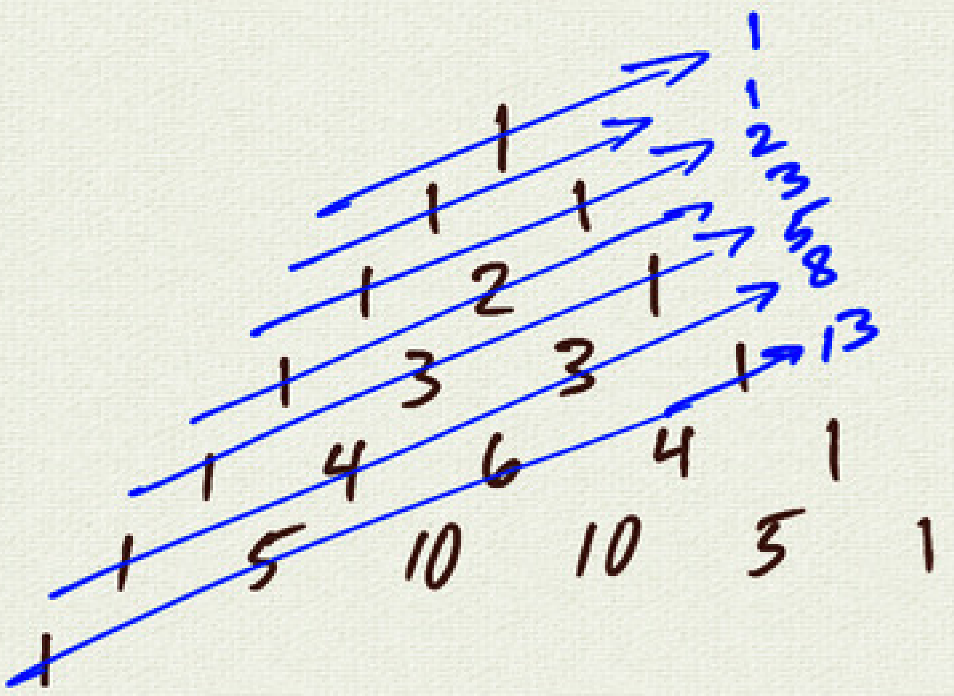
pizzas w/ 3 possible toppings $\Longrightarrow$ $2^3 = 8$ total

(# subsets of set of toppings)

$$\binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3} = 2^3$$

$$\begin{array}{cccc} 1 & 3 & 3 & 1 \end{array}$$

(pick 1 topping)



Fibonacci sequence

$1, 1, 2, 3, 5, 8, 13, 21, 34$

$\underline{8} + \underline{13} = 21$

---

sequence: infinite list of numbers

$0, 0, 0, 0, 0, 0, \ldots \to 0$

$1, 2, 3, 4, 5, \ldots \to \infty$

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \to 0$

$1, -1, 1, -1, 1, -1, \ldots$ no limit

notation:

$a_1, a_2, a_3, \ldots$

n^{th} term a_n

sequence $\{a_n\}_{n=1}^{\infty}$

$\{a_n\}$

---

example:

$a_n = 2^n \leftarrow$ explicit formula

$a_1 = 2^1$

$a_2 = 2^2$

$a_3 = 2^3$

$\ldots$

index value

a_1	a_2	a_3	a_4	a_5	
2	4	8	16	32	$\ldots$

$\lim_{n\to\infty} a_n = \infty$ (does not exist)

---

example: sequence

$a_k = 3 + \left(\frac{1}{3}\right)^k$

index k	value a_k
1	$3\frac{1}{3}$
2	$3\frac{1}{9}$
3	$3\frac{1}{27}$
4	$3\frac{1}{81}$

a_1	a_2	a_3	a_4	$\ldots$
$3\frac{1}{3}$	$3\frac{1}{9}$	$3\frac{1}{27}$	$3\frac{1}{81}$	

$\lim_{k\to\infty} a_k = 3$

$\lim_{n\to\infty} a_n = 3$

arithmetic sequence

$a_1 \xrightarrow{+d} a_2 \xrightarrow{+d} a_3 \xrightarrow{+d} a_4$

d
common
difference

example

$a_1 = 5$
$d = 3$

$\Longrightarrow$

a_1	a_2	a_3	a_4	
5	8	11	14	$\cdots$

$5 \xrightarrow{+3} 8 \xrightarrow{+3} 11 \xrightarrow{+3} 14$

recursive definition:

$$a_1 = 5$$

$$a_{n+1} = a_n + 3$$

explicit definition:

$$a_n = a_1 + d(n-1)$$

$$\boxed{a_n = 5 + 3(n-1)}$$

$$a_n = 5 + 3n - 3$$

$$\boxed{a_n = 3n+2}$$

n	$a_n = 3n+2$
1	5
2	8
3	11
4	14

geometric sequence

$a_1 \xrightarrow{\cdot r} a_2 \xrightarrow{\cdot r} a_3 \xrightarrow{\cdot r} a_4 \quad a_5$

r common ratio

example:

$1 \xrightarrow{1/2} \frac{1}{2} \xrightarrow{1/2} \frac{1}{4} \xrightarrow{1/2} \frac{1}{8}, \frac{1}{16}, \ldots$

recursive def: $\boxed{\begin{array}{l} a_1 = 1 \\ a_{n+1} = a_n \cdot \frac{1}{2} \end{array}}$

explicit def:

$a_n = a_1 \cdot r^{n-1}$

$a_n = 1\left(\frac{1}{2}\right)^{n-1}$

$\boxed{a_n = \left(\frac{1}{2}\right)^{n-1}}$

n	$a_n = \left(\frac{1}{2}\right)^{n-1}$
1	$(1/2)^0 = 1$
2	$(1/2)^1 = \frac{1}{2}$
3	$(1/2)^2 = \frac{1}{4}$

example: geometric sequence

		4				2500	
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8

find recursive def + explicit def.

explicit formula:

$a_n = a_1 \cdot r^{n-1}$

$a_7 = a_1 \cdot r^6$

$a_3 = a_1 \cdot r^2$ divide

$\frac{a_7}{a_3} = r^4 \quad (a_7 = a_3 \cdot r^4)$

$\frac{2500}{4} = r^4$

$625 = r^4$

$5^4 = r^4$

$r = 5$

4/25	4/5	4	20	100	500	2500
a_1	a_2	a_3				a_7

$\Rightarrow$ recursive: $a_1 = 4/25$

$a_{n+1} = 5a_n$

explicit: $a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = \left(\frac{4}{25}\right) \cdot 5^{n-1}}$

Fibonacci:

$$1, 1, 2, 3, 5, 8, \ldots$$

$$a_{n+2} = a_n + a_{n+1}$$ recursive

$$\left(\text{or:} \quad a_k = a_{k-1} + a_{k-2}\right)$$