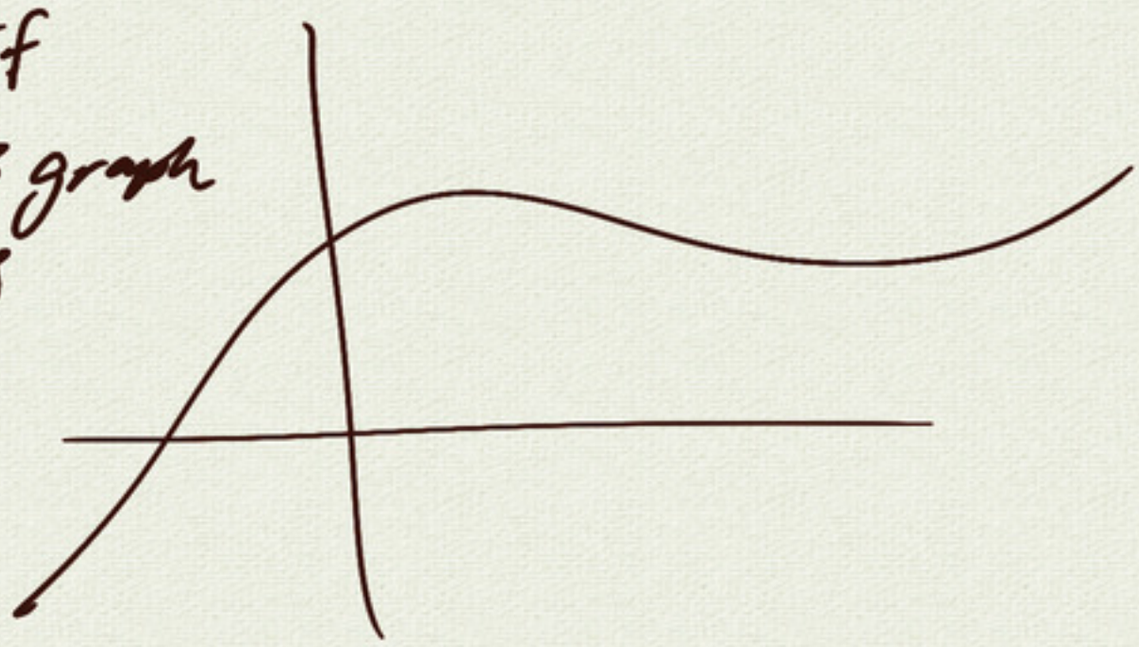
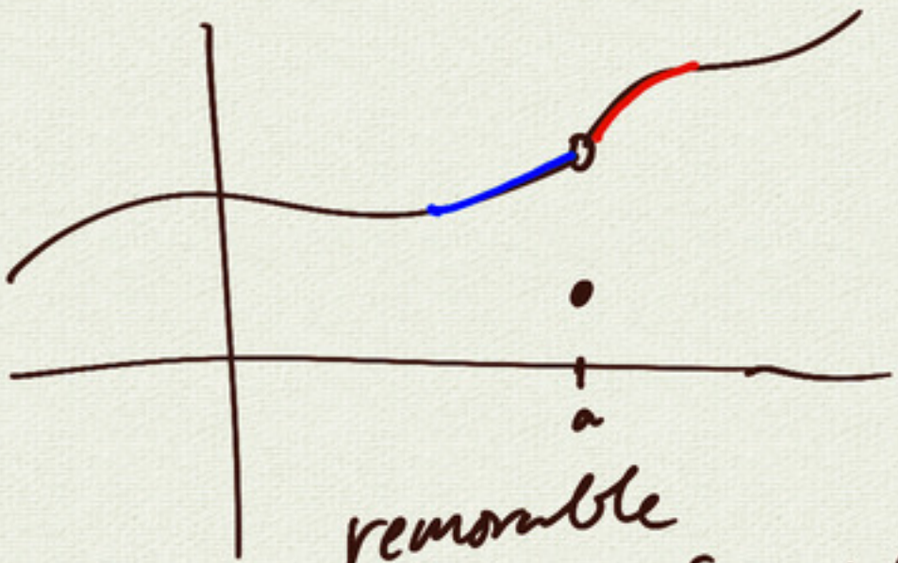
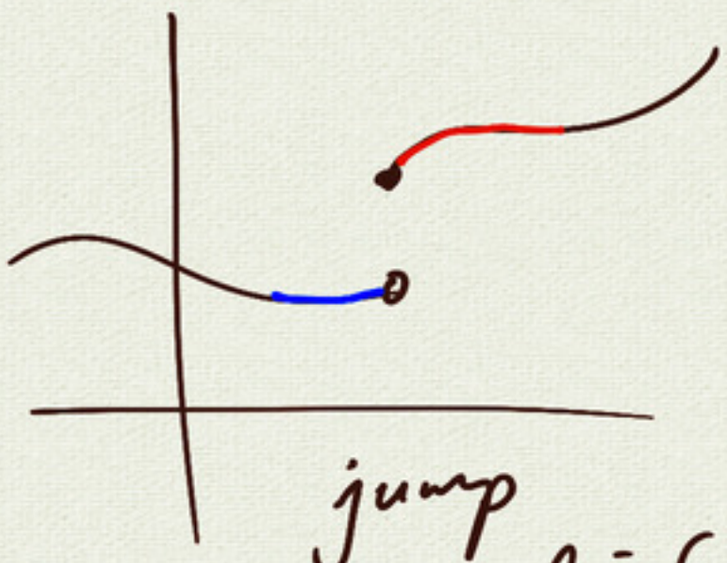
8.2 Continuity

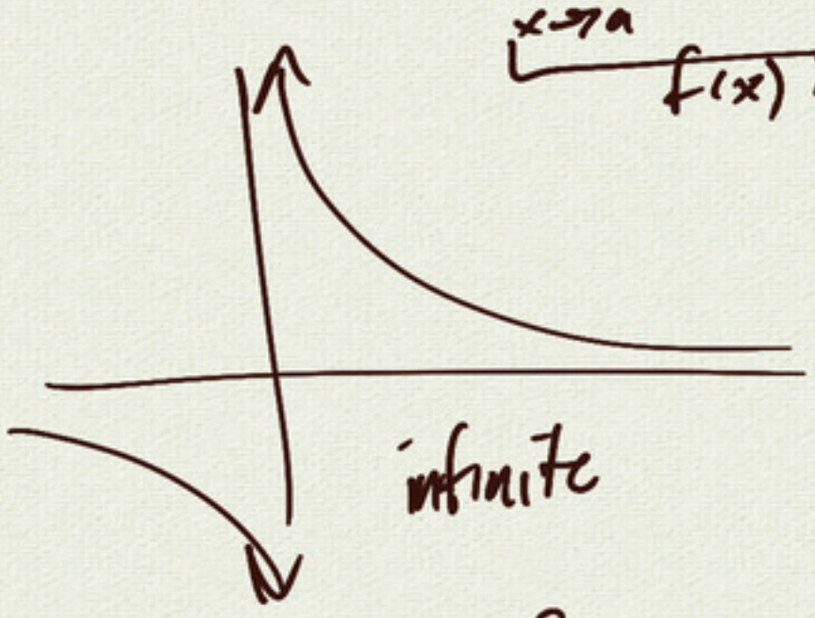
f is continuous if we can draw its graph without lifting our pencil



discontinuities:







definition: f is continuous at $x = a$

(1) $\lim_{x \to a} f(x)$ exists (left limit = right limit)

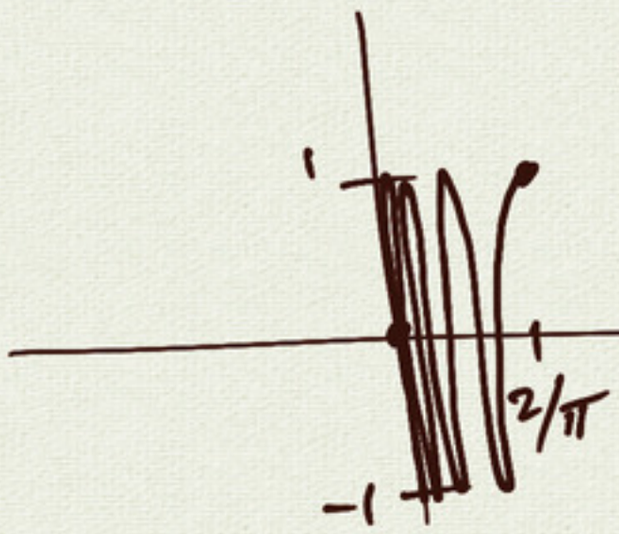
(2) $f(a)$ exists

(3) $\lim_{x \to a} f(x) = f(a)$

example:

$$f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

at $x = 0$



at $x = \frac{2}{\pi}$:

$$f\left(\frac{2}{\pi}\right) = \sin\frac{\pi}{2} = 1$$

as $x \to 0$

$\frac{1}{x} \to \infty$

$\lim_{x \to 0} f(x)$ does not exist

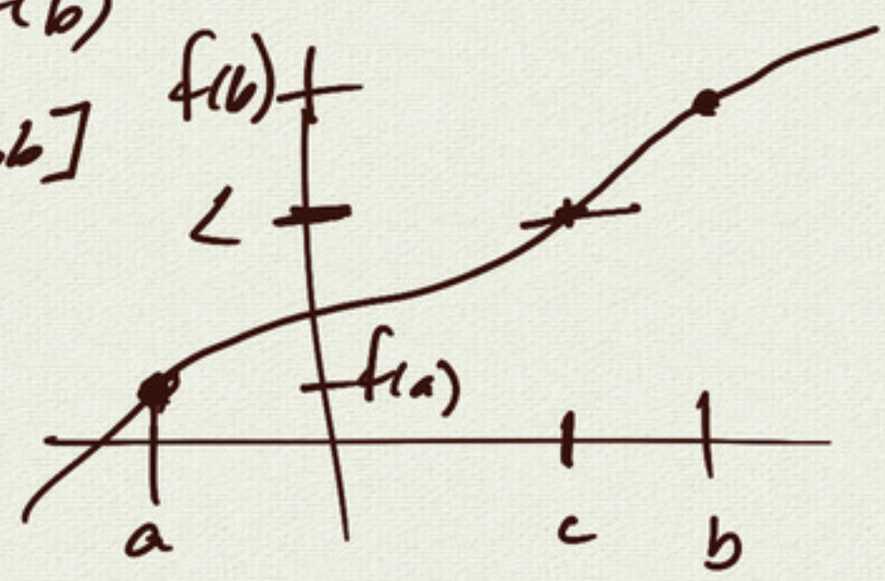
Intermediate Value Theorem

if f is continuous on $[a, b]$

and $f(a) \leq L \leq f(b)$

then there is a $c \in [a, b]$

such that $f(c) = L$



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f hits every intermediate value between $f(a)$ and $f(b)$

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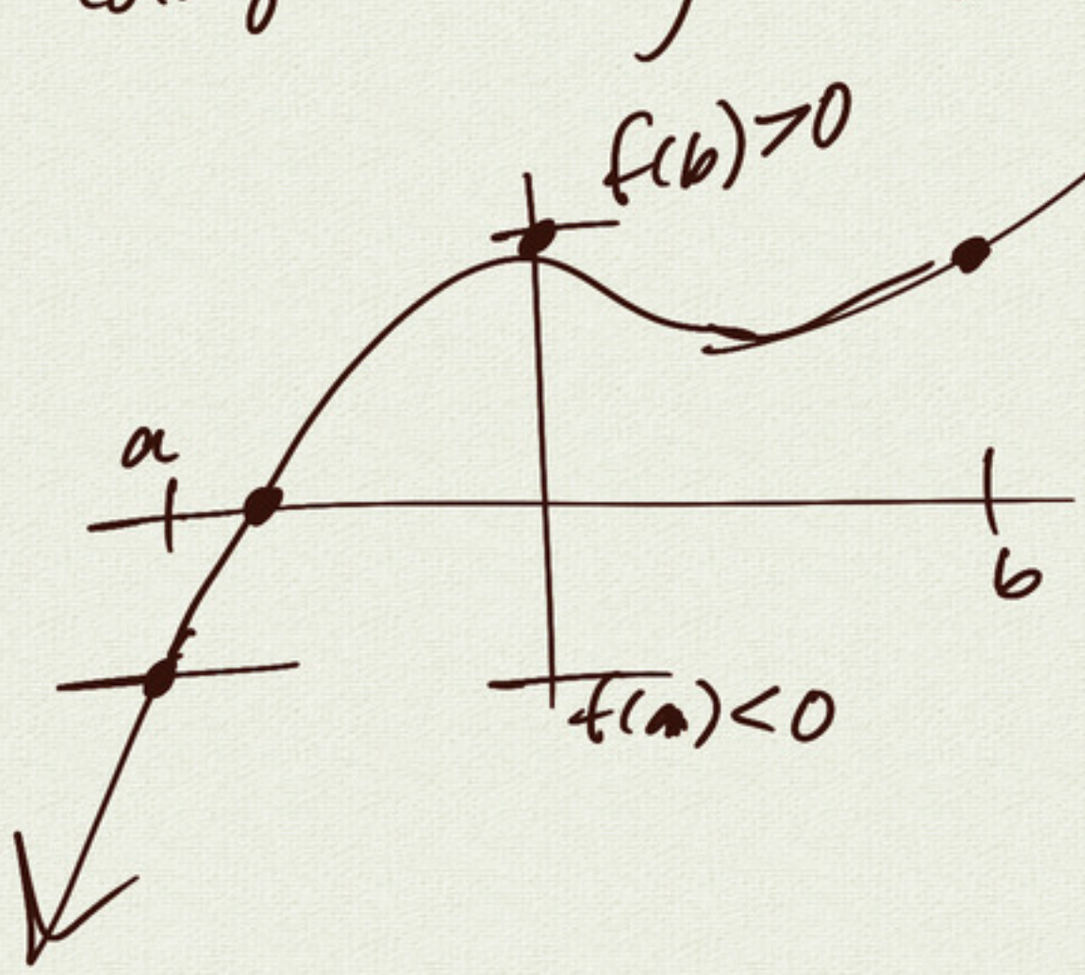
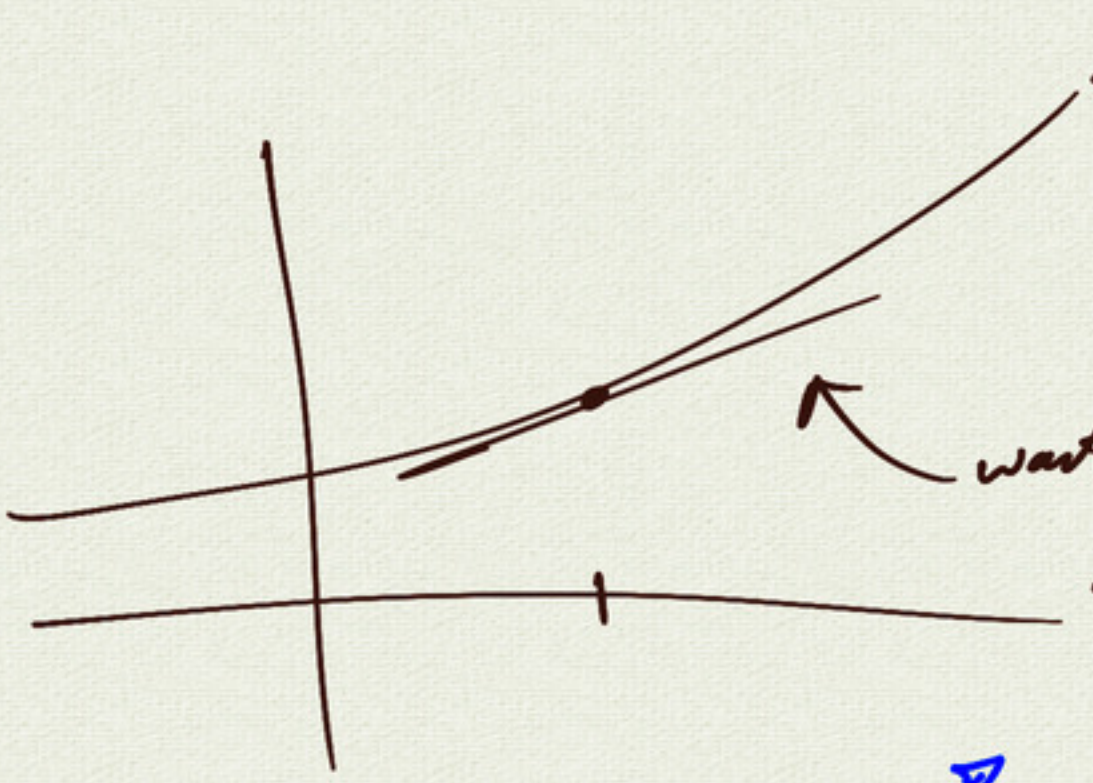
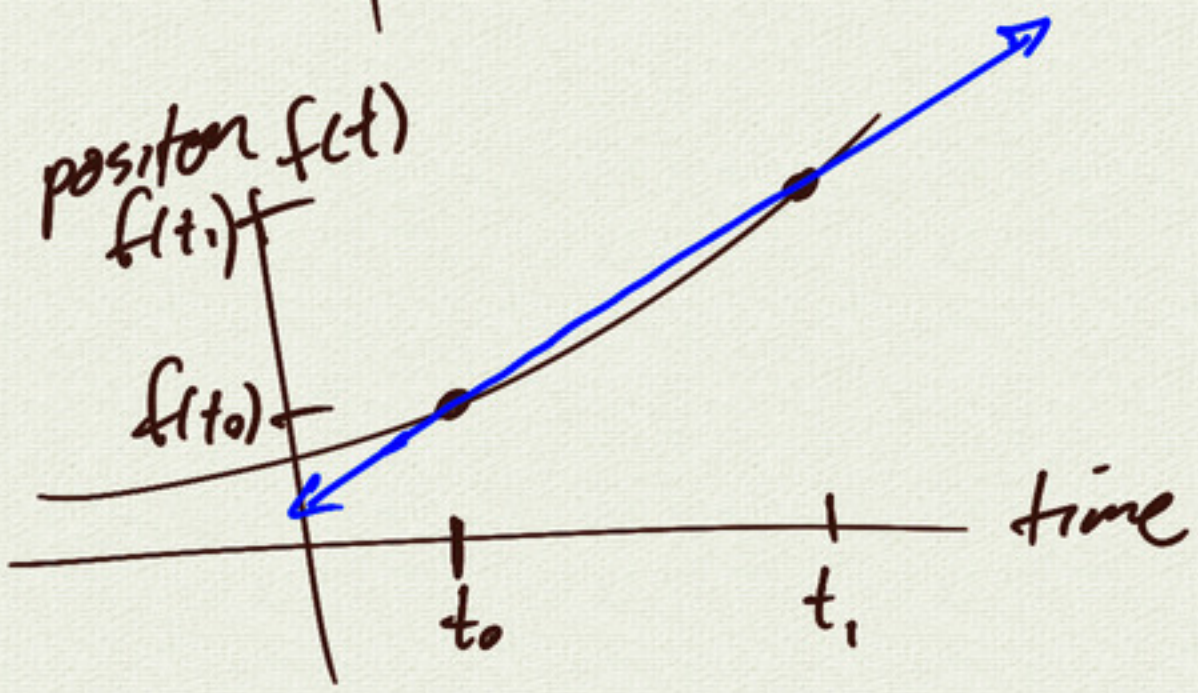
consequence: every odd degree polynomial has a zero

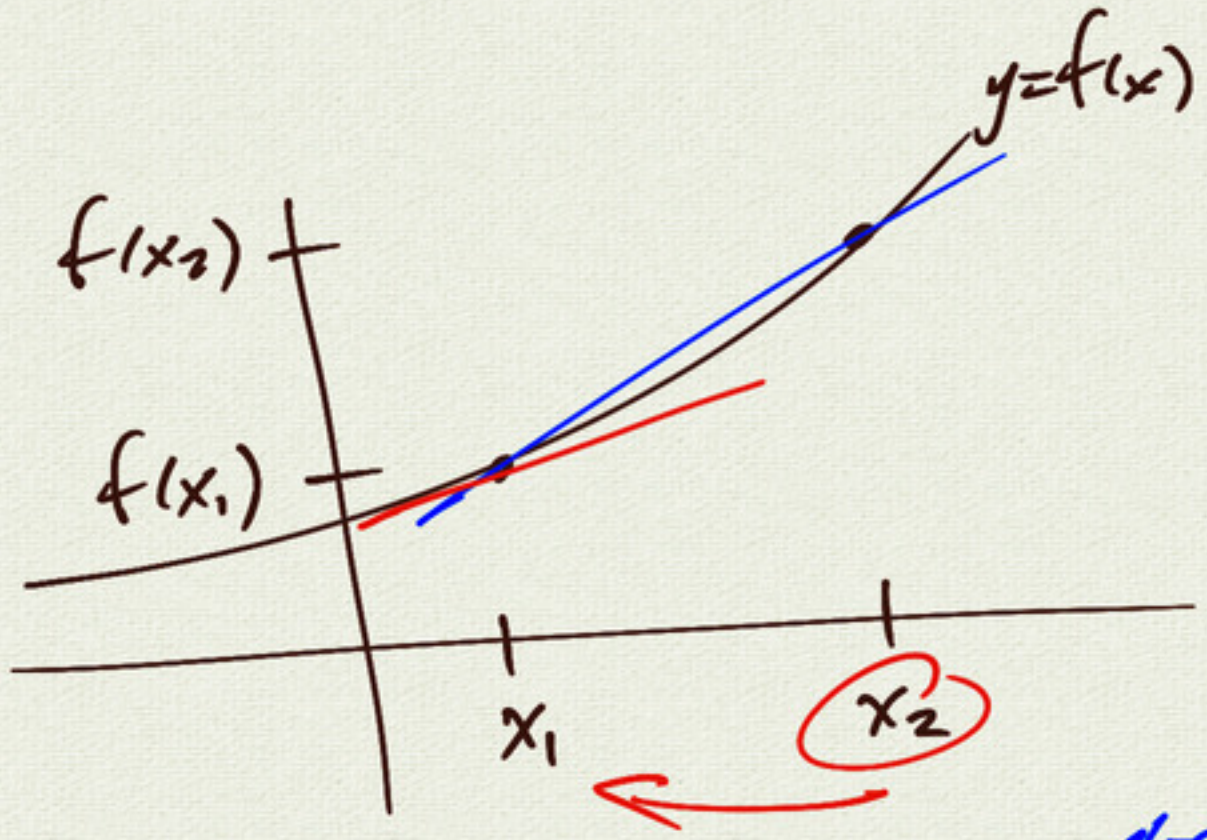




want to define the slope of a function



average speed
= slope of secant line
$= \frac{f(t_1)-f(t_0)}{t_1-t_0}$



average rate of change
= slope of secant line
$= \frac{f(x_2)-f(x_1)}{x_2-x_1}$