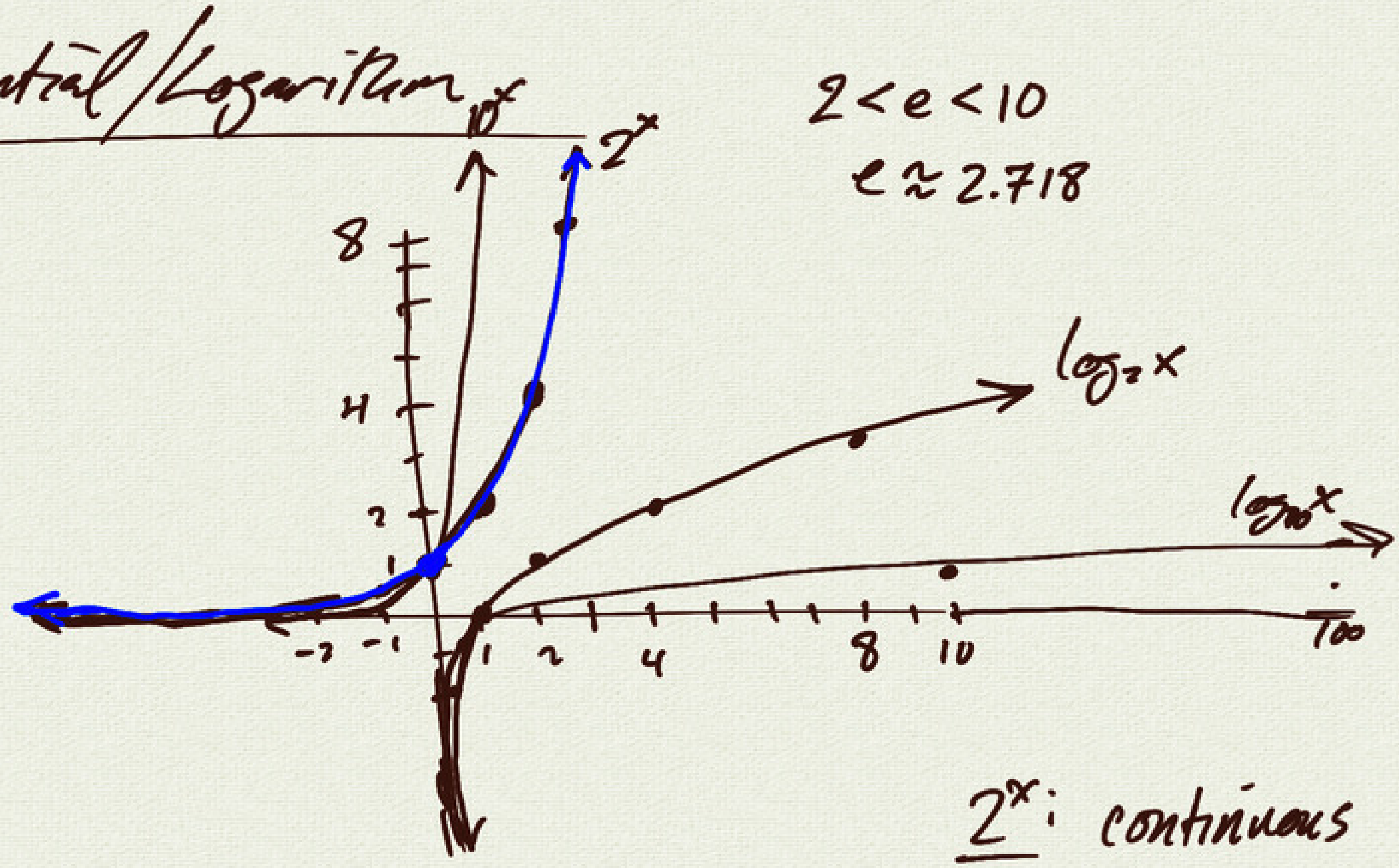
7.1 Exponential/Logarithm

$2 < e < 10$

$e \approx 2.718$

$y = 2^x$

x	2^x
0	1
1	2
2	4
3	8
-1	$1/2$
-2	$1/4$



x	$\log_2 x$
1	0
2	1
4	2
8	3
$1/2$	-1
$1/4$	-2

$x = \log_2 y \Longleftrightarrow 2^x = y$

$$\left.\begin{aligned} \log_2(2^x) &= x \\ 2^{\log_2 x} &= x \end{aligned}\right]$$ inverse functions

$\underline{2^x}$: continuous

$\lim_{x \to -\infty} 2^x = 0$

increasing

domain $\mathbb{R}$

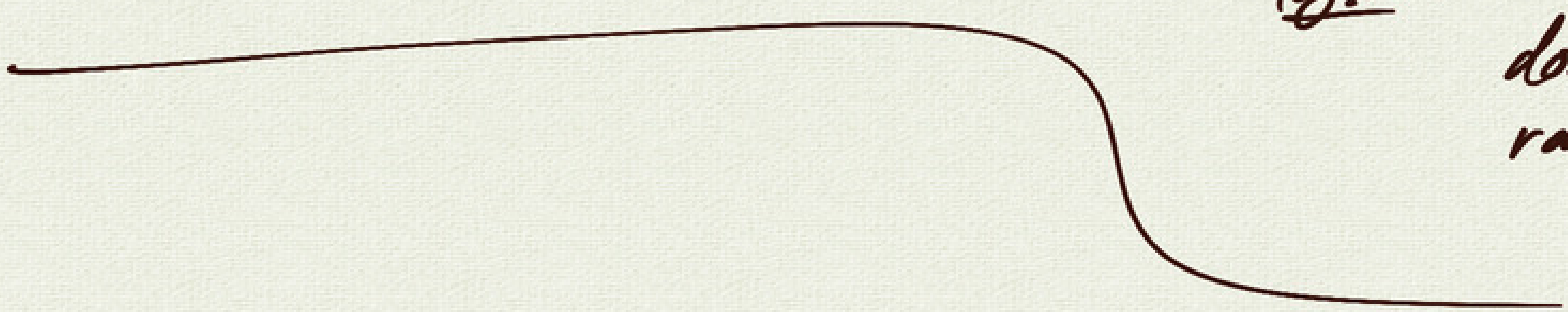
range $(0, \infty)$

$2^x > 0$ always

$\underline{\log_2 x}$: increasing

domain $(0, \infty)$

range $\mathbb{R}$



properties of b^x $\quad (b=2,10,e)$

$$b^x b^y = b^{x+y} \qquad (\underbrace{b\cdots b}_{x})(\underbrace{b\cdots b}_{y}) = \underbrace{b\cdots b}_{x+y}$$

$$(b^x)^y = b^{xy} \qquad \underbrace{(\underbrace{b\cdots b}_{x})(\underbrace{b\cdots b}_{x})\cdots(\underbrace{b\cdots b}_{x})}_{y} = b^{xy}$$

$$2 \xrightarrow{\;10^x\;} 10^2 = 100$$

$$3 \longrightarrow 10^3 = 1000$$

$$2+3 = 5 \longrightarrow 10^5 = (10^2)(10^3)$$

add $\longrightarrow$ multiplication

add $\xleftarrow{\;\log_{10}\;}$ multiplication

log properties:

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b(x^n) = n\log_b x$$

base change

compute $\log_b y = x$

$$y = b^x$$

$$\ln(y) = \ln(b^x)$$

$$\ln y = x \ln b$$

$$x = \frac{\ln y}{\ln b}$$

$$\boxed{\log_b y = \frac{\ln y}{\ln b} \quad \left(= \frac{\log_{10} y}{\log_{10} b} \right)}$$ base change

example: $\log_3(57) = \frac{\ln(57)}{\ln(3)} \quad \left(= \frac{\log_{10} 57}{\log_{10} 3} \right)$

$\ln x = \log_e x$

math books:

$\log x = \log_{10} x$

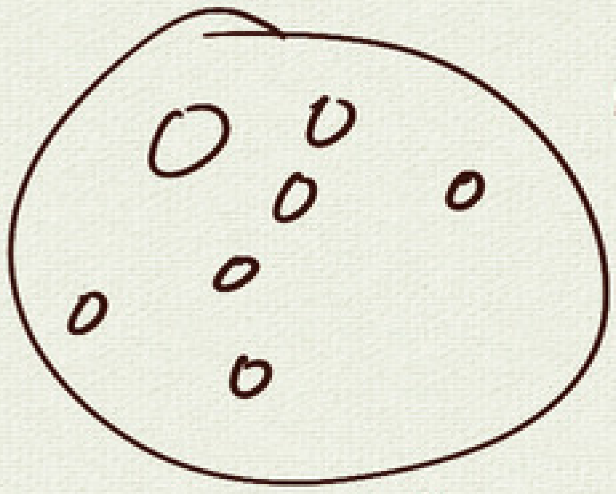
common log

coding:

$\log x = \ln x$

natural log

application: population growth



bacteria

initial population $P_0 = 10000$

doubling time 4 hours

model population $P(t)$

t	$P(t)$
0	10000
4	$20000 = 10000 \cdot 2$
8	$40000 = 10000 \cdot 2^2$
12	$80000 = 10000 \cdot 2^3$

$$\boxed{P(t) = 10000 \cdot 2^{t/4}}$$ model (function)

$$P(t) = P_0 \cdot 2^{t/T}$$

doubling time

$$P(16) = 10000 \cdot 2^4 = 160000$$

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alternate: $P(t) = P_0 e^{kt}$ find P_0, k

$P_0 = 10000$
doubling time 4 hours

$t=0 \Rightarrow P(0) = P_0 = 10000$

$$P(t) = 10000 e^{kt}$$

$$P(4) = 10000 e^{k \cdot 4} = 20000$$

$$e^{4k} = 2$$

$$4k = \ln 2$$

$$k = \frac{\ln 2}{4}$$

$$\boxed{P(t) = 10000 e^{\frac{\ln 2}{4} t}}$$

$\ln(e^{4k}) = 4k$

$\ln(e^{4k})$

$= 4k \underbrace{(\ln e)}_{1}$