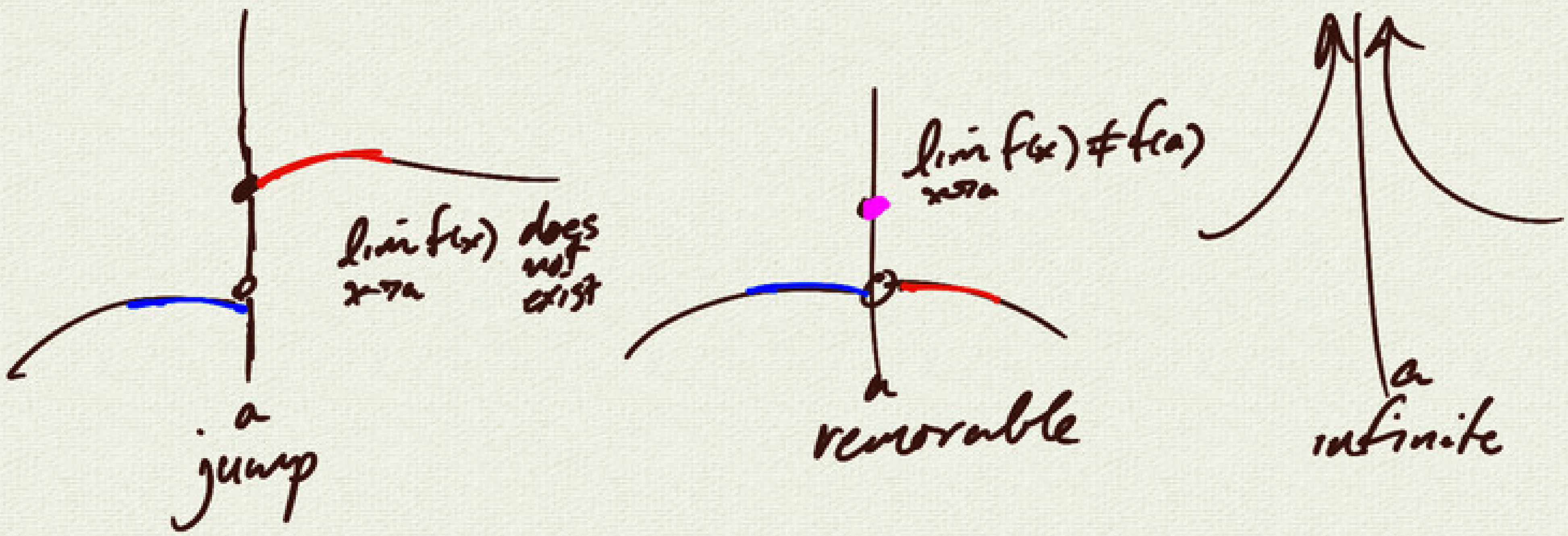
8.3 The Derivative

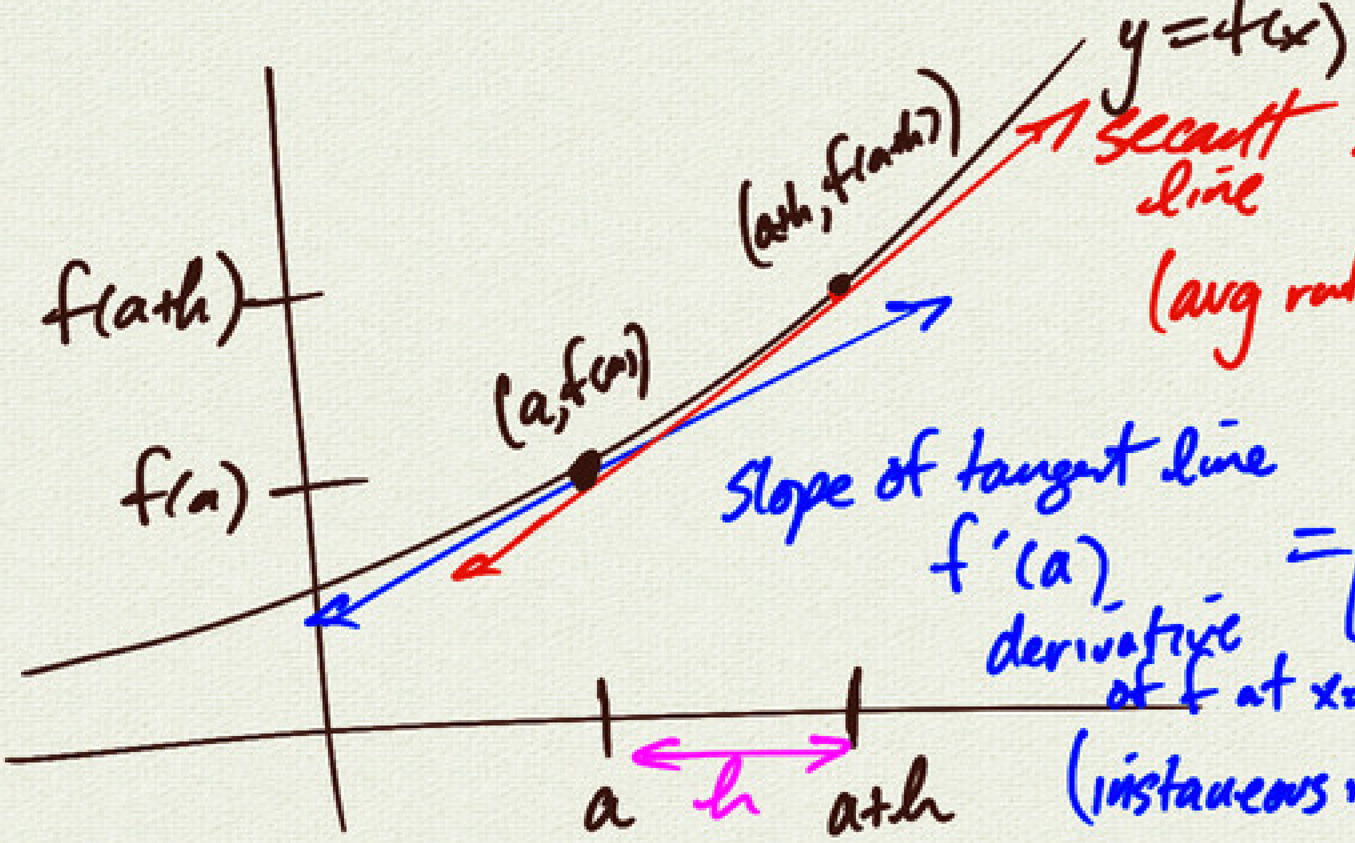


f is <u>continuous</u> at a if

① $\lim_{x\to a} f(x)$ exists

② $f(a)$ exists

③ $\boxed{\lim_{x\to a} f(x) = f(a)}$

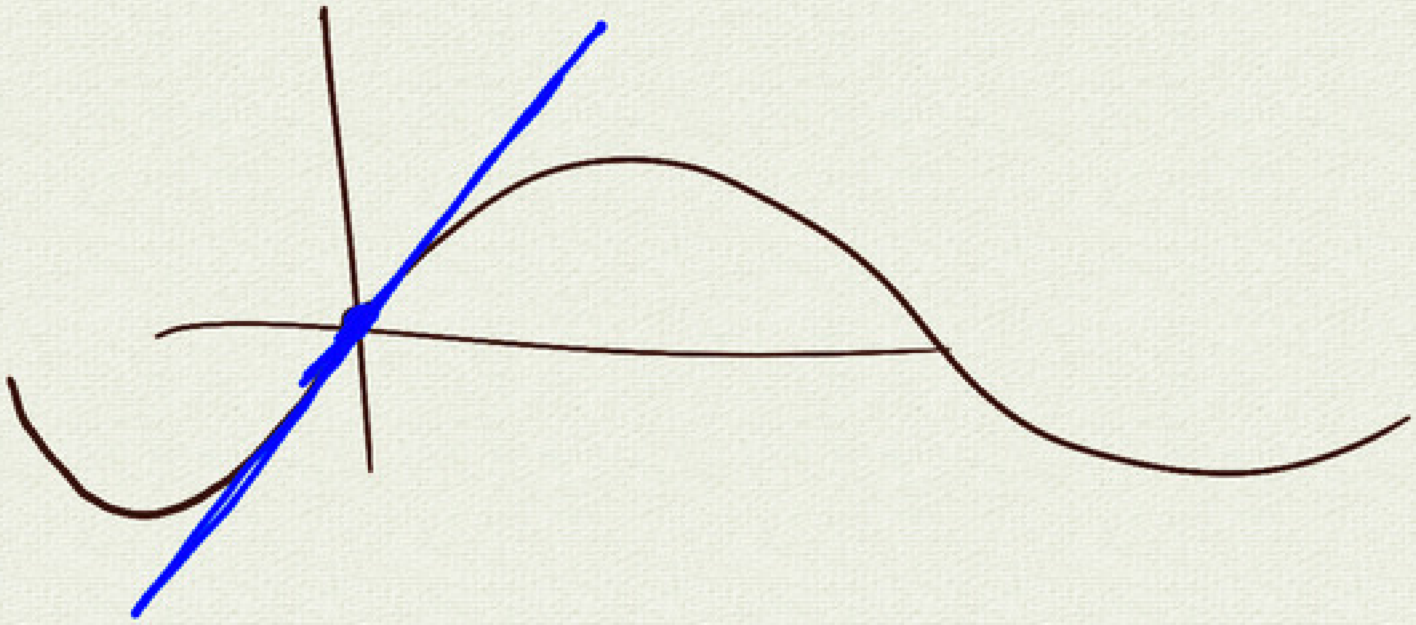


secant line slope $\dfrac{f(a+h)-f(a)}{(a+h)-a}$ h

(avg rate of change)

slope of tangent line $f'(a)$ derivative of f at x=a $= \boxed{\lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}}$

(instantaneous rate of change)

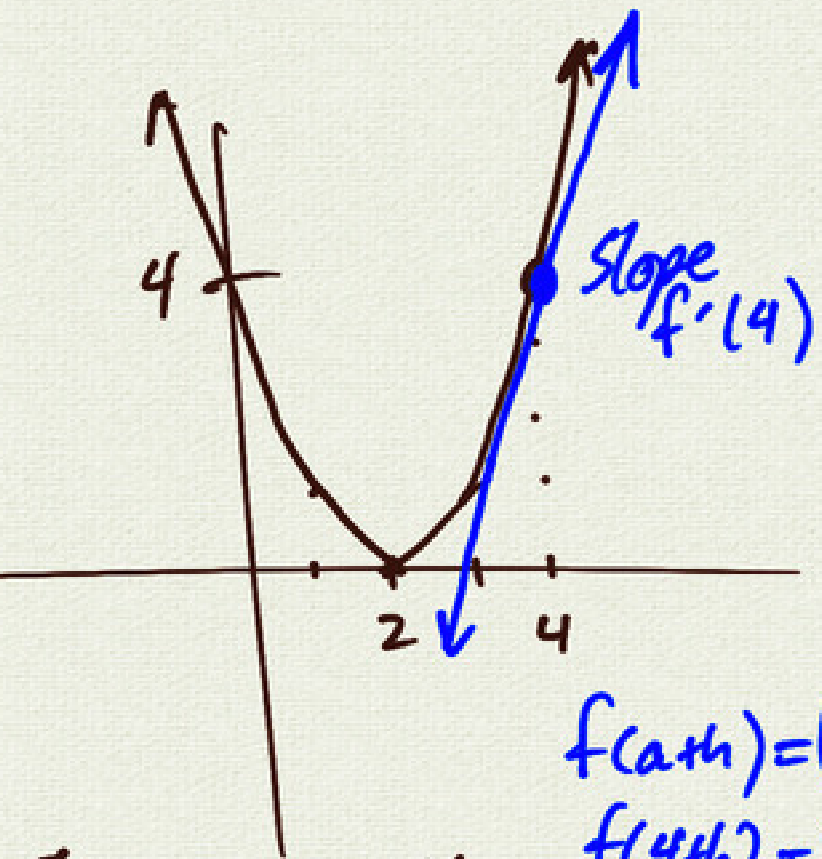


example $f(x) = (x-2)^2$

$= \boxed{x^2-4x+4}$

$a = 4$

Find $f'(4)$



$f(a+h) = (a+h)^2 - 4(a+h) + 4$

$f(4+h) = (4+h)^2 - 4(4+h) + 4$

$$f'(4) = \lim_{h\to 0} \boxed{\frac{f(a+h)-f(a)}{h}}$$

$$= \lim_{h\to 0} \frac{\left[(4+h)^2 - 4(4+h) + 4\right] - \left[4^2 - 4\cdot 4 + 4\right]}{h}$$

$$= \lim_{h\to 0} \frac{\left[4^2 + 8h + h^2 - 16 - 4h + 4\right] - 4}{h}$$

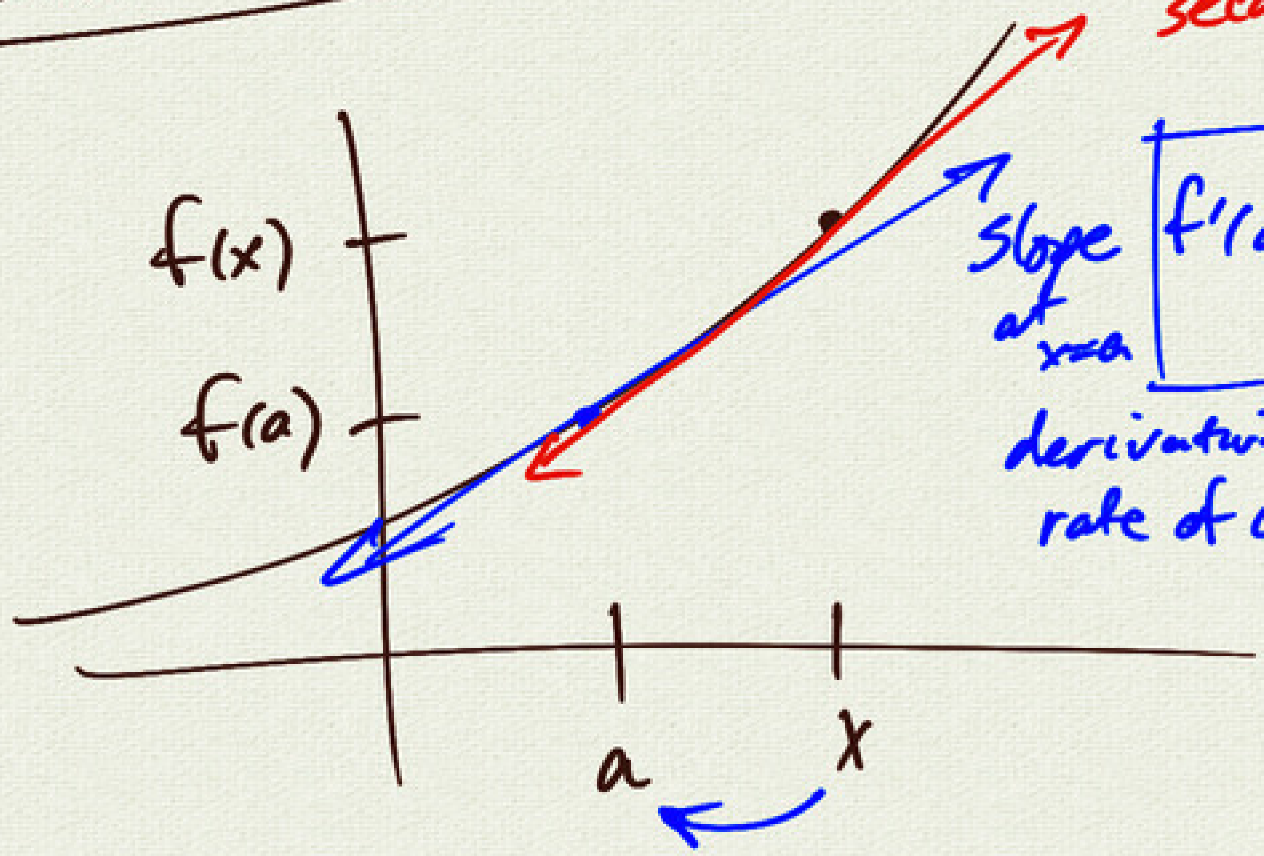
$$= \lim_{h\to 0} \frac{h^2 + 4h}{h}$$

$$= \lim_{h\to 0} \frac{h(h+4)}{h}$$

$$= \lim_{h\to 0} (\underbrace{h}_{0} + 4)$$

$$= 4$$

alternate definition



slope of secant $\frac{f(x)-f(a)}{x-a}$

slope at $x=a$ $\boxed{f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}}$

derivative of f at a

rate of change of f at a

same example: $f(x) = (x-2)^2$

$a = 4$

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x \to 4} \frac{f(x)-f(4)}{x-4}$$

$$= \lim_{x \to 4} \frac{(x-2)^2-(4-2)^2}{x-4}$$

$$= \lim_{x \to 4} \frac{x^2-4x+4-4}{x-4}$$

$$= \lim_{x \to 4} \frac{x^2-4x}{x-4}$$

$$= \lim_{x \to 4} \frac{x(x-4)}{(x-4)}$$

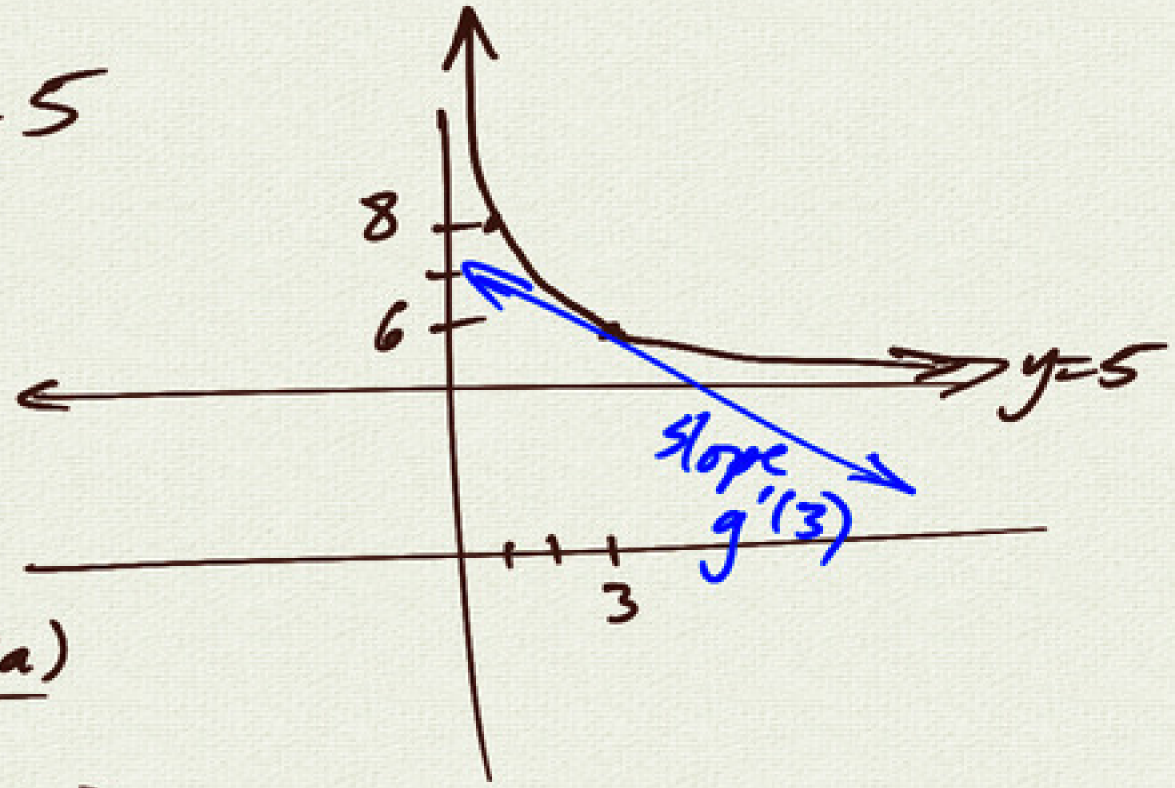
$$= \lim_{x \to 4} x$$

$$= 4$$

example: $g(x) = \frac{3}{x} + 5$

$a = 3$

find $g'(3)$



$$g'(3) = \lim_{h \to 0} \frac{g(a+h) - g(a)}{h}$$

$$= \lim_{h \to 0} \frac{g(3+h) - g(3)}{h}$$

$$= \lim_{h \to 0} \left(\left[\frac{3}{3+h} + 5\right] - \left[\frac{3}{3} + 5\right] \right) \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{3}{3+h} - 1 \right] \quad \longleftarrow \text{common denominator}$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{3 - (3+h)}{3+h} \right]$$

$$= \lim_{h \to 0} \frac{-1}{3+h}$$

$$= -\frac{1}{3}$$

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alt def:

$$g'(3) = \lim_{x \to 3} \frac{g(x) - g(3)}{x - 3}$$

$$g'(a) = \lim_{x \to a} \frac{g(x) - g(a)}{x - a}$$

$$= \lim_{x \to 3} \frac{\left(\frac{3}{x} + \underline{5}\right) - \left(\frac{3}{3} + \underline{5}\right)}{x - 3}$$

$$= \lim_{x \to 3} \frac{1}{x-3} \left(\frac{3}{x} - 1 \right)$$

$$= \lim_{x \to 3} \frac{1}{x-3} \left(\frac{3-x}{x} \right)$$

$$3 - x = -(x-3)$$

$$= \lim_{x \to 3} -\frac{1}{x}$$

$$= -\frac{1}{3}$$

$f'(a)$ derivative of f at a

$f'(x)$ function

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$