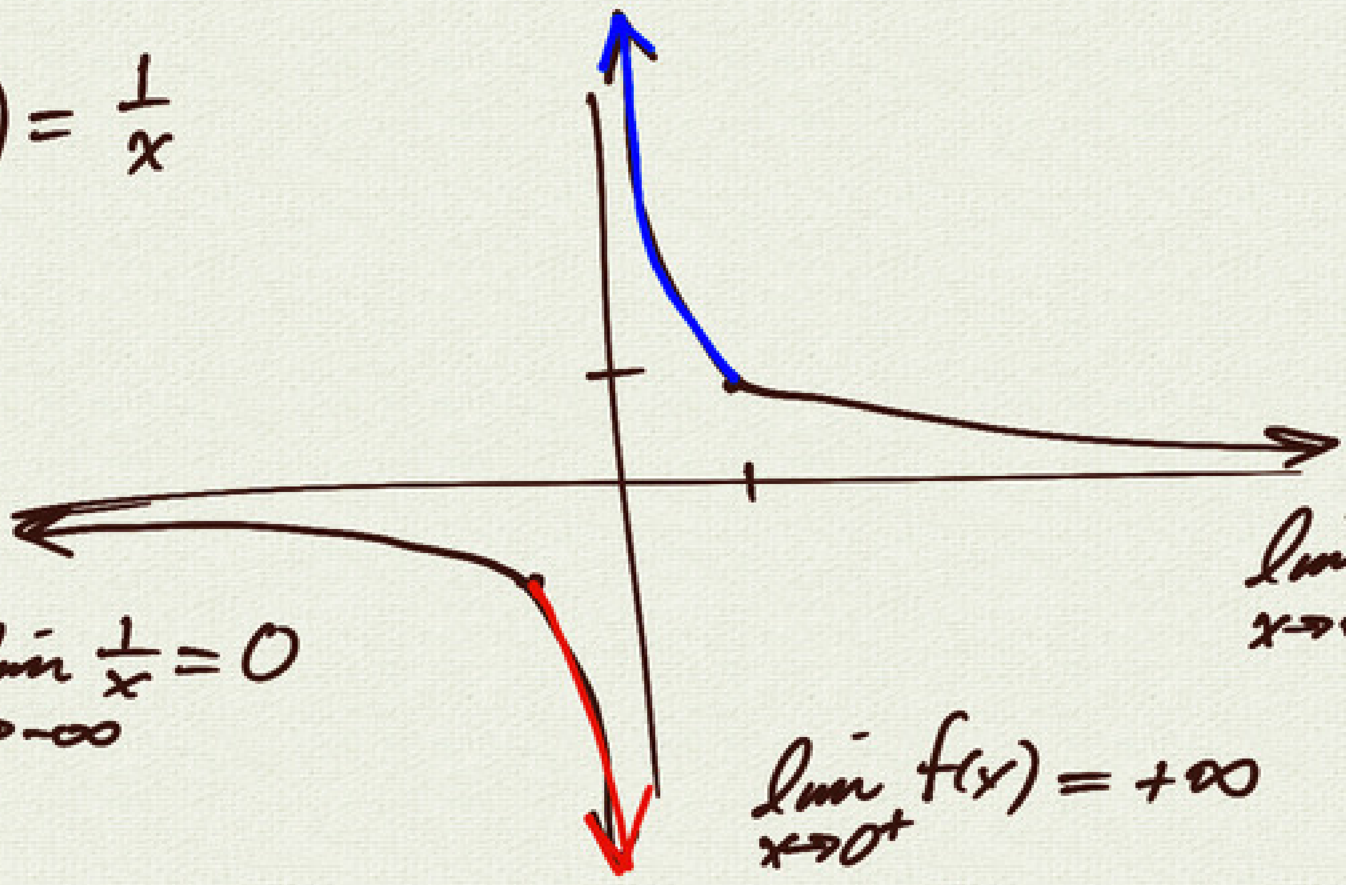
8.1 Limits

$f(x) = \frac{1}{x}$



$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} \frac{1}{x} = 0$

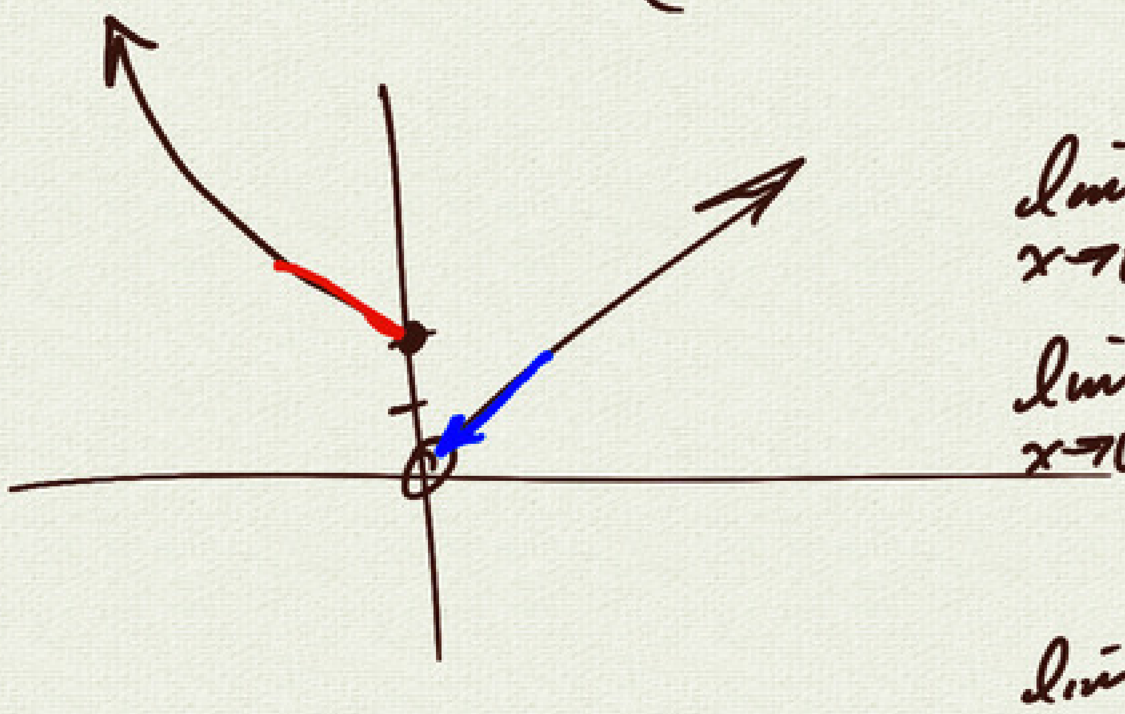
$\lim_{x \to 0^+} f(x) = +\infty$

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

end behavior
behavior near asymptotes

$$\boxed{\lim_{x \to a} f(x) = L}$$

example: $f(x) = \begin{cases} x^2+2 & \text{if } x \le 0 \\ x & \text{if } x > 0 \end{cases}$



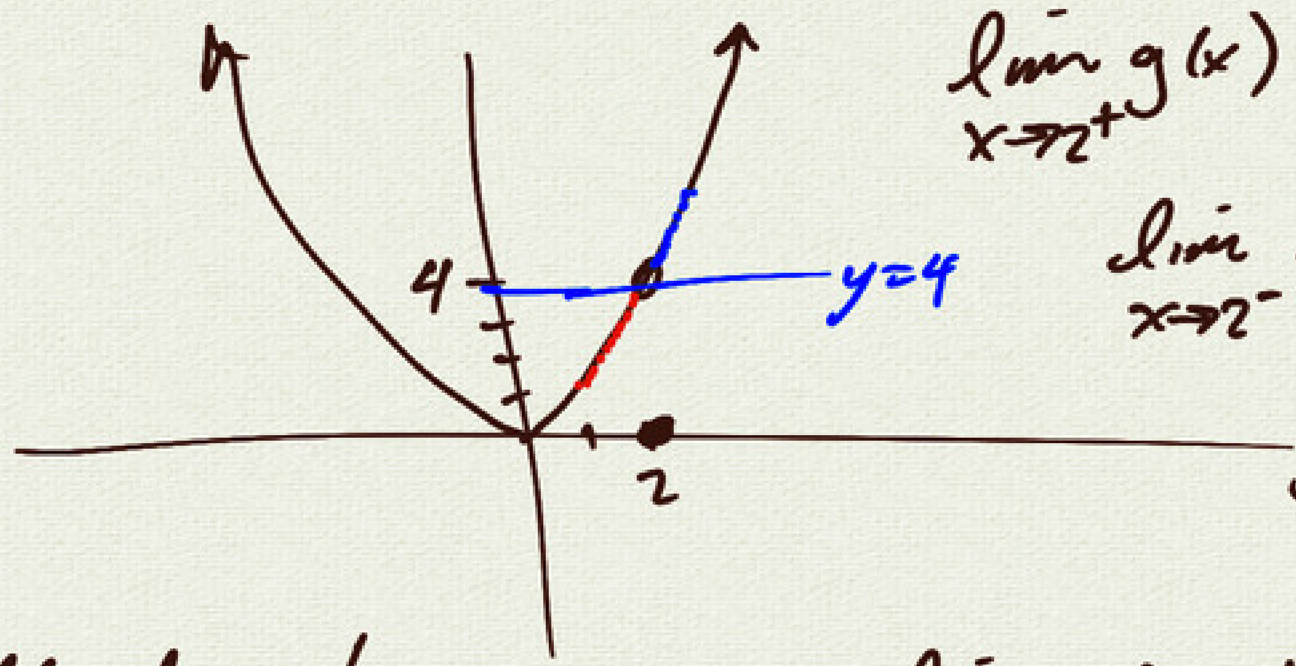
$\lim_{x \to 0^+} f(x) = 0$ right

$\lim_{x \to 0^-} f(x) = 2$ left

$\lim_{x \to 0} f(x)$ does not exist (2 sided)

left limit $\neq$ right limit

$g(x) = \begin{cases} x^2 & \text{if } x \neq 2 \\ 0 & \text{if } x = 2 \end{cases}$



$\left.\begin{array}{l} \lim_{x \to 2^+} g(x) = 4 \\ \lim_{x \to 2^-} g(x) = 4 \end{array}\right\}$ equal

$g(2)$ does not matter

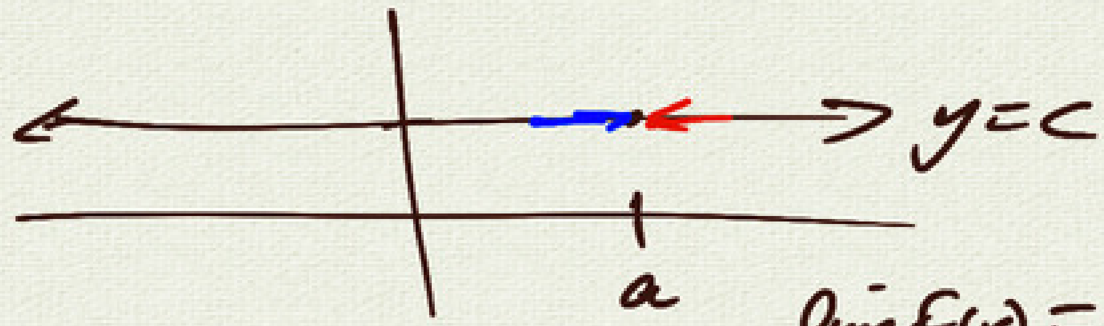
removable discont at $x=2$:
redefine $g(2) = 4$

$\lim_{x \to 2} g(x) = 4$
(2 sided limit exist)

$$\lim_{x\to a} f(x) = L \iff f(x) \text{ gets closer to } L \text{ as } x \text{ approaches } a$$

rules for limits:

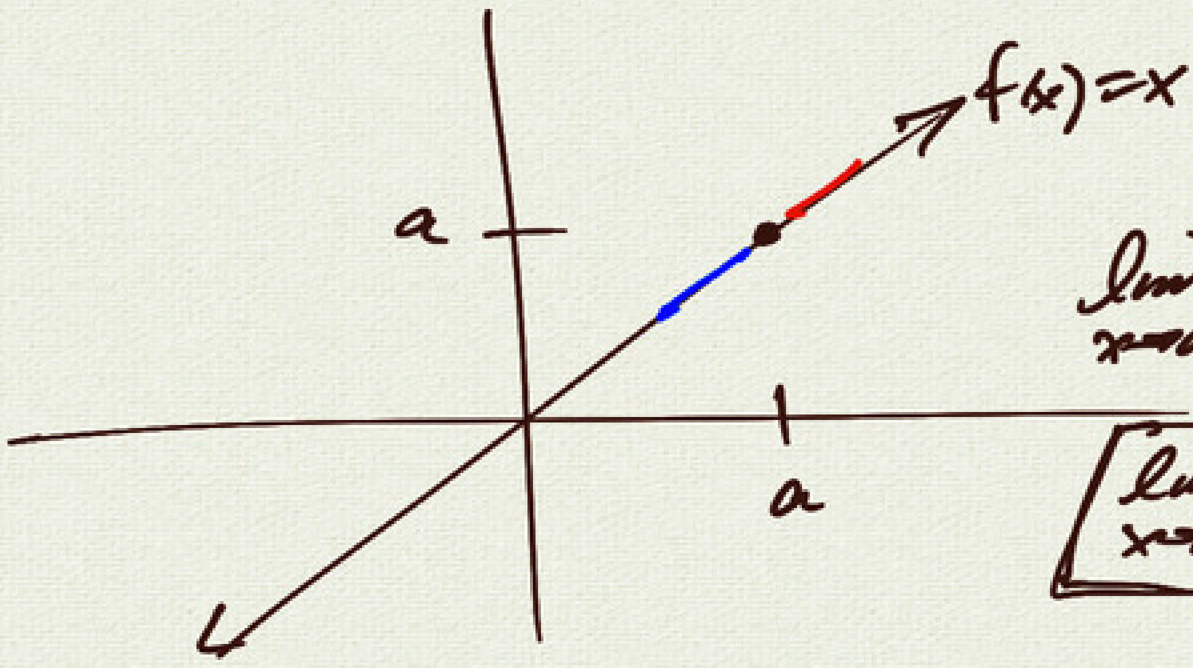
$f(x) = C$



$$\lim_{x\to a} f(x) = c$$

$$\boxed{\lim_{x\to a} c = c}$$

$f(x) = x$

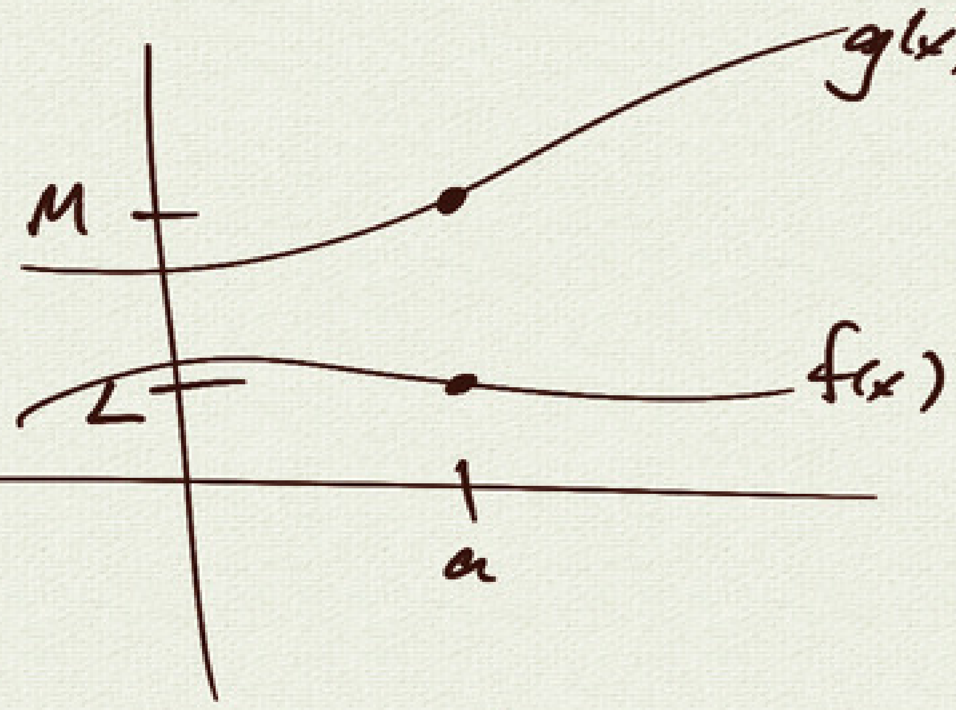


$$\lim_{x\to a} f(x) = a$$

$$\boxed{\lim_{x\to a} x = a}$$

$$\lim_{x\to a} f(x) = L$$

$$\lim_{x\to a} g(x) = M$$



$$\boxed{\lim_{x\to a} (f+g)(x) = L+M}$$

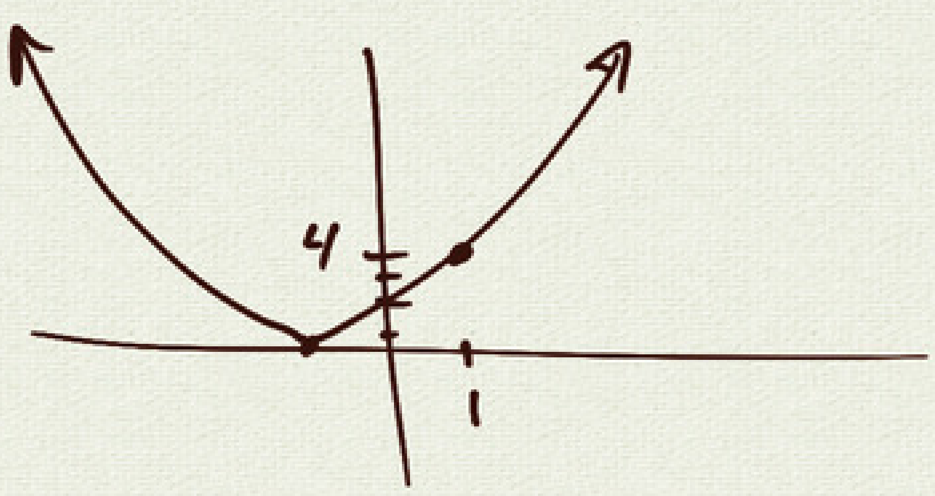
limits add

$$\lim_{x\to a} (fg)(x) = LM$$

$$\lim_{x\to a} (f/g)(x) = L/M \quad (\text{if } M \neq 0)$$

example: $f(x) = x^2 + 2x + 1$
$= (x+1)^2$

$$\lim_{x\to 1} f(x) = 4$$



we can plug in to find limit:

- any polynomial
- rational function
- trig functions
- exp/log
- $\sqrt{x}$

what can go wrong?

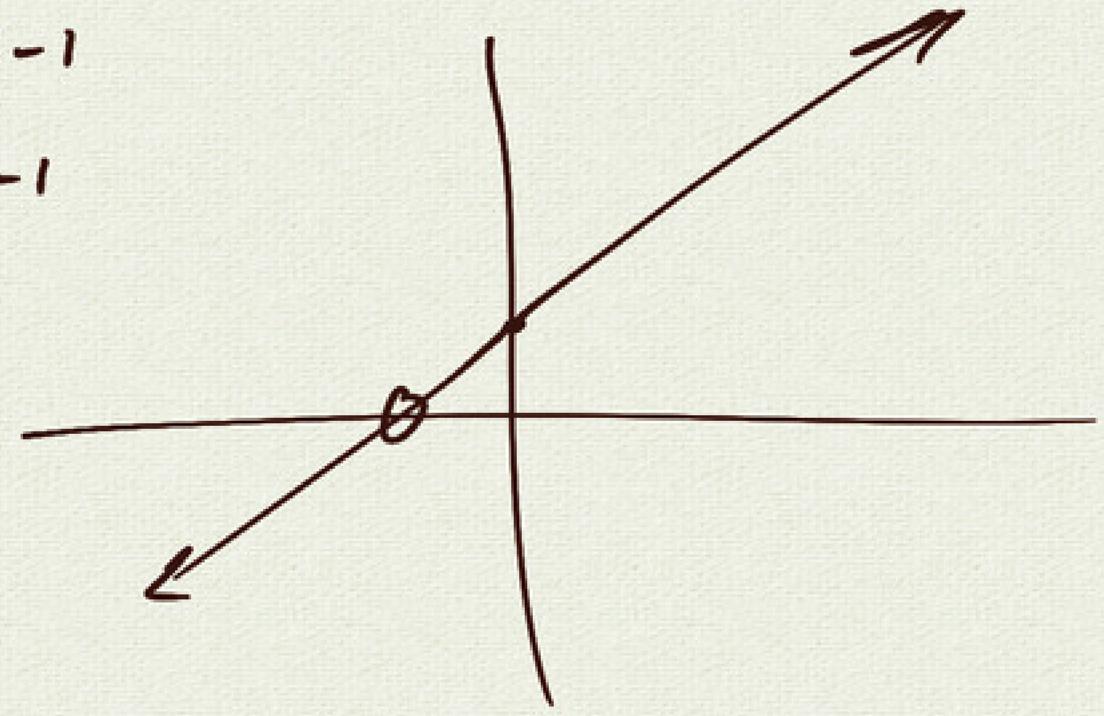
$$g(x) = \frac{x^2+2x+1}{x+1}$$ ← plug in: $\lim_{x\to -1} g(x) = \frac{0}{0}$ ← I don't know

$$= \frac{(x+1)^2}{(x+1)}$$

$$= \begin{cases} x+1 & \text{if } x \neq -1 \\ \text{undef} & x = -1 \end{cases}$$

plug in here

$$\lim_{x\to -1} g(x) = 0$$



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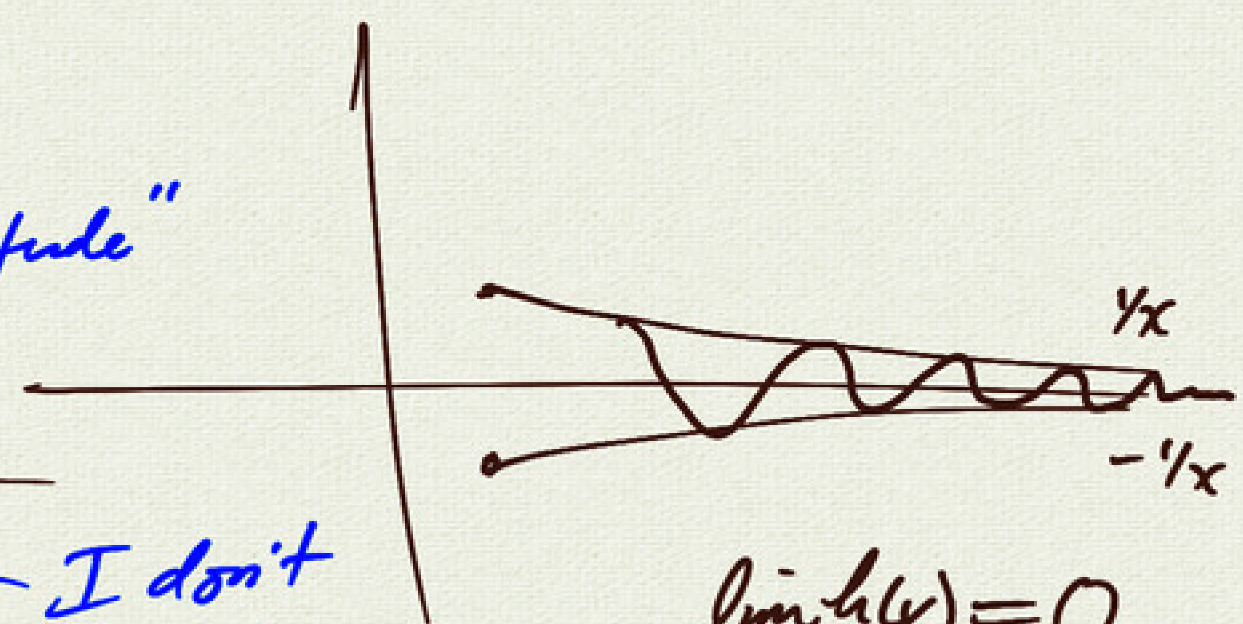
example:

$x \to \infty$

$$h(x) = \frac{\sin x}{x}$$

$$= \boxed{\frac{1}{x}} \sin x$$

"amplitude"



$$\lim_{x\to 0} \frac{\sin x}{x} = \frac{0}{0}$$ ← I don't know

$$\lim_{x\to\infty} h(x) = 0$$

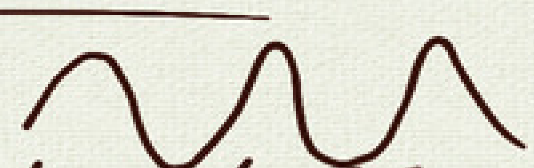
by sandwich theorem

squeeze theorem

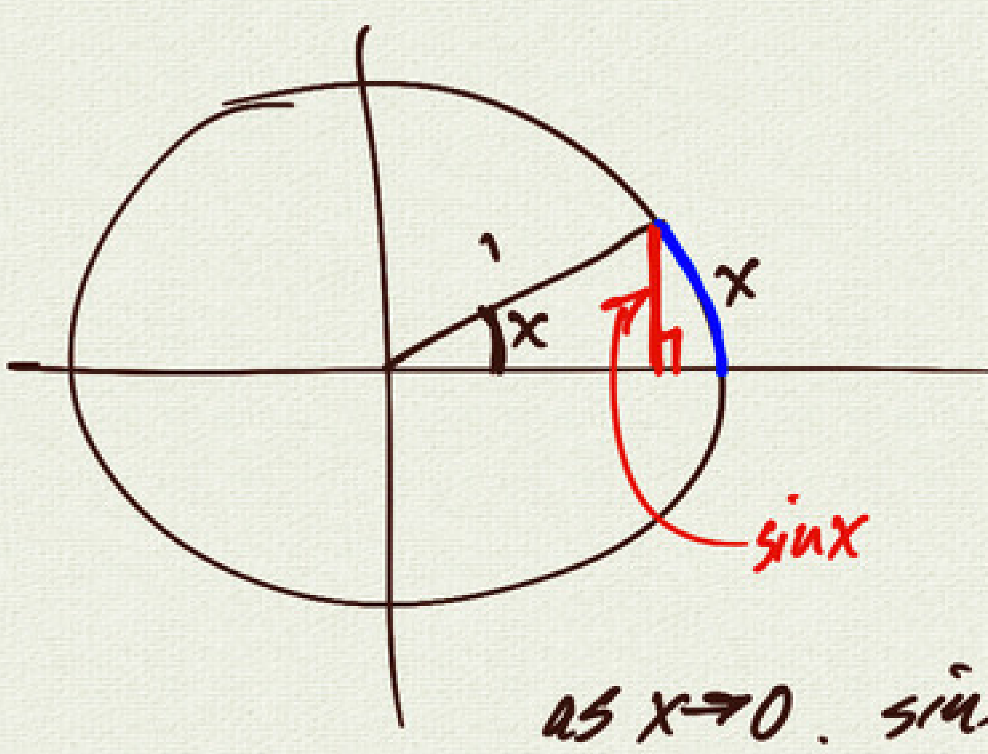
special limit:

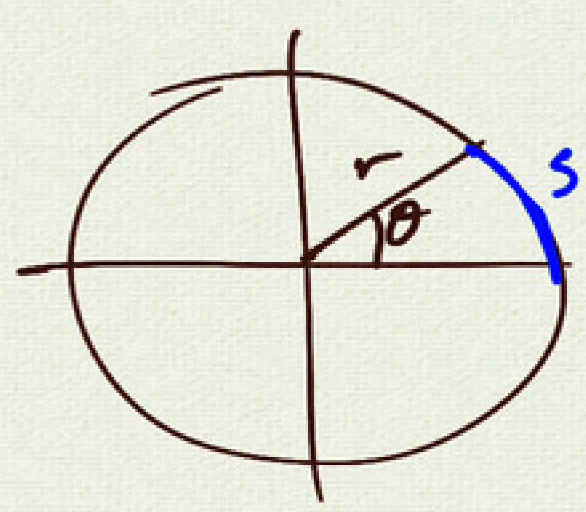
$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

contrast:



$\lim_{x\to\infty} \sin x$ does not exist





$$s = \left(\frac{\theta}{2\pi}\right) 2\pi r$$
$$= \theta r$$

as $x \to 0$. $\sin x \approx x$

$$\frac{\sin x}{x} \approx 1$$

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$