coin flip $\longrightarrow$ heads $\frac{1}{2}$

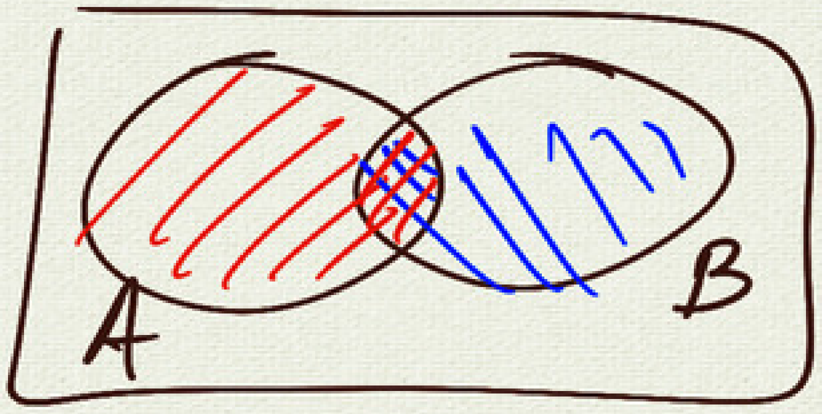
pick 1 card $\longrightarrow$ club $\frac{1}{4}$

		card: $\frac{1}{4}$ club	card: $\frac{3}{4}$ no club	
coin	$\frac{1}{2}$ H	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
coin	$\frac{1}{2}$ T	$\frac{1}{8}$	$\frac{3}{8}$	
		$\frac{1}{4}$		

$$P(\text{heads or club}) = \frac{1}{8} + \frac{1}{8} + \frac{3}{8} = \frac{5}{8}$$

A, B <u>independent</u> $\longleftrightarrow$ $P(A \cap B) = P(A)P(B)$

when is $\boxed{P(A \cup B) = P(A) + P(B)}$? no overlap

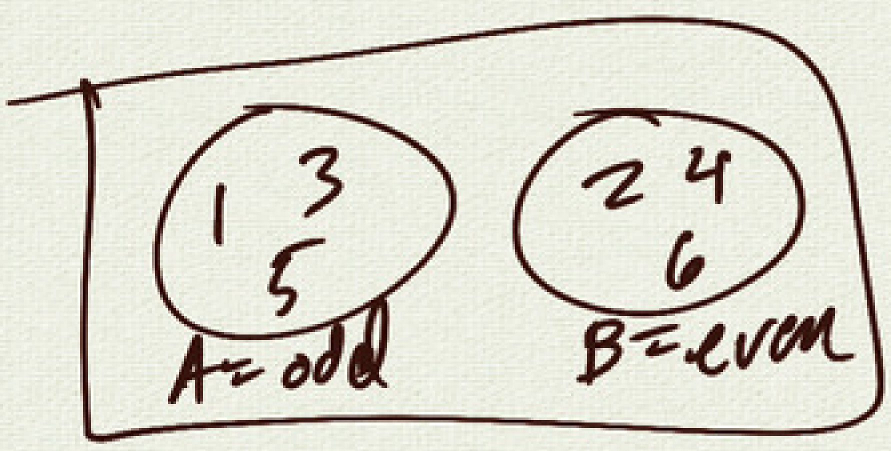


$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$A = \text{odd} = \{1, 3, 5\}$ $\quad P(A) = \frac{1}{2}$

$B = \text{big} = \{4, 5, 6\}$ $\quad P(B) = \frac{1}{2}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\frac{1}{2} + \frac{1}{2} - \frac{1}{6}$$
$$= 5/6$$



$$P(\text{odd or even}) = \frac{1}{2} + \frac{1}{2} = 1$$

no overlap intersection $\longleftrightarrow$ mutually exclusive

group work: 9 black] pick 5
3 white]

replacement $\rightarrow$ binomial
(independent)

$$P(\text{all black}) = \left(\frac{3}{4}\right)^5$$

---

no replacement $\rightarrow$ dependency
hypergeometric

$$P(\text{all black}) = \frac{\binom{9}{5}\binom{3}{0}}{\binom{12}{5}} = \frac{\frac{9\cdot 8\cdot 7\cdot 6\cdot 5}{5!}}{\frac{12\cdot 11\cdot 10\cdot 9\cdot 8}{5!}}$$

$$P(\text{all black}) = \frac{9}{12}\cdot\frac{8}{11}\cdot\frac{7}{10}\cdot\frac{6}{9}\cdot\frac{5}{8}$$

---

end, substitute

$$\sum_{k=3}^{5} 2^k = 2^3 + 2^4 + 2^5$$

index, start

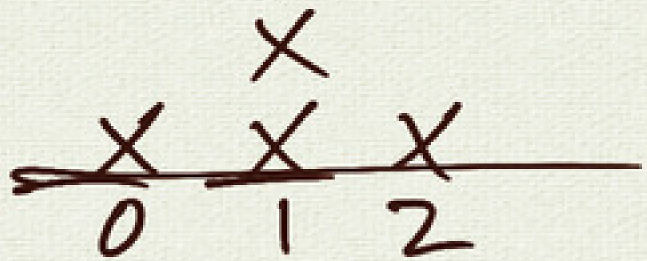
$$\sum_{k=1}^{5} 1 = 1 + 1 + 1 + 1 + 1$$

P(2 red
4 white)

7 red 3 white	pick 6

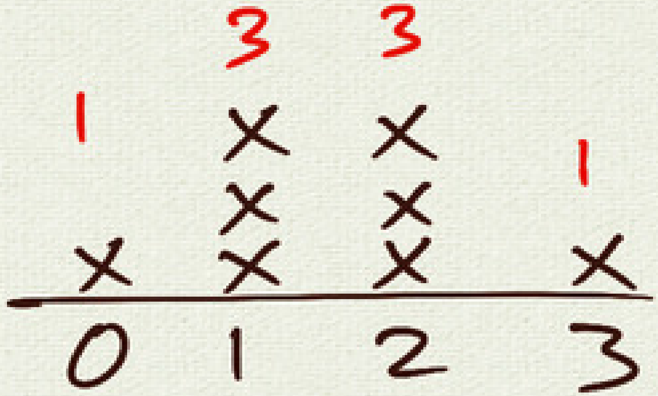
probability

2 coins:

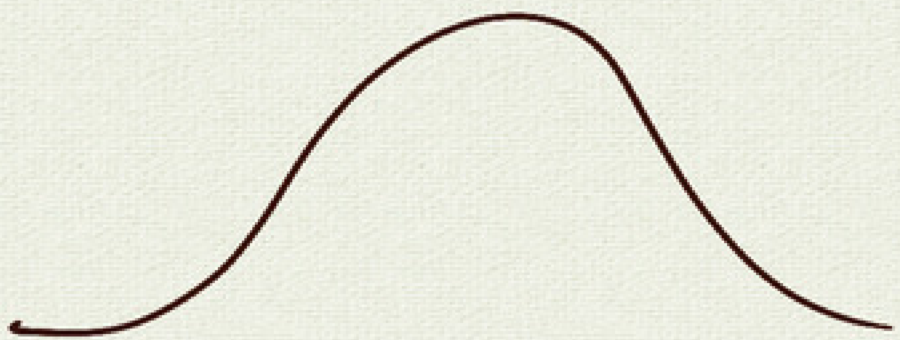


heads

3 coins



$n \to \infty$



bell curve
normal
distribution
Gaussian
e^{-x^2}

Central Limit Theorem

set theory – Cantor

x ⟷ •
x ⟷ •
x ⟷ •
x ⟷ •
x ⟷ •

2 sets are the same size

⇕

1-1 correspondence

infinite sets:

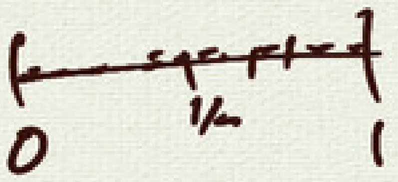
0, 1, 2, 3, 4, ...

..., -3, -2, -1, 0, +1, 2, 3, ...

0 ⟷ 0	0 ⟷ 2
1 ⟷ +1	1 ⟷ 4
2 ⟷ −1	2 ⟷ 6
3 ⟷ +2	3 ⟷ 8
4 −2	⋮
⋮ +3	
−3	even #s

a set is <u>countable</u> if it is the same size as $\mathbb{Z}$

$\mathbb{Q}$ (set of rational #'s) is countable



(1st diagonal argument)

	1	2	3	4	5 ...
1	1/1	1/2	1/3	1/4	...
2	2/1	2/2	2/3	...	
3	3/1	3/2	3/3		
4	4/1	4/2	4/3 ...		
5					

$\mathbb{R}$ (set of real #s) is uncountable

Proof: assume $\mathbb{R}$ is countable

$\Rightarrow$ we can put every real # in a list:

.134565
.24768
.35712
.41295
⋮

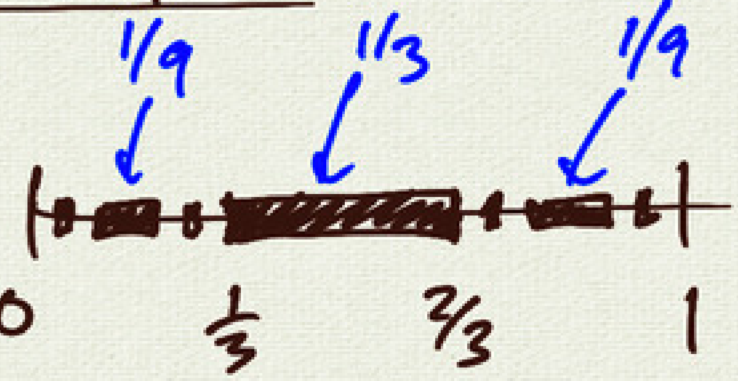
.2568 . . $\Leftarrow$ a real # not on list (contradiction)

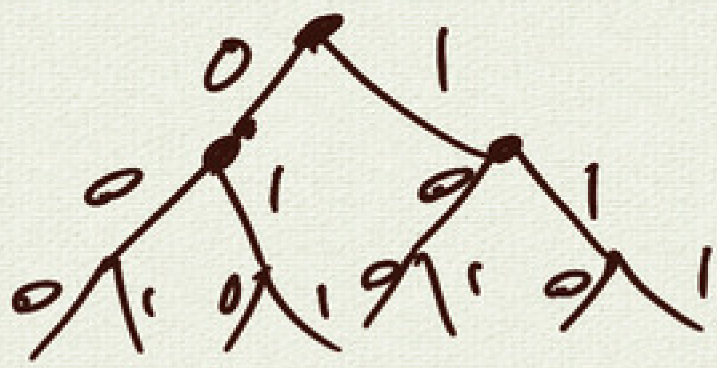
$\Rightarrow$ $\mathbb{R}$ is uncountable

$\mathbb{Q}$ countable

most #s are irrational

Cantor Set





point in Cantor set

$\Updownarrow$

infinite binary sequence

{infinite binary sequences} $\leftarrow$ uncountable

what did we remove?

$$S = \frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \dots$$ geometric

$a_1 = \frac{1}{3}$

$r = \frac{2}{3}$

$$S = \frac{a_1}{1-r} = \frac{\frac{1}{3}}{1-2/3} = 1$$

Cantor Set: uncountable, "measure zero"

big small

$\mathbb{Q}$: countable, "dense"

small big