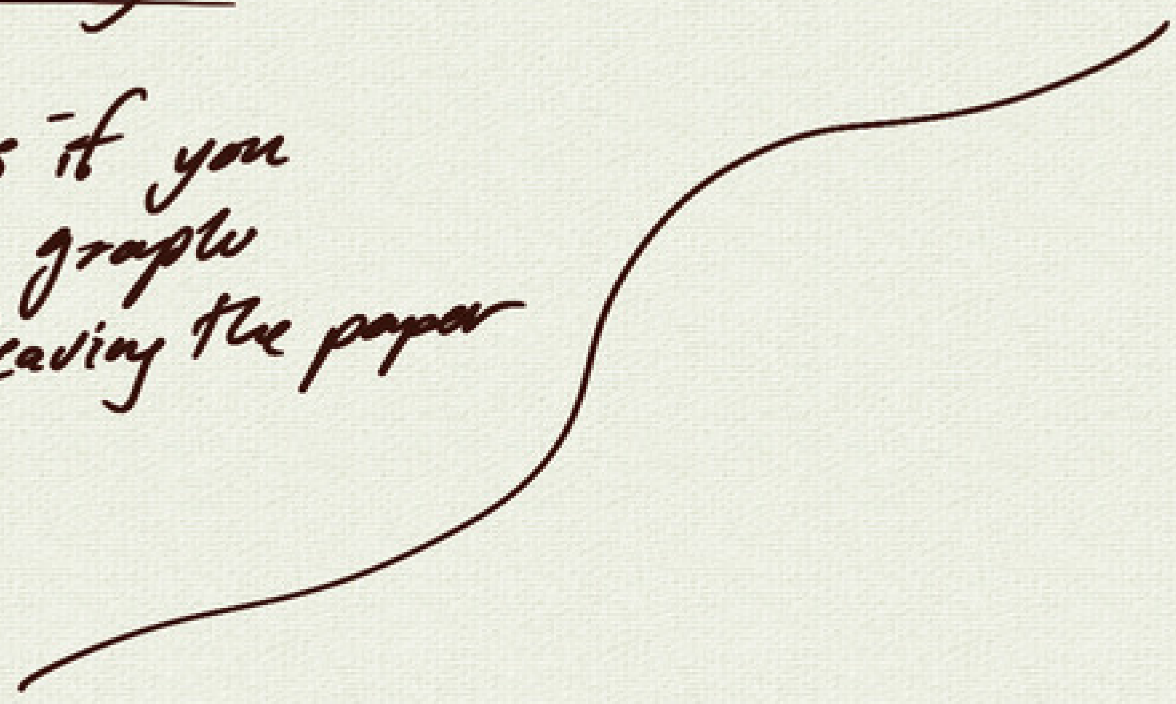
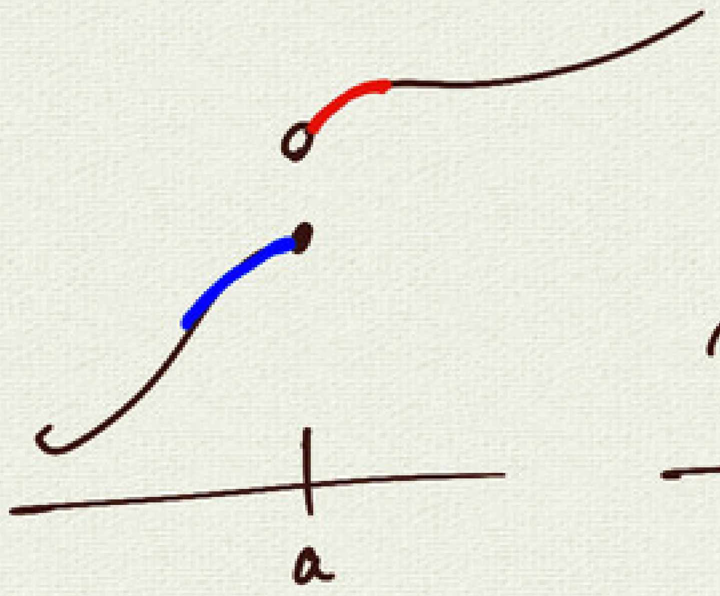
8.2 Continuity

f is continuous if you can draw its graph without leaving the paper



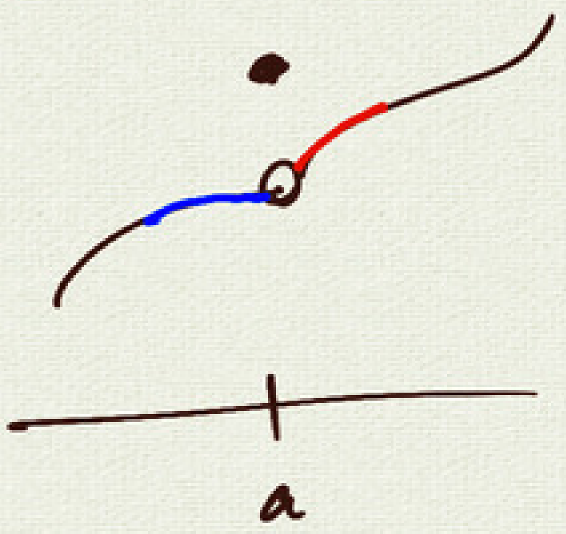
discontinuities:

jump



$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$

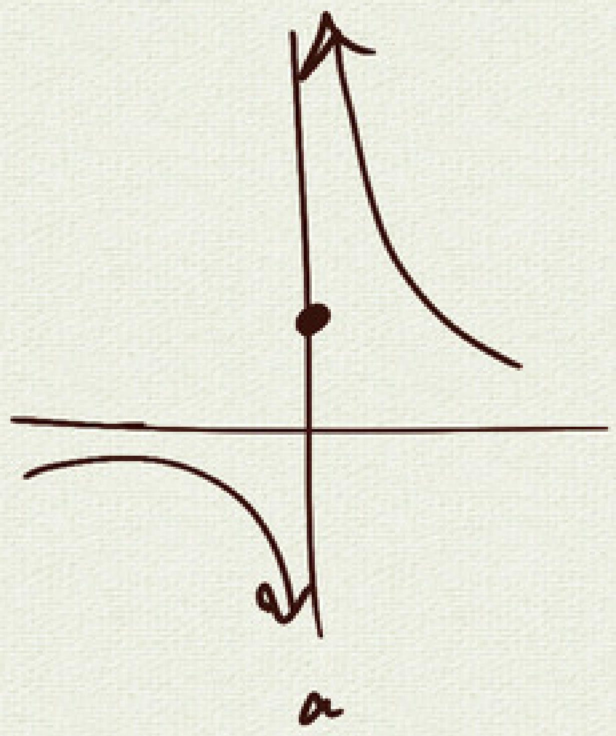
removable



$\lim_{x \to a} f(x)$ exists

but $\neq f(a)$

infinite



$\lim_{x \to a} f(x)$ does not exist

f is continuous at $x = a$ if

1. $\lim_{x \to a} f(x)$ exists
2. $f(a)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

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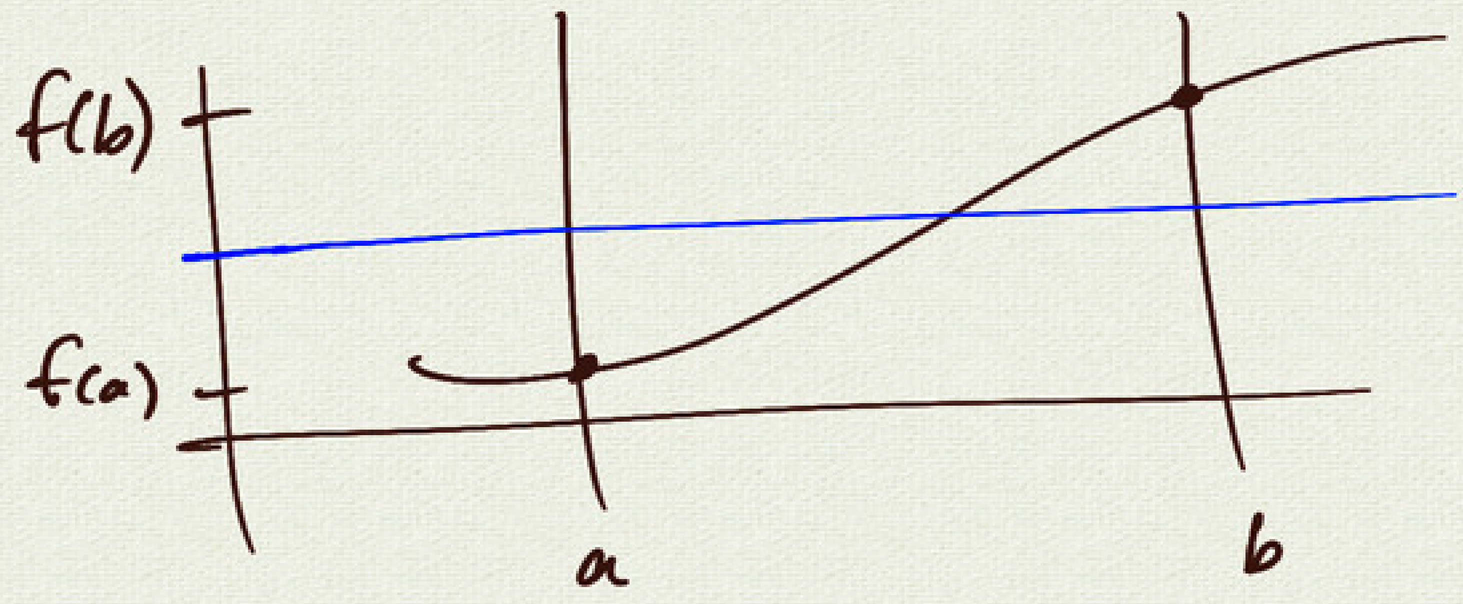
consequence: Intermediate Value Theorem

if f is continuous on $[a, b]$

then f takes on all values between $f(a)$ and $f(b)$



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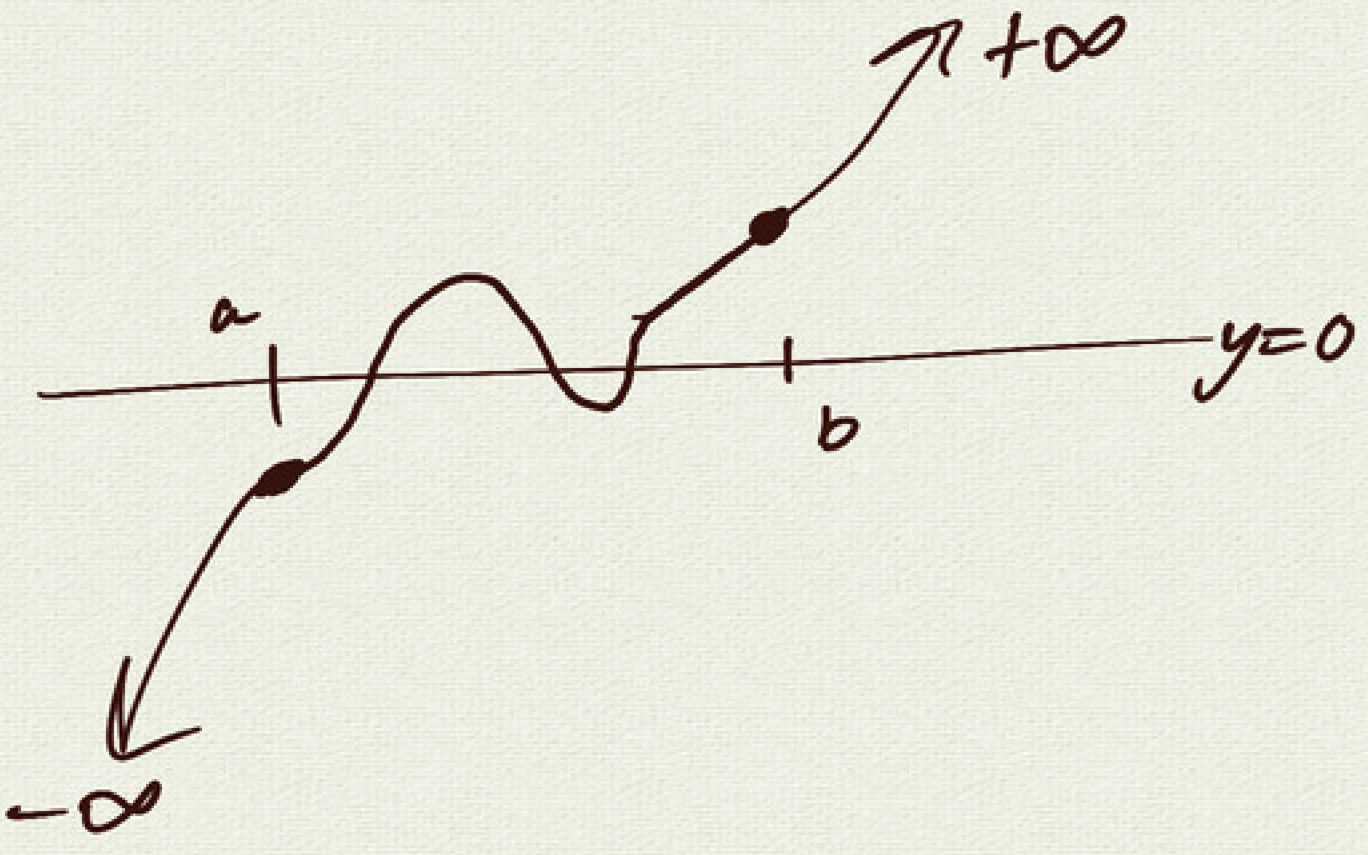
example: odd degree polynomials must have a root

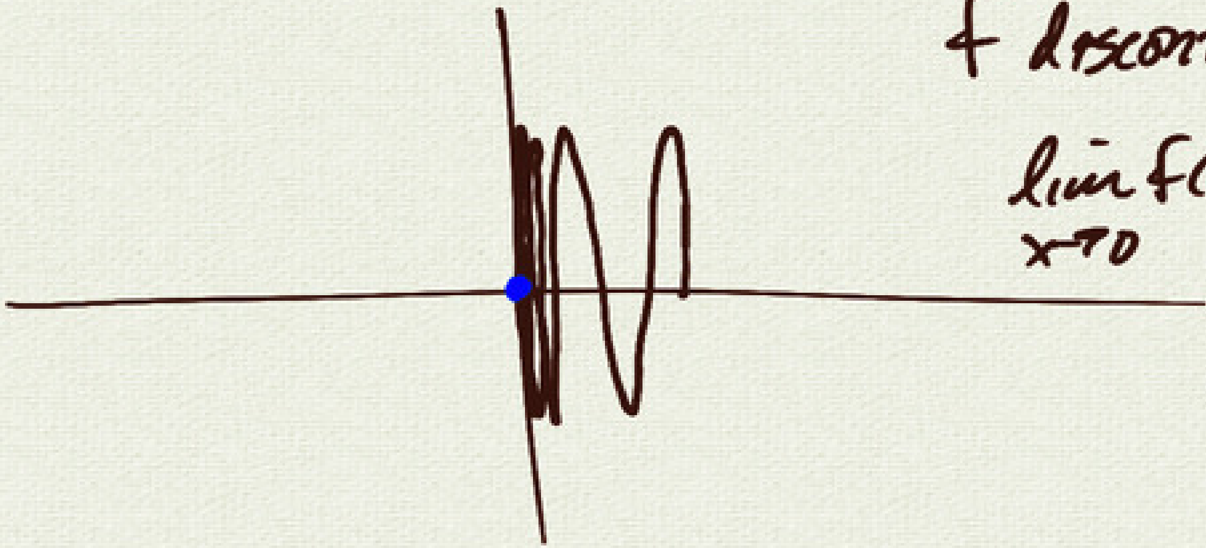


weird example:

$$f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

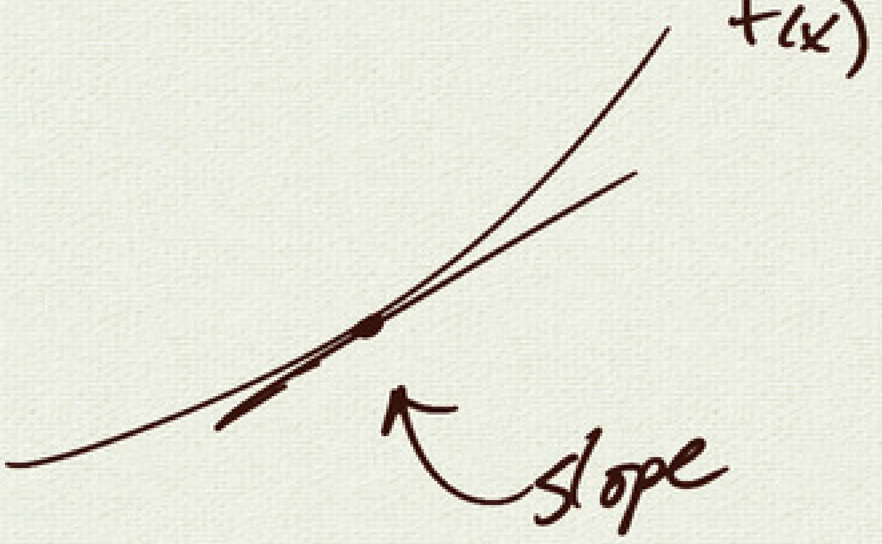
what happens as $x \to 0$?

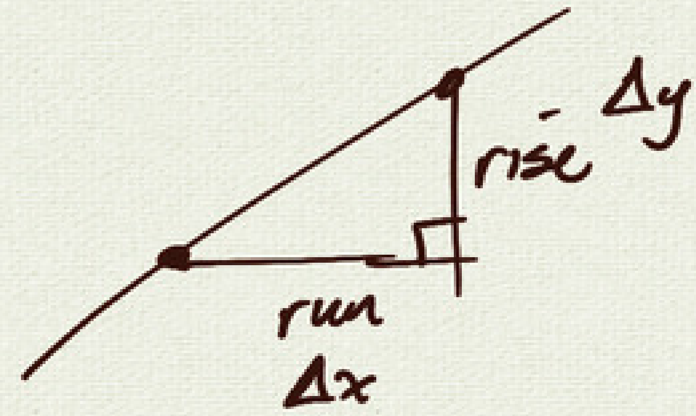
$\frac{1}{x} \to \infty$



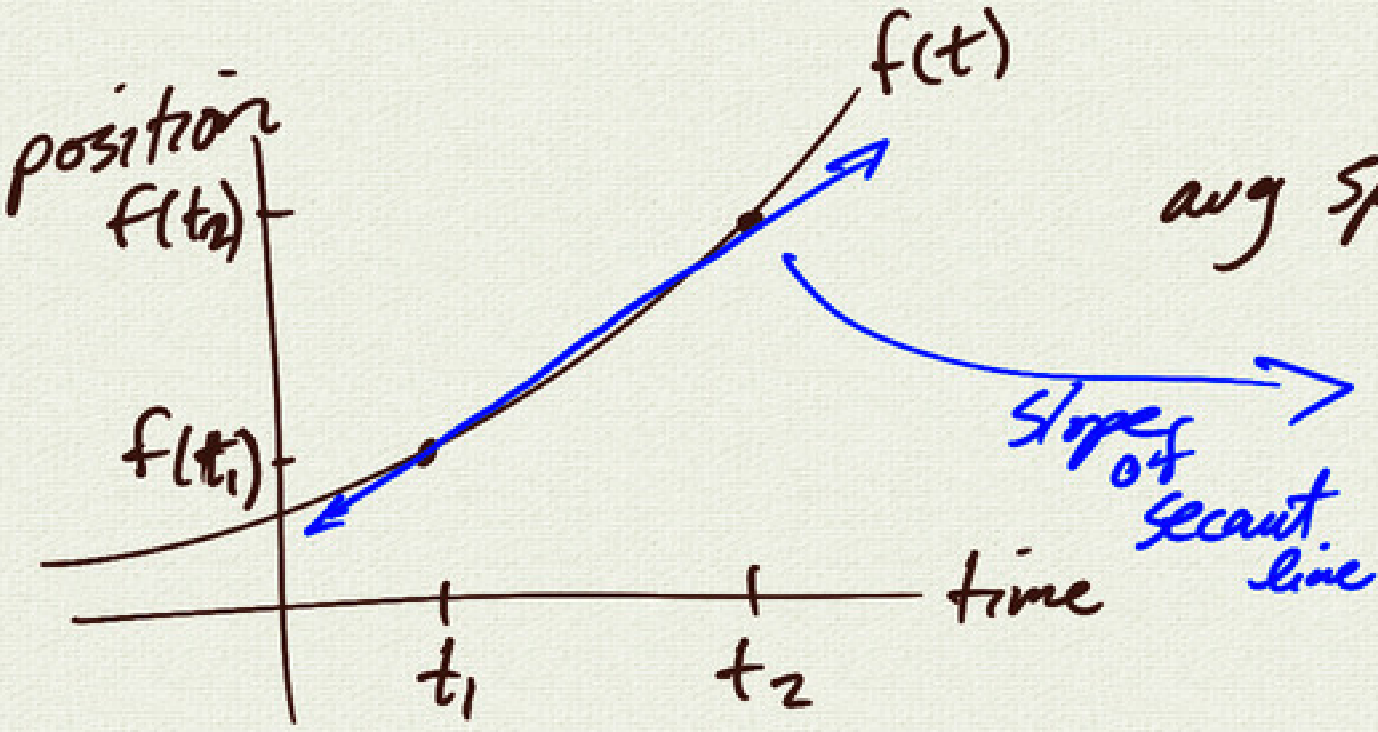
f discontinuous at $x=0$

$\lim_{x \to 0} f(x)$ does not exist



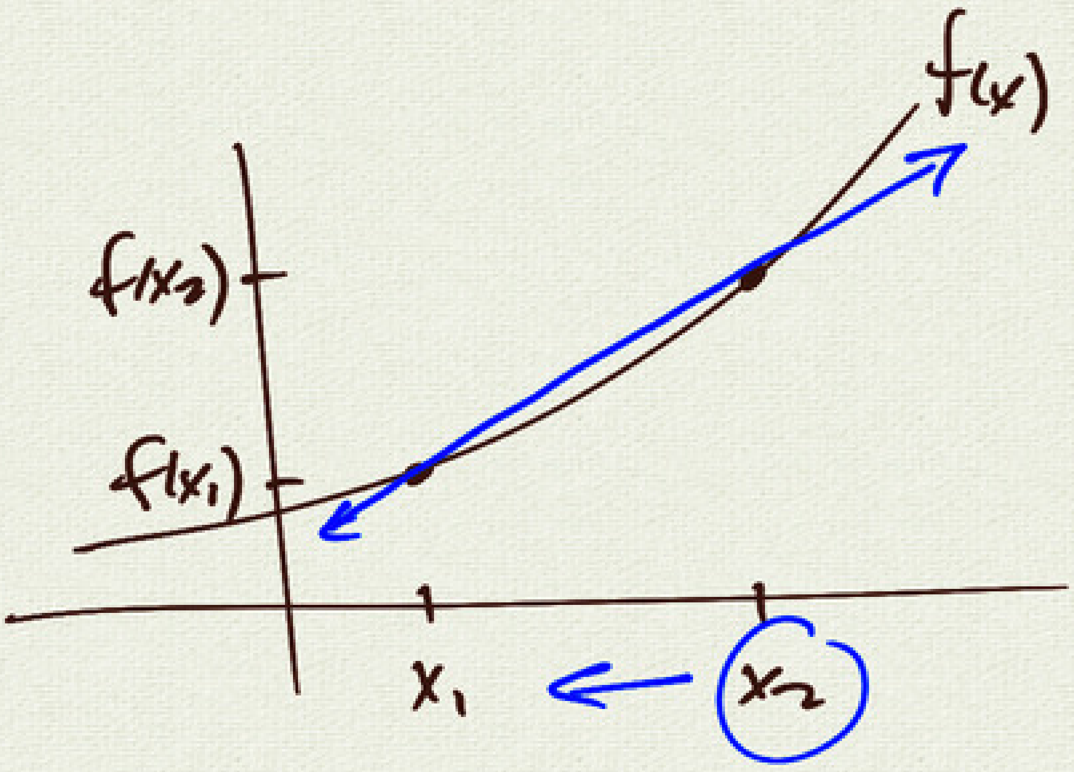


$$\text{slope} = \frac{\Delta y}{\Delta x}$$



$$\text{avg speed} = \frac{\Delta \text{position}}{\Delta \text{time}}$$

$$= \frac{f(t_2) - f(t_1)}{t_2 - t_1}$$



avg. rate of change

$$= \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

= slope of secant line