marbles 9 black
3 white

pick 5

replacement
→ independant devices
→ binomial dist

no replacement
→ dependency
→ hypergeometric

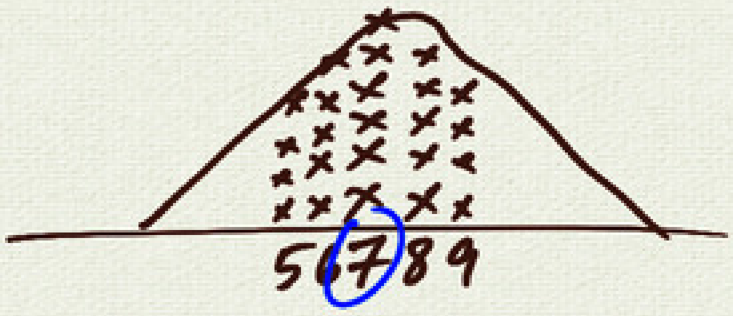
$$P(5\,\text{black}) = \frac{\binom{9}{5}\cdot\binom{3}{0}}{\binom{12}{5}} = \frac{\frac{9\cdot 8\cdot 7\cdot 6\cdot 5}{5!}}{\frac{12\cdot 11\cdot 10\cdot 9\cdot 8}{5!}}$$

another view:

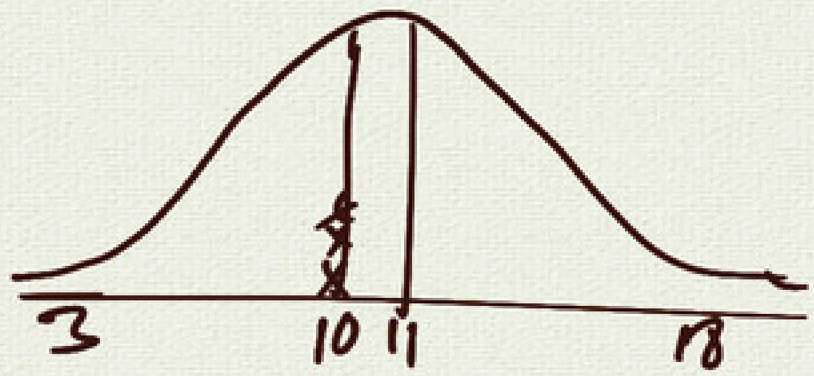
$$\left(\frac{9}{12}\right)\left(\frac{8}{11}\right)\left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{5}{8}\right)$$

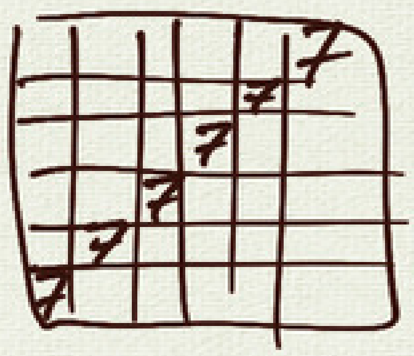
Probability

dice: 2 (sum)

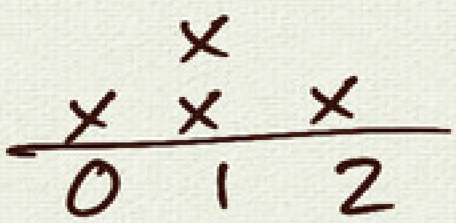


3 dice (sum)

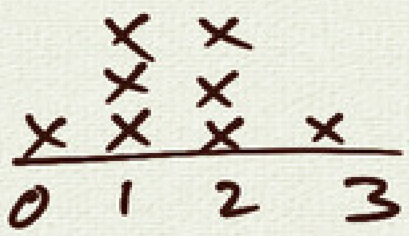




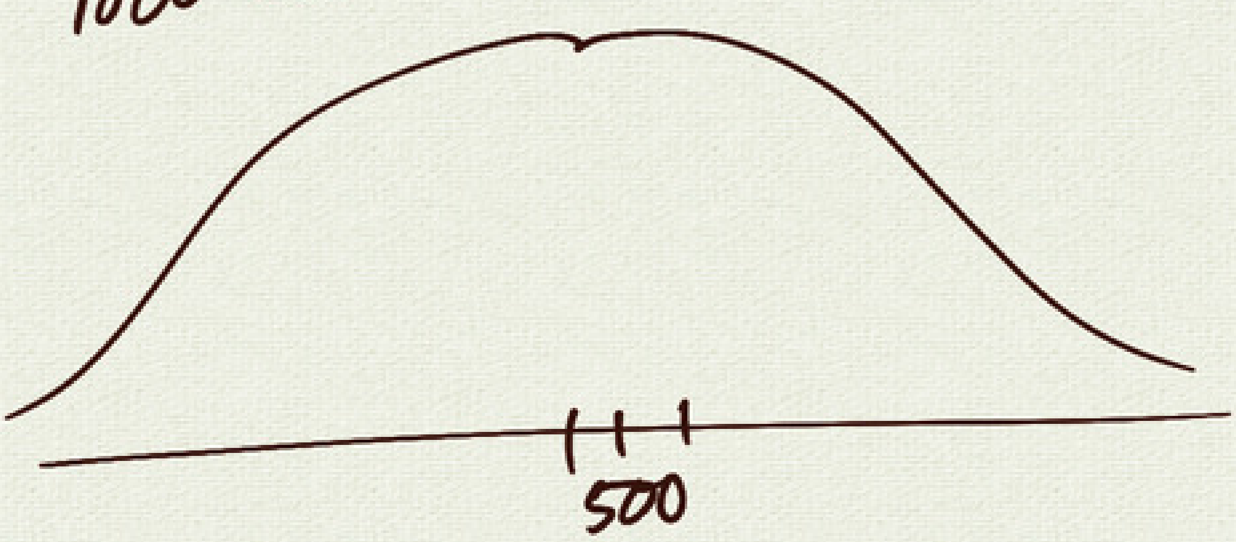
coins: 2 (#heads)



3 (#heads)



1000 coins



Central Limit Theorem

add lots of random things $\xrightarrow[n\to\infty]{}$ bell curve

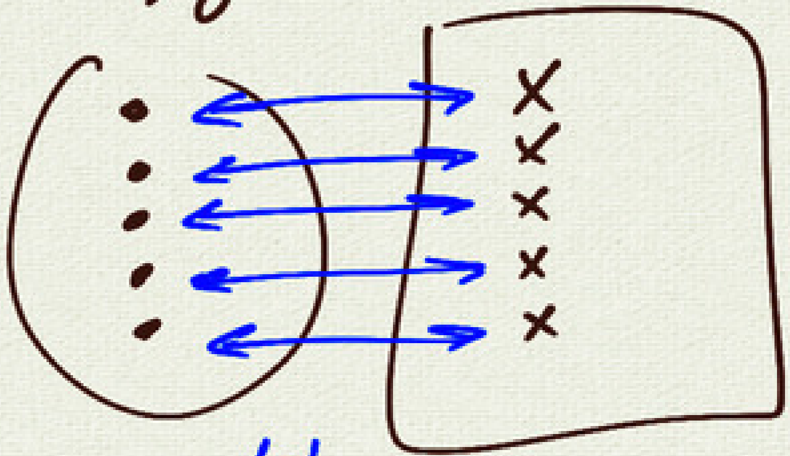
normal distribution

Gaussian

e^{-x^2}

Set Theory – Cantor

size of a set



1-1 correspondence $\longleftrightarrow$ same size

$0, 1, 2, 3, 4, \ldots$ natural #'s

$\ldots, -2, -1, 0, 1, 2, \ldots$ integers $\mathbb{Z}$

$$\begin{array}{ccc} 0 & \longleftrightarrow & 0 \\ 1 & \longleftrightarrow & 1 \\ 2 & \longleftrightarrow & -1 \\ 3 & \longleftrightarrow & 2 \\ \vdots & & -2 \\ & & 3 \\ & & -3 \\ & & \vdots \end{array}$$

$0, 2, 4, 6, 8, \ldots$

---

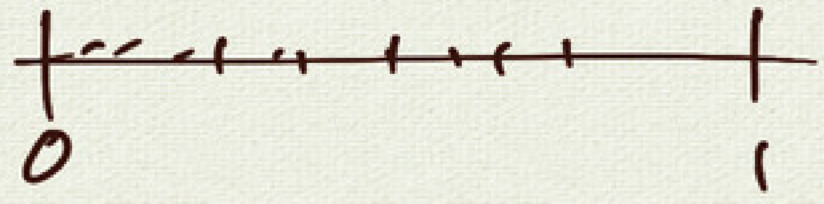
a set is <u>countable</u> if same size as $\mathbb{Z}$
(we can put all elements in a list)

---

rational #s $\mathbb{Q}$ (fractions)

	1	2	3	4	5
1	1/1	1/2	1/3	1/4	
2	2/1	2/2			
3	3/1	3/2	3/3		
4					
5					

$\longrightarrow$ $\mathbb{Q}$ is countable



claim: $\mathbb{R}$ (set of real numbers) is uncountable

Proof:

assume $\mathbb{R}$ countable

$\Rightarrow$ put all real #s in a list:

.134567...

.235123...

.141592...

.567812...

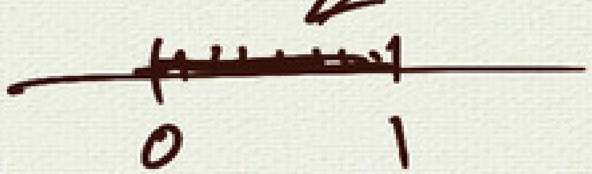
$\vdots$

.2227... $\Leftarrow$ I can find a # not on the list. (contradiction)

$\Rightarrow$ $\mathbb{R}$ uncountable.

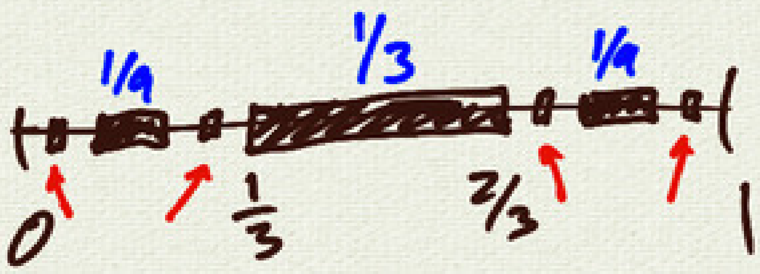
---

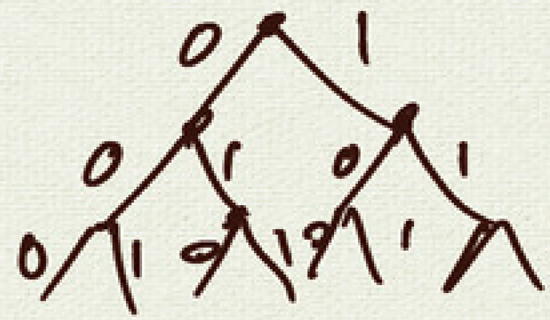
most real #s are irrational



{infinite binary sequences} uncountable

Cantor Set





Cantor set

$\updownarrow$

{ infinite binary sequences }

$\Longrightarrow$ Cantor Set is uncountable

how much is not in the Cantor Set:

$$\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \cdots$$

geometric: $a_1 = \frac{1}{3}$

$r = 2/3$

$$\text{sum} = \frac{a_1}{1-r} = \frac{1/3}{1-2/3} = 1$$

Cantor Set: uncountable, "measure zero"

$\mathbb{Q}$: countable, "dense"
