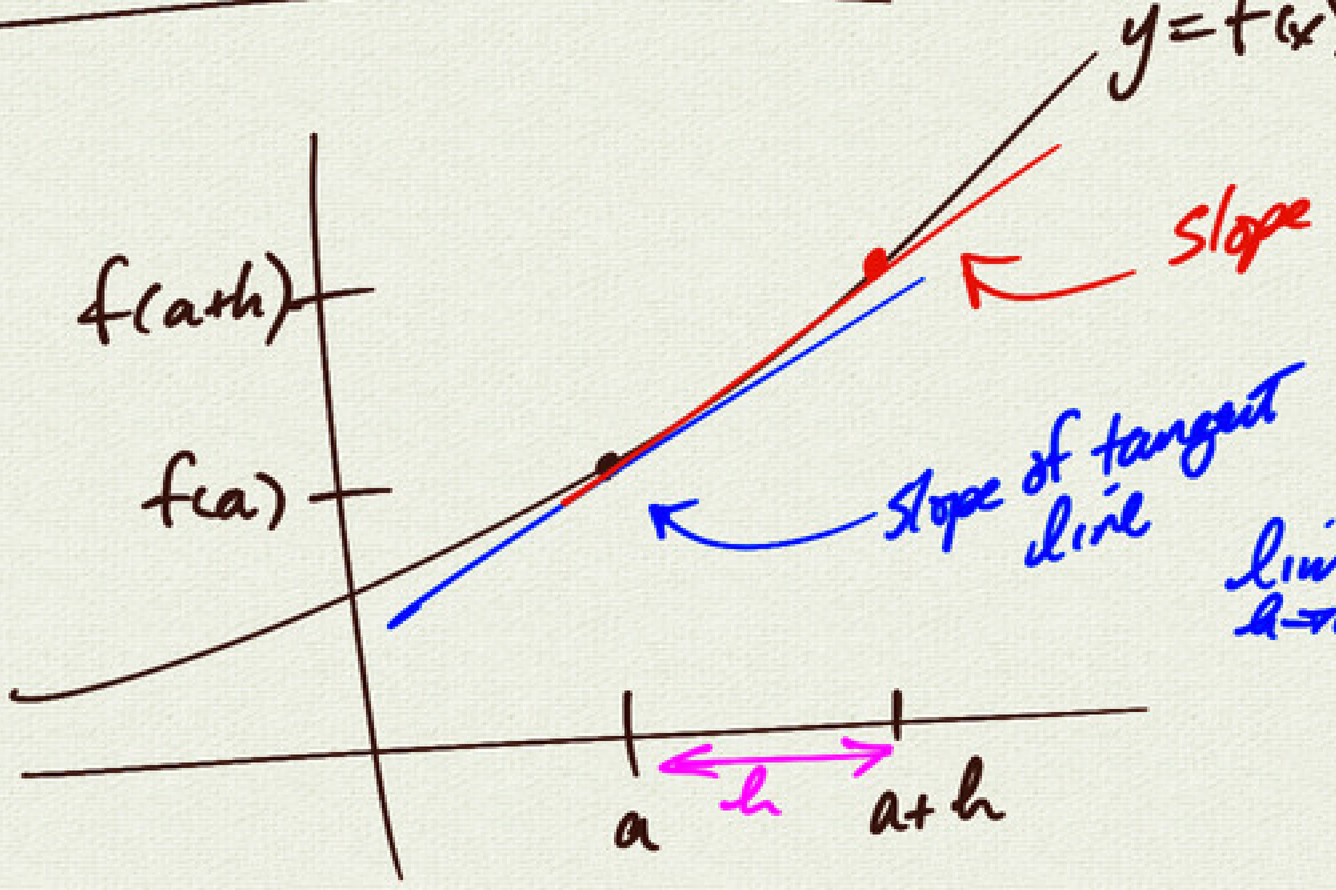
8.3 The Derivative



$$\frac{f(a+h)-f(a)}{\underbrace{a+h-a}_{h}}$$ average rate of change

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = f'(a)$$

derivative of f at a

instantaneous rate of change

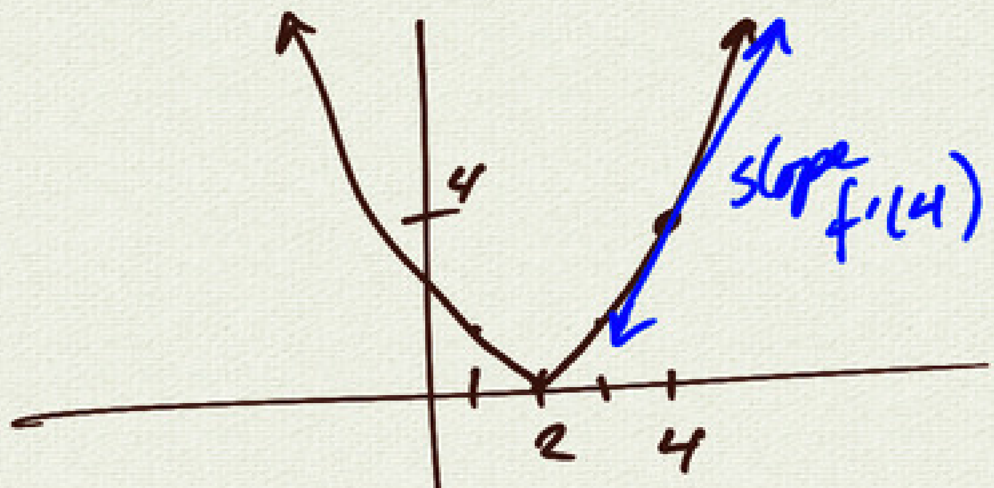
example:

$f(x) = x^2 - 4x + 4$

$= (x-2)^2$

let $a = 4$

find $f'(a) = f'(4)$



$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$$f'(4) = \lim_{h\to 0}\frac{f(4+h)-f(4)}{h} \to \frac{0}{0}$$

$$= \lim_{h\to 0}\frac{(4+h)^2 - 4(4+h) + 4 - (4^2 - 4\cdot 4 + 4)}{h}$$

$$= \lim_{h\to 0}\frac{(16+8h+h^2) - 16 - 4h + 4 - 4}{h}$$

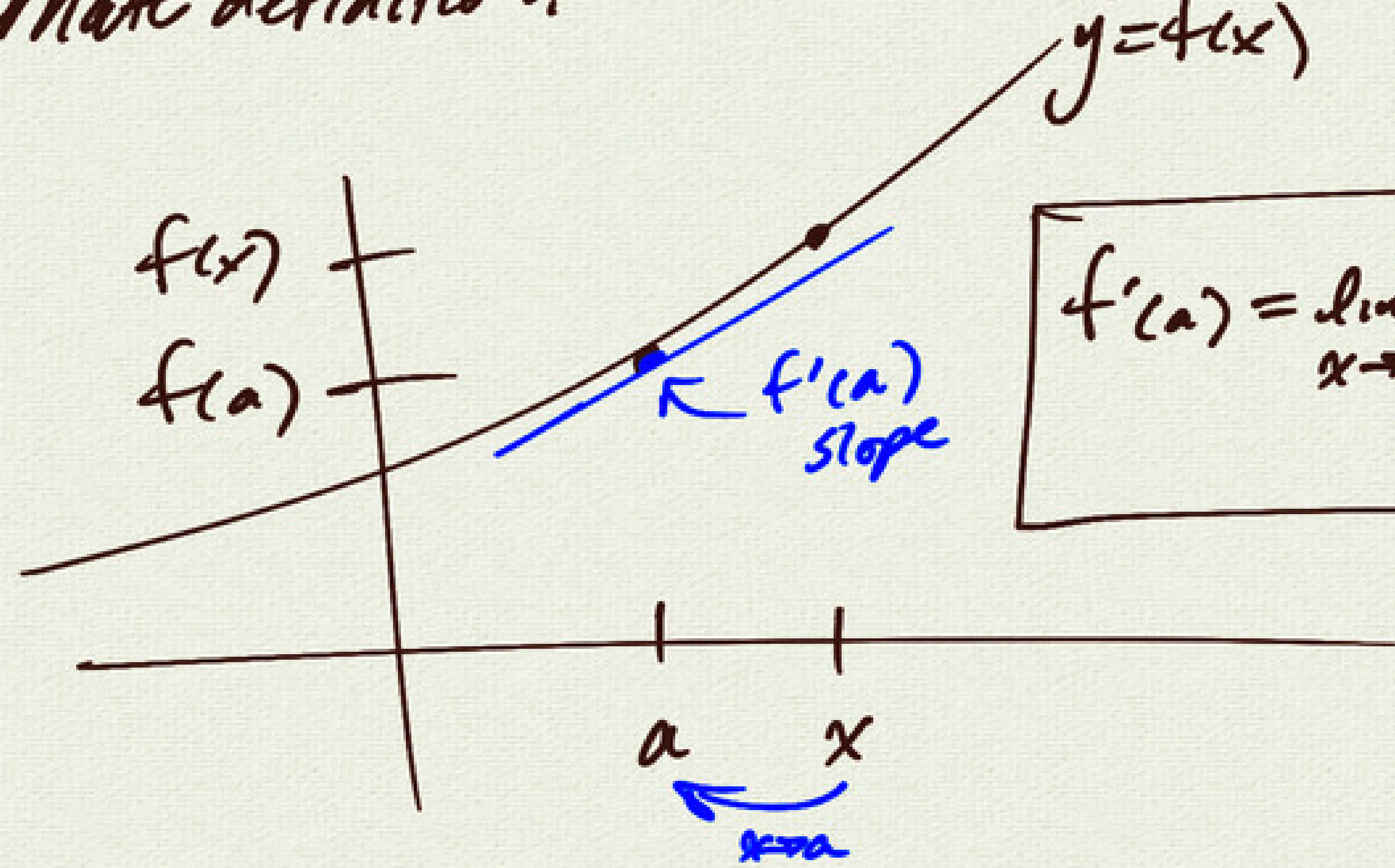
$$= \lim_{h\to 0}\frac{h^2+4h}{h}$$

$$= \lim_{h\to 0}\frac{h(h+4)}{h}$$

$$= \lim_{h\to 0}(h+4)$$

$$= 4$$

alternate definition:



$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

example: $f(x) = (x-2)^2 = x^2 - 4x + 4$

$a = 4$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$a=4$

$$= \lim_{x \to a} \frac{(x^2 - 4x + 4) - (a^2 - 4a + 4)}{x - a}$$

$$= \lim_{x \to a} \frac{x^2 - a^2 - 4x + 4a}{x - a}$$

$$= \lim_{x \to 4} \frac{x^2 - 16 - 4x + 16}{x - 4}$$

$$= \lim_{x \to 4} \frac{x^2 - 4x}{x - 4}$$

$$= \lim_{x \to 4} \frac{x(x-4)}{x-4}$$

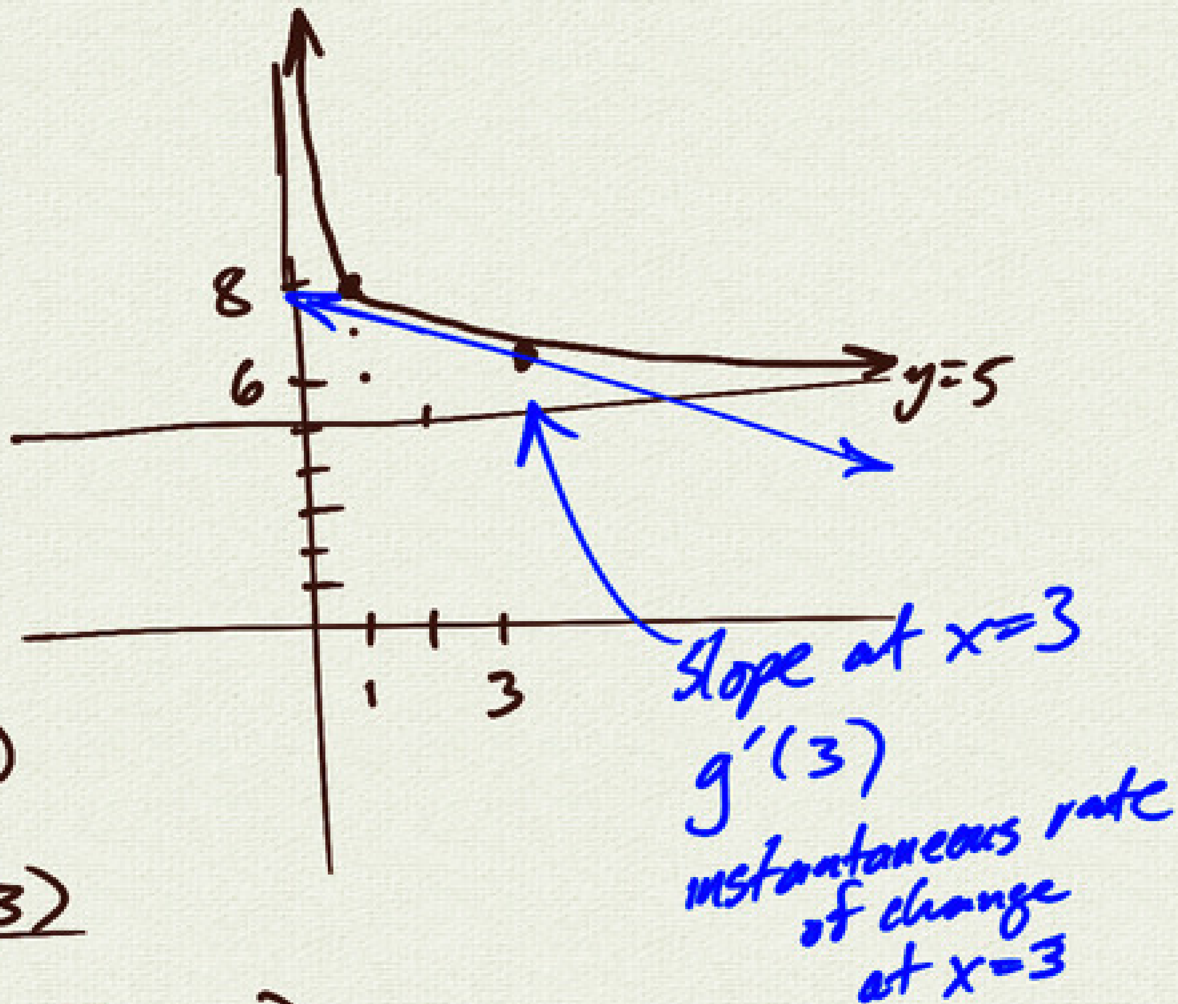
$$= \lim_{x \to 4} x$$

$$= 4$$

Example 2

$$g(x) = \frac{3}{x} + 5$$

find $g'(3)$



$$g'(3) = \lim_{h \to 0} \frac{g(a+h) - g(a)}{h}$$

$$= \lim_{h \to 0} \frac{g(3+h) - g(3)}{h}$$

$$= \lim_{h \to 0} \frac{\left(\frac{3}{3+h} + 5\right) - \left(\frac{3}{3} + 5\right)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{3}{3+h} - 1\right]$$

common denominator

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{3 - (3+h)}{3+h}\right]$$

$$= \lim_{h \to 0} \frac{-h}{h(3+h)}$$

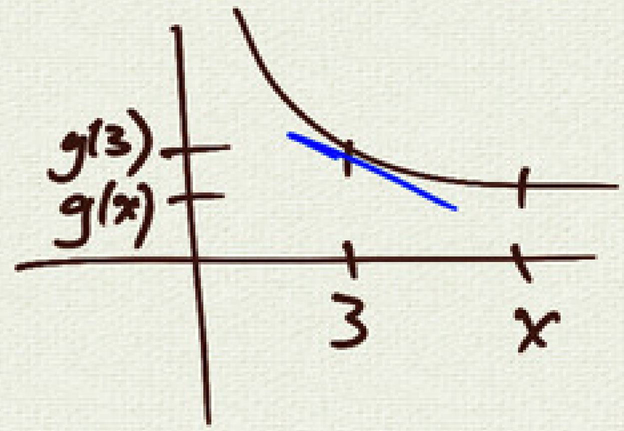
$$= \lim_{h \to 0} -\frac{1}{3+h}$$

$$= -\frac{1}{3}$$

$$g(x) = \frac{3}{x} + 5$$

alternate definition:

$$g'(3) = \lim_{x\to 3} \frac{g(x) - g(3)}{x-3}$$



$$= \lim_{x\to 3} \frac{\left(\frac{3}{x} + 5\right) - \left(\frac{3}{3} + 5\right)}{x-3}$$

$$= \lim_{x\to 3} \frac{1}{x-3}\left[\frac{3}{x} - 1\right]$$

$$= \lim_{x\to 3} \frac{1}{x-3}\left[\frac{3-x}{x}\right]$$

$$3 - x = -(x-3)$$

$$= \lim_{x\to 3} -\frac{1}{x}$$

$$= -\frac{1}{3}$$

$f'(a) =$ derivative of f at $x=a$

slope

rate of change

$f'(x)$ derivative of f (at x)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

function definition