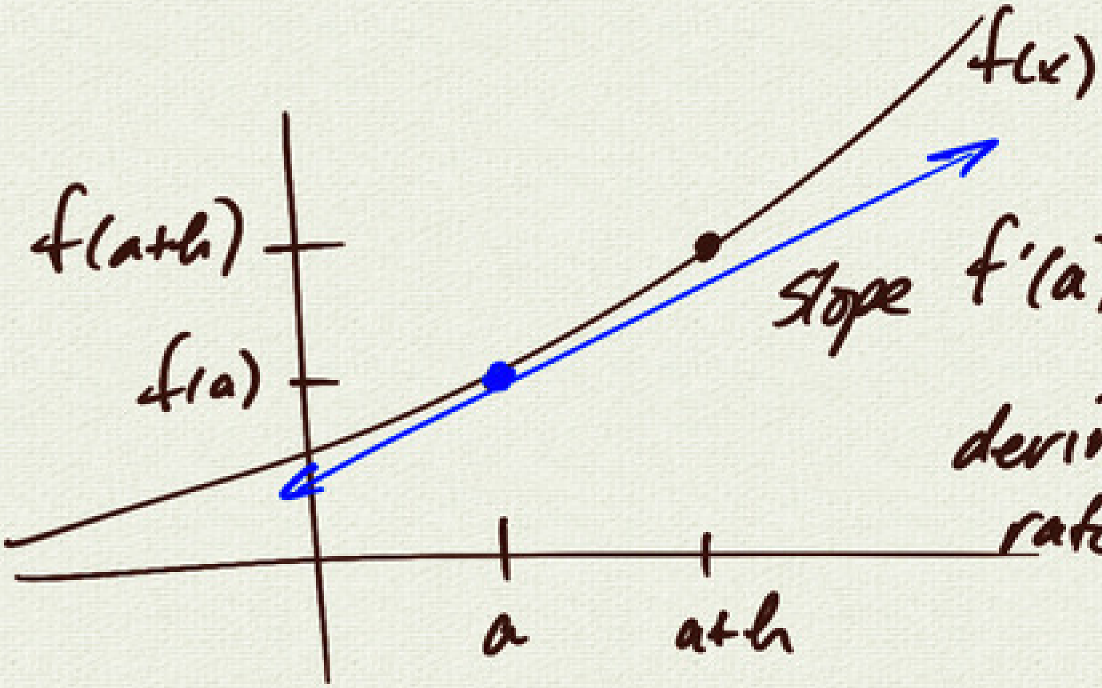
8.4 Derivative Rules

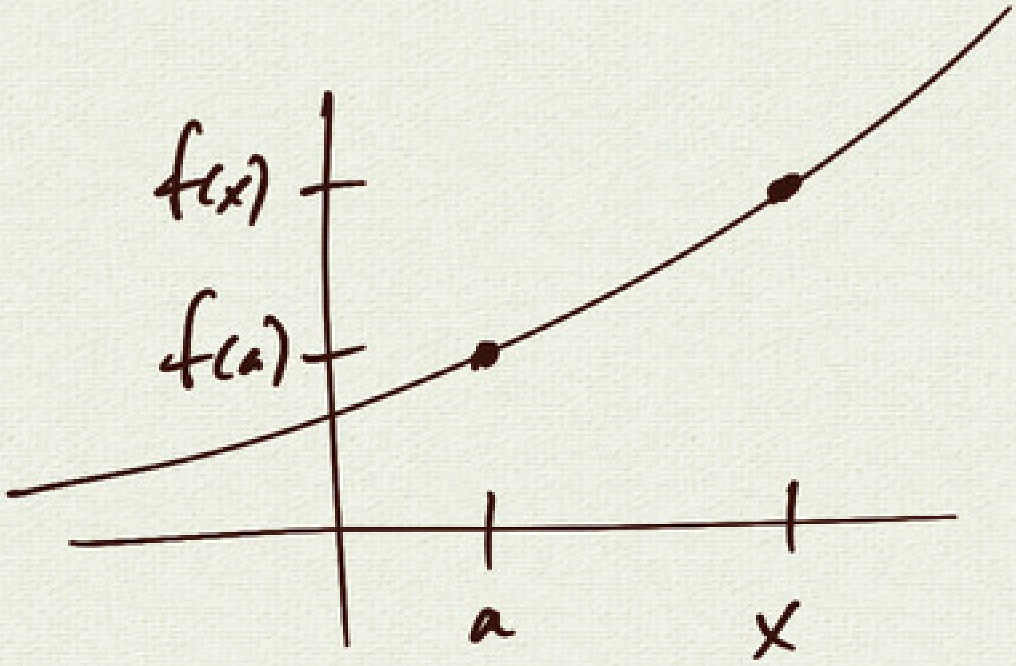


slope $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

derivative of f at a

rate of change

alternate def:



$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

function def:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

notation: $f'(x)$, $\frac{df}{dx}$, $\frac{dy}{dx}$, $D_x(f)$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

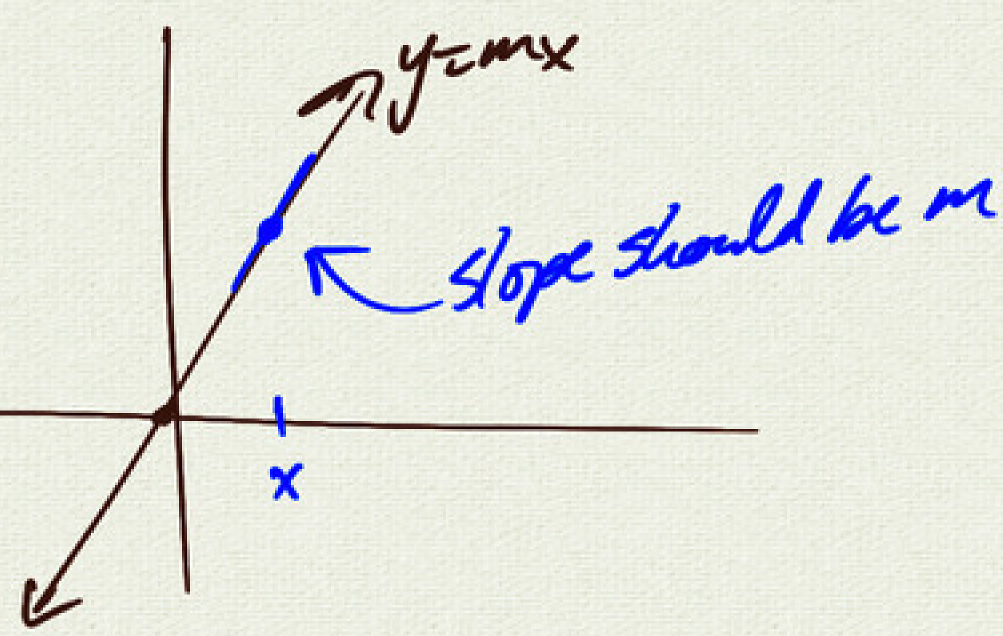
simplest example:

$$f(x) = c$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{c - c}{h} \quad \Delta y = 0$$
$$= \lim_{h \to 0} 0$$
$$= 0$$

$$\boxed{f(x) = \text{const} \Rightarrow f'(x) = 0}$$

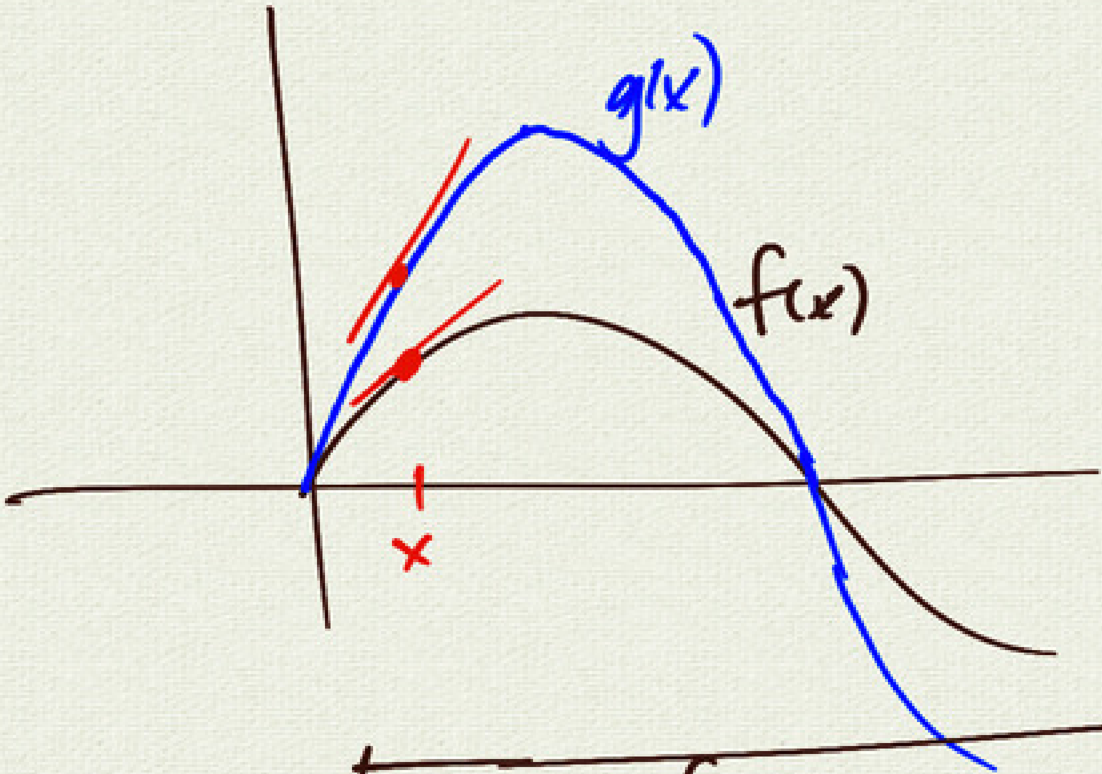
$$g(x) = mx$$



$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$
$$= \lim_{h \to 0} \frac{m(x+h) - mx}{h}$$
$$= \lim_{h \to 0} \frac{mh}{h}$$
$$= m$$

$$\frac{d(mx)}{dx} = m$$

$$g(x) = 2f(x)$$

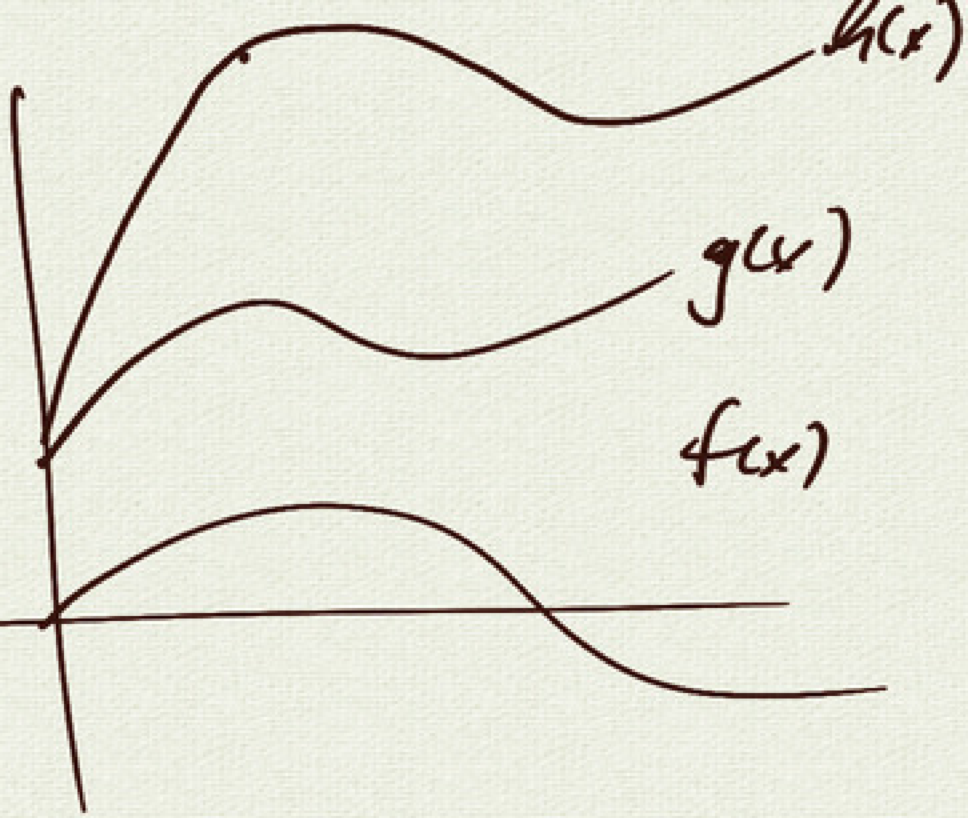


$$\Rightarrow g'(x) = 2f'(x)$$

rule: $\boxed{g(x) = cf(x) \Rightarrow g'(x) = cf'(x)}$

$$\frac{d(cf)}{dx} = c\frac{df}{dx}$$

$$h(x) = (f+g)(x)$$
$$= f(x) + g(x)$$



$$\Rightarrow h'(x) = f'(x) + g'(x)$$

$\boxed{(f+g)'(x) = f'(x) + g'(x)}$ sum rule

$$\frac{d(f+g)}{dx} = \frac{df}{dx} + \frac{dg}{dx}$$

$f(x)=x^2$

$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to0}\frac{(x+h)^2-x^2}{h}$$
$$=\lim_{h\to0}\frac{x^2+2xh+h^2-x^2}{h}$$
$$=\lim_{h\to0}\frac{2xh+h^2}{h}$$
$$=\lim_{h\to0}(2x+h)$$
$$=2x$$

$f(x)=x^2 \Rightarrow f'(x)=2x$

$$\frac{d(x^2)}{dx}=2x$$

$g(x)=x^n$

$$g'(x)=\lim_{h\to0}\frac{g(x+h)-g(x)}{h}$$
$$=\lim_{h\to0}\left[(x+h)^n-x^n\right]\frac{1}{h}$$
$$=\lim_{h\to0}\frac{1}{h}\left[\left(x^n+nx^{n-1}h+\boxed{}h^2+\dots\right)-x^n\right]$$
$$=\lim_{h\to0}\frac{1}{h}\left[nx^{n-1}h+\boxed{}h^2+\dots\right]$$
$$=\lim_{h\to0}\left[nx^{n-1}+\underbrace{\boxed{}h+\boxed{}h^2+\dots}_{\to 0 \text{ as } h\to0}\right]$$
$$=nx^{n-1}$$

$$(x+h)^n=x^n+\binom{n}{1}x^{n-1}h^1+\binom{n}{2}x^{n-2}h^2+\dots+h^n$$

n

$$\boxed{f(x)=x^n \Rightarrow f'(x)=nx^{n-1}}$$ power rule

$$\frac{d(x^n)}{dx}=nx^{n-1}$$

example:

$$\frac{d(x^2)}{dx}=2x$$
$$\frac{d}{dx}(x^3)=3x^2$$
$$\frac{d}{dx}(x^7)=7x^6$$
$$\frac{d}{dx}(x)=1x^0=1$$

example:

$$f(x) = 5x^4 + 6x^3 + 3x + 1$$

$$f'(x) = 5(4x^3) + 6(3x^2) + 3$$
$$= 20x^3 + 18x^2 + 3$$

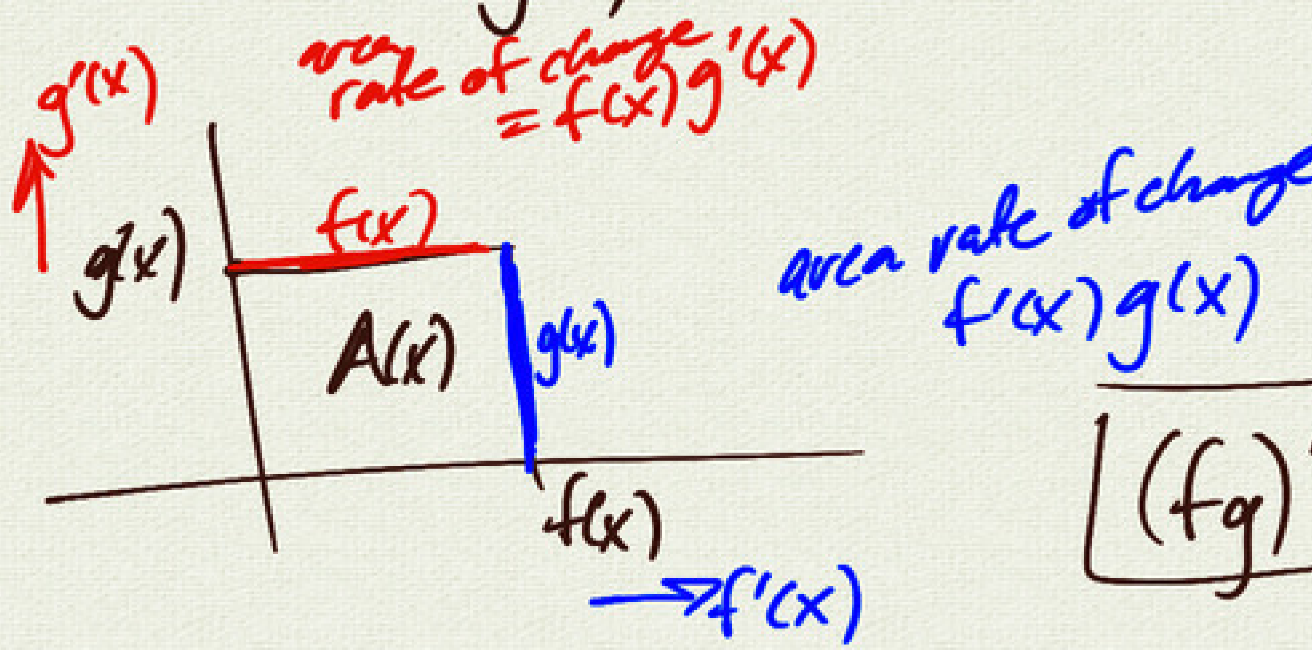
$$g(x) = 20x^4 + 3x^3 + 5x^2 + 7$$

$$\boxed{g'(x) = 80x^3 + 9x^2 + 10x}$$

$$\frac{d}{dx}(20x^4) = 20\frac{d}{dx}(x^4)$$
$$= 20(4x^3)$$
$$= 80x^3$$

$$A(x) = (fg)(x)$$
$$= f(x)g(x)$$

$$A'(x) = f'(x)g(x) + f(x)g'(x)$$



$$\boxed{(fg)'(x) = f'(x)g(x) + f(x)g'(x)}$$

product rule

example:

$$f(x) = (\underbrace{2x+1}_{f})(\underbrace{5x^2+x}_{g})$$

① expand:

$$f(x) = 10x^3 + 2x^2 + 5x^2 + x$$
$$= 10x^3 + 7x^2 + x$$

$$f'(x) = 30x^2 + 14x + 1$$

② product rule: $(fg)' = f'g + fg'$

$$f'(x) = 2(5x^2+x) + (2x+1)(10x+1)$$
$$= 10x^2 + 2x + 20x^2 + 12x + 1$$
$$= 30x^2 + 14x + 1$$

quotient rule:

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

don't memorize

example:

$$h(x) = \frac{\overset{f(x)}{1}}{\underset{g(x)}{x}}$$

$$h'(x) = \frac{0 \cdot x - 1(1)}{x^2}$$
$$= -\frac{1}{x^2}$$

interesting:

$$h(x) = x^{-1}$$

power rule:

$$h'(x) = -1x^{-2}$$
$$= -x^{-2}$$