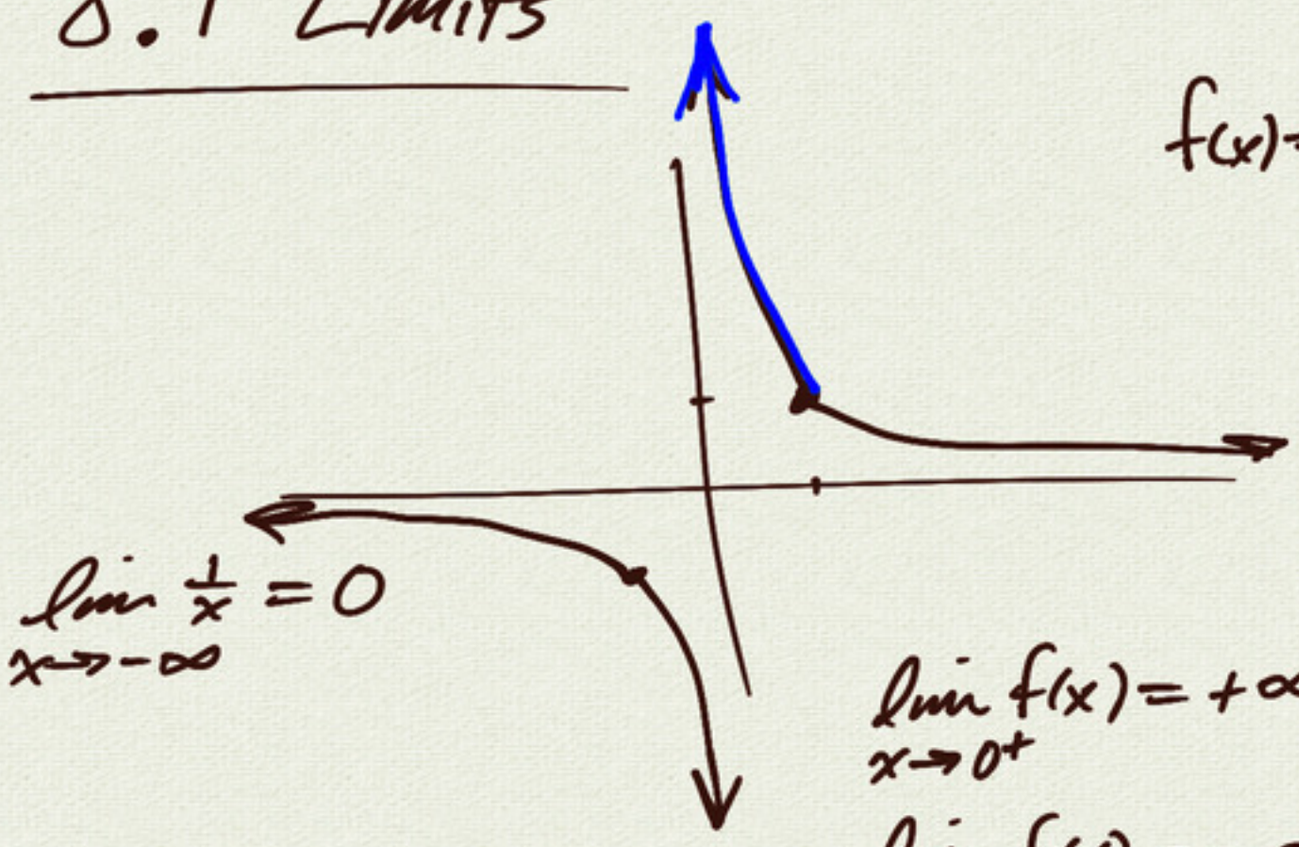
8.1 Limits



$f(x) = \frac{1}{x}$

$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} \frac{1}{x} = 0$

$\lim_{x \to 0^+} f(x) = +\infty$

$\lim_{x \to 0^-} f(x) = -\infty$

limits to describe:

- end behavior
- behavior near (vertical) asymptotes

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general: describe behavior of $f(x)$ as $x \to a$

$\lim_{x \to a} f(x) = L$

$f(x)$ approaches L "is close to" as $x \to a$

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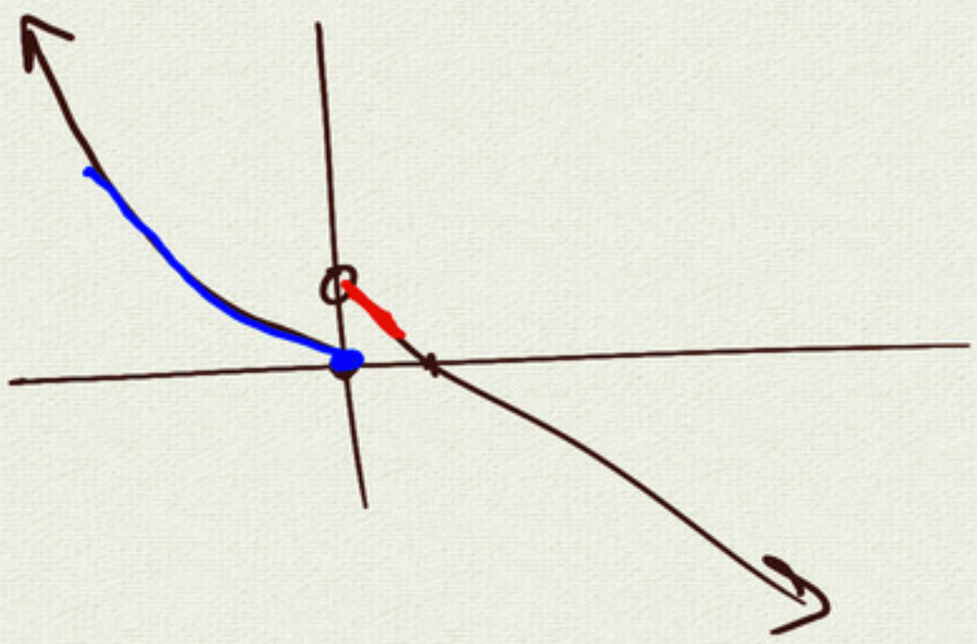
piecewise function:

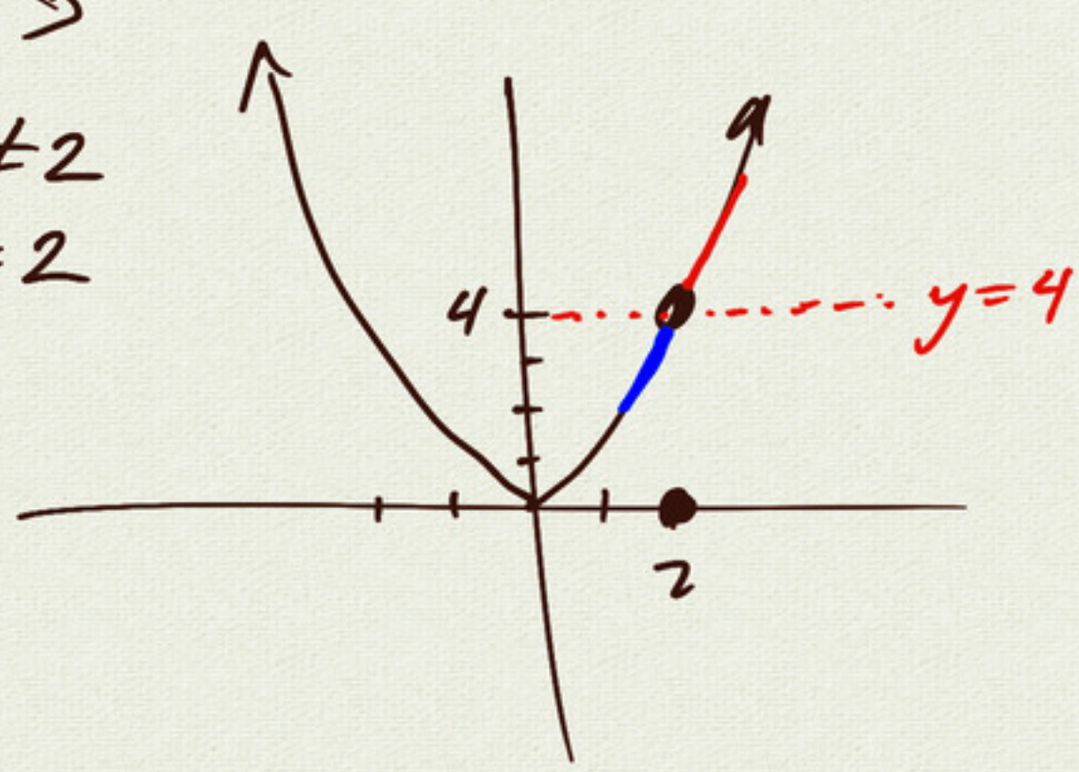
$$f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ 1 - x & \text{if } x > 0 \end{cases}$$



$\lim_{x \to 0^-} f(x) = 0$

$\lim_{x \to 0^+} f(x) = 1$

$$g(x) = \begin{cases} x^2 & \text{if } x \neq 2 \\ 0 & \text{if } x = 2 \end{cases}$$



$\lim_{x \to 2^-} g(x) = 4$

$\lim_{x \to 2^+} g(x) = 4$

$\lim_{x \to 2} g(x) = 4$ (2 sided limit exists)

(left limit = right limit)

$\lim_{x \to a} f(x) = L$ "$f(x)$ gets close to L as x approaches a"

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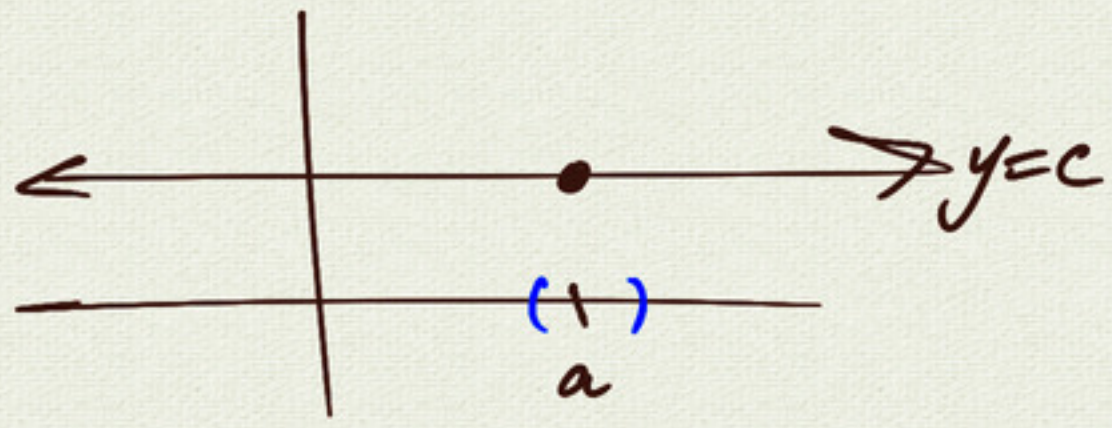
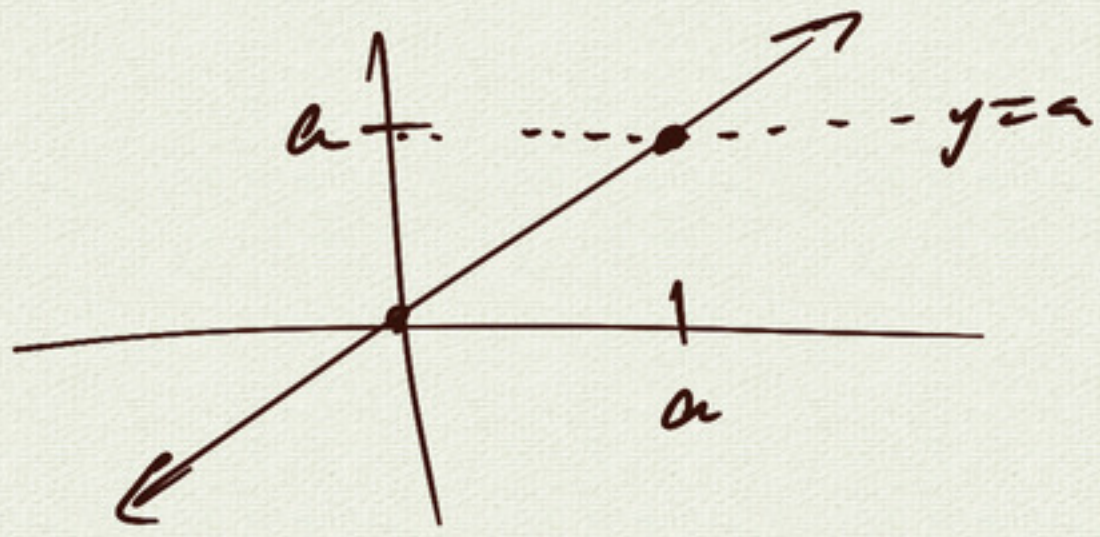
rules for limits

$f(x) = c$ (const.)



$\lim_{x \to a} f(x) = c$

$f(x) = x$

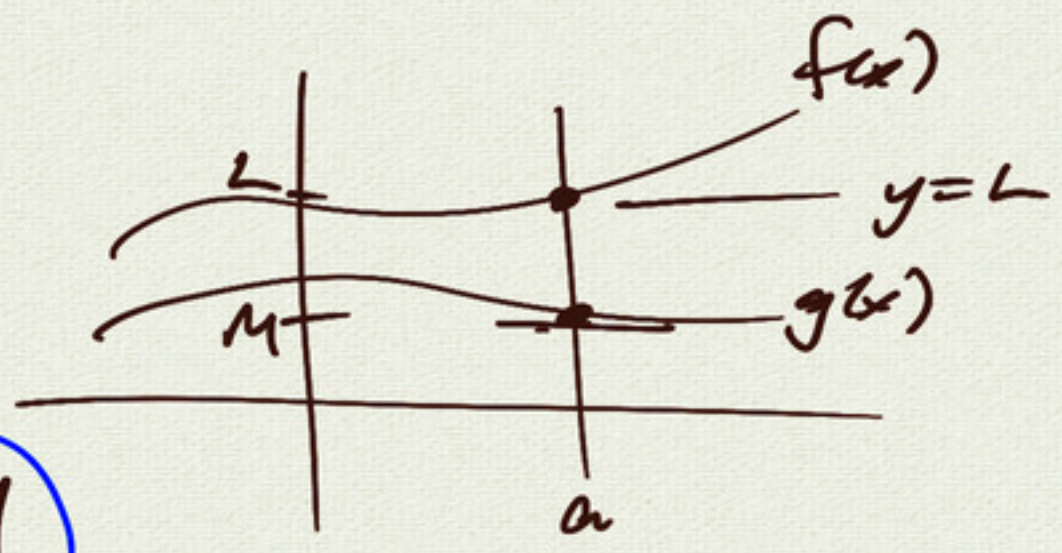


$\lim_{x \to a} f(x) = a$

$\lim_{x \to a} x = a$

$\lim_{x \to a} f(x) = L$

$\lim_{x \to a} g(x) = M$



$\lim_{x \to a} (f+g)(x) = L+M$ (limits add)

also:

$\lim_{x \to a} (fg)(x) = LM$

$\lim_{x \to a} (f/g)(x) = L/M$ (as long as $M \neq 0$)

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example: $p(x) = x^2 + 2x + 1 = (x+1)^2$

$\lim_{x \to 1} p(x) = p(1) = 4$ ⟸ plug in

$\lim_{x \to -1} p(x) = 0$

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when can we plug in?

polynomials rational functions trig functions exp/log $\sqrt[n]{x}$	continuous functions

$\lim_{x \to \pi/2} \sin(x) = 1$

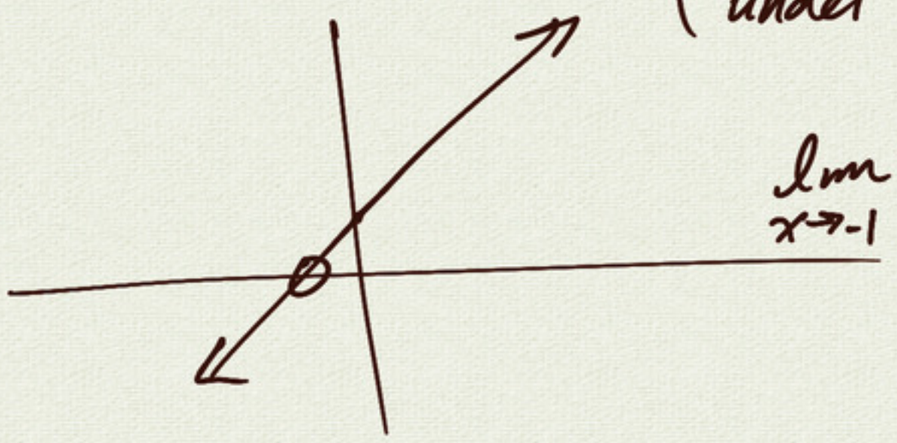
example:

$$f(x) = \frac{x^2+2x+1}{x+1}$$

$$\lim_{x\to -1} f(x) = \frac{0}{0} \quad]\ ?$$

cancel, plug in

$$f(x) = \frac{(x+1)^2}{(x+1)} = \begin{cases} x+1 & x+1\neq 0 \\ & x\neq -1 \\ \text{undef} & x=-1 \end{cases}$$



$$\lim_{x\to -1} f(x) = 0$$

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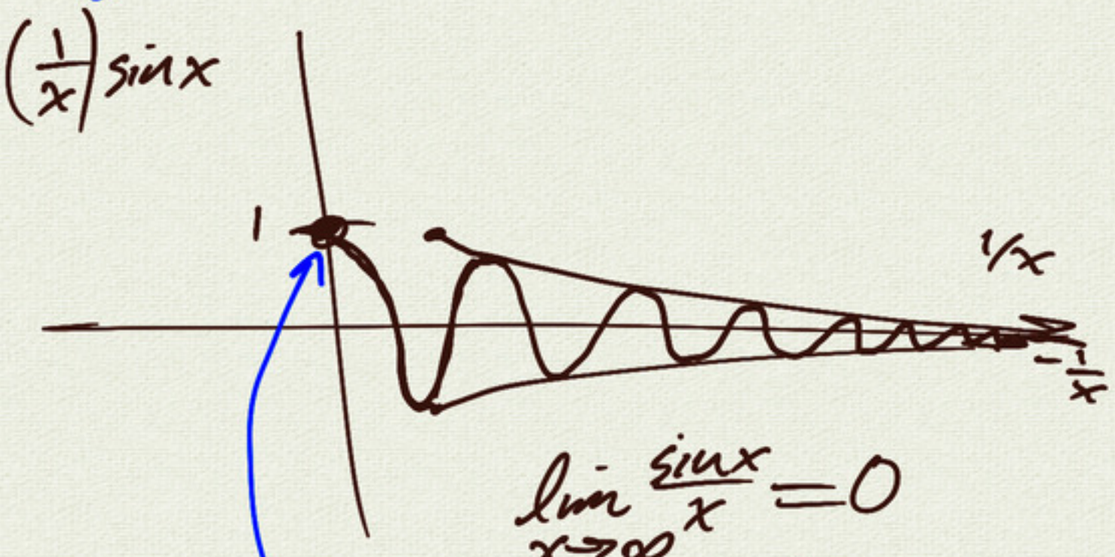
example:

amplitude

$$f(x) = \frac{\sin x}{x} = \left(\frac{1}{x}\right)\sin x$$

$(x\to\infty)$



$$\lim_{x\to\infty} \frac{\sin x}{x} = 0$$

by "squeeze theorem"
"sandwich theorem"

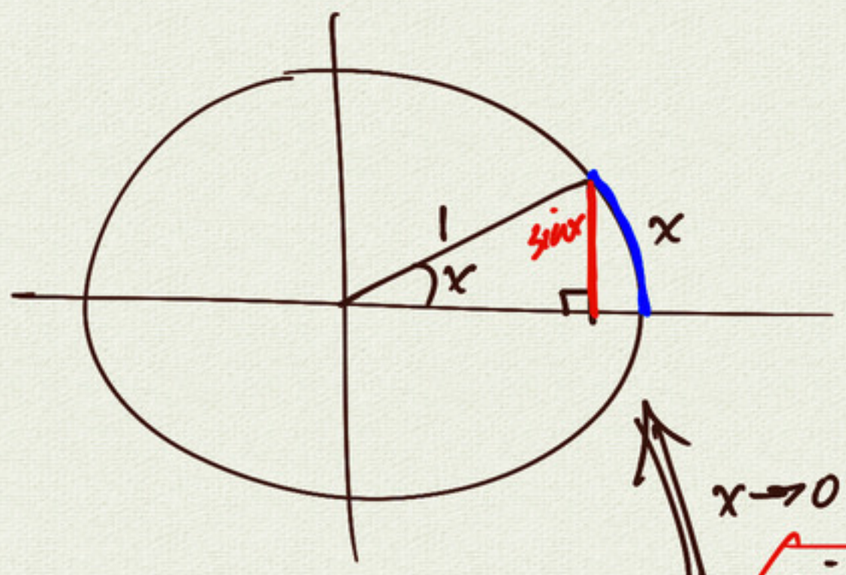
near $x=0$?

$$\lim_{x\to 0} \frac{\sin x}{x} = \frac{0}{0} = ?$$

removable discont.

special limit: $\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$

why?

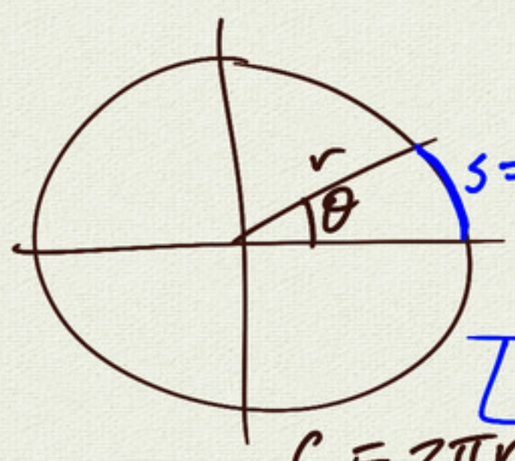


$x\to 0$

$\sin x \approx x$

$\frac{\sin x}{x} \approx 1$

$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$



$s = C\cdot\frac{\theta}{2\pi}$

$= \frac{2\pi r\cdot\theta}{2\pi}$

$\boxed{s = r\theta}$

$C = 2\pi r$