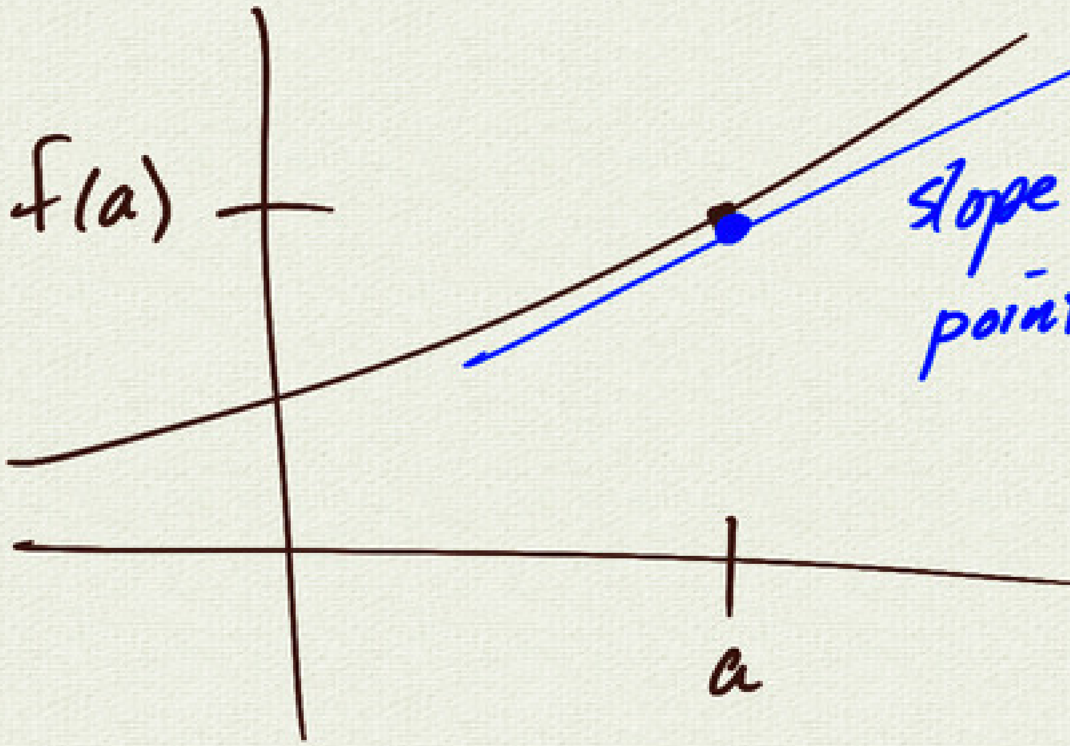


slope $f'(a)$
point $(a, f(a))$

slope-point:

a

$f(a)$

$$y - \underline{\underline{y_1}} = m(x - \underline{\underline{x_1}})$$

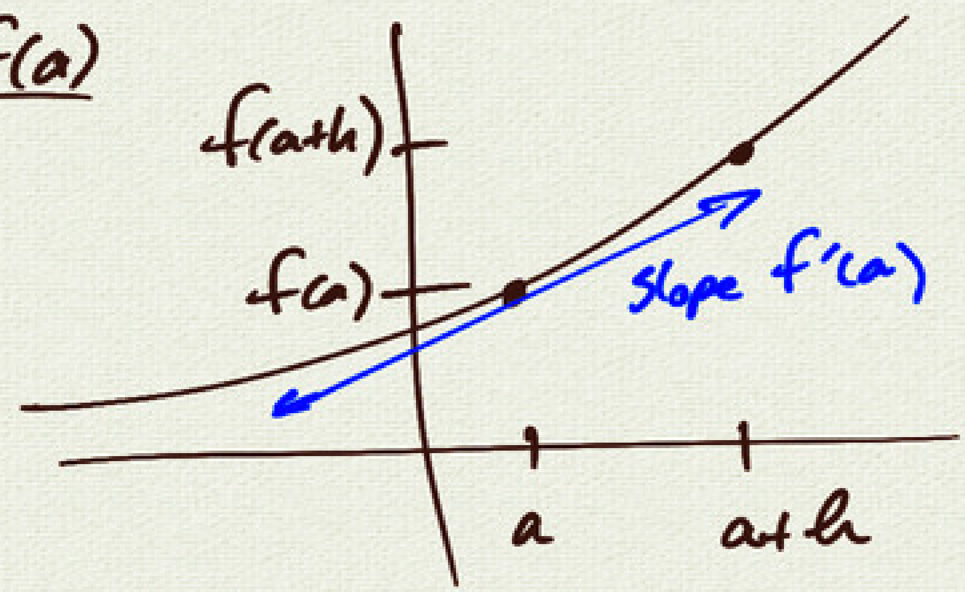
$$y - k = m(x - h)$$

translation of $y = mx$
to center (h, k)
$(a, f(a))$

8.2 Rules for Differentiation

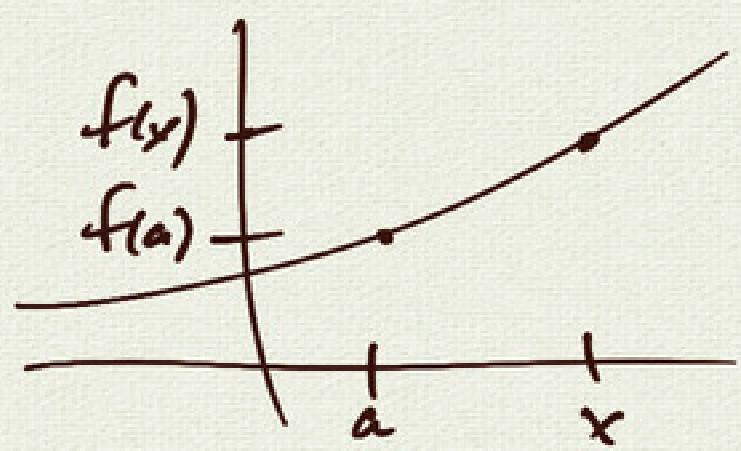
differentiate f $\iff$ find the derivative f'

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



alternate def:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$



function def:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

notation: $f'(x)$

$\frac{df}{dx}$

$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

$D_x f$

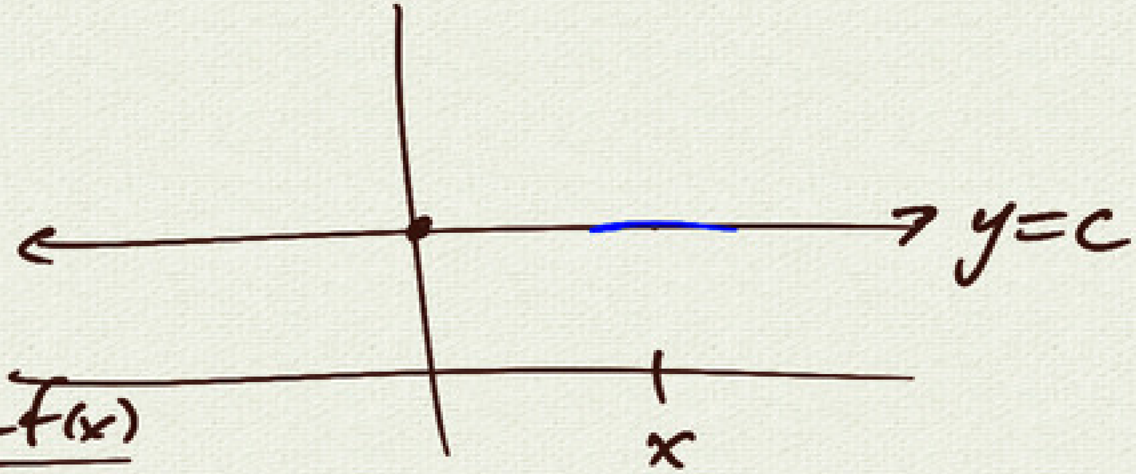
$g(t) \Rightarrow g'(t)$ derivative

$\frac{dg}{dt}$

$\frac{dy}{dt}$

some simple functions:

$$f(x) = c$$



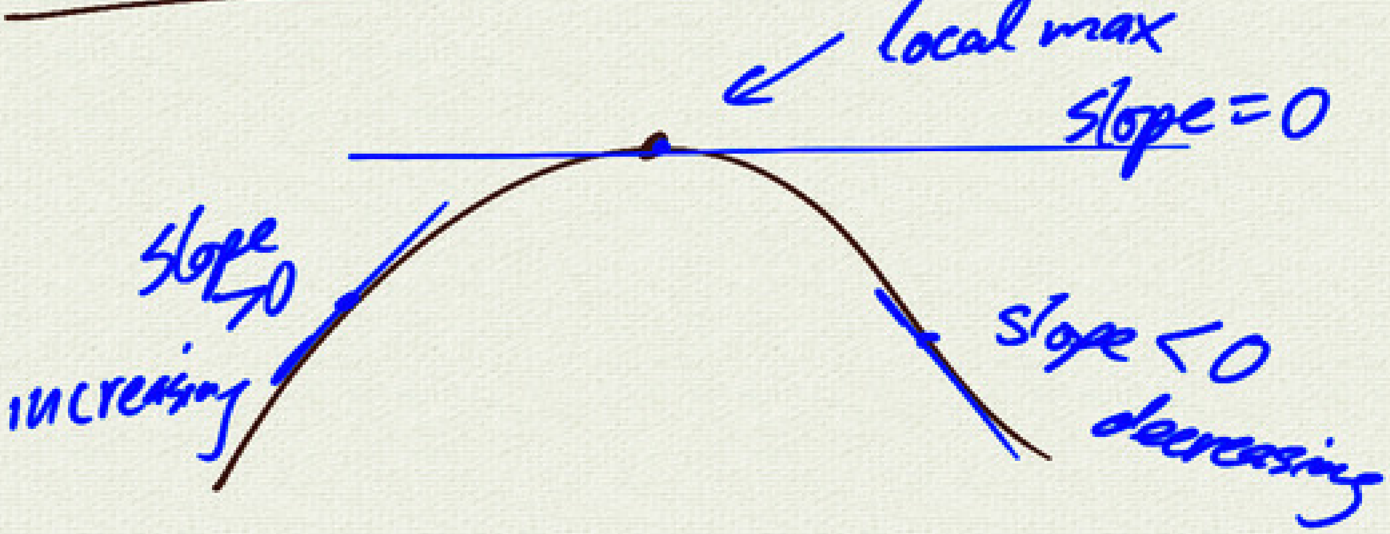
$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{c - c}{h}$$

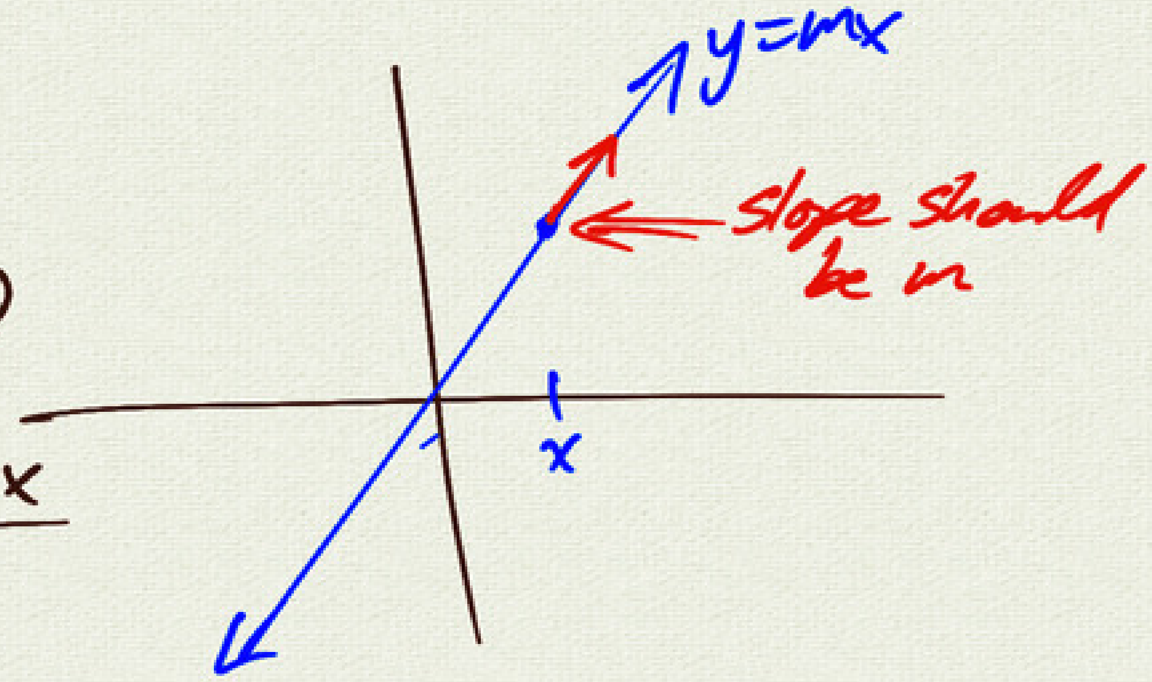
$$= \lim_{h \to 0} 0$$

$$= 0$$

$$\boxed{f(x) = \text{const} \Rightarrow f'(x) = 0}$$



$g(x) = mx$

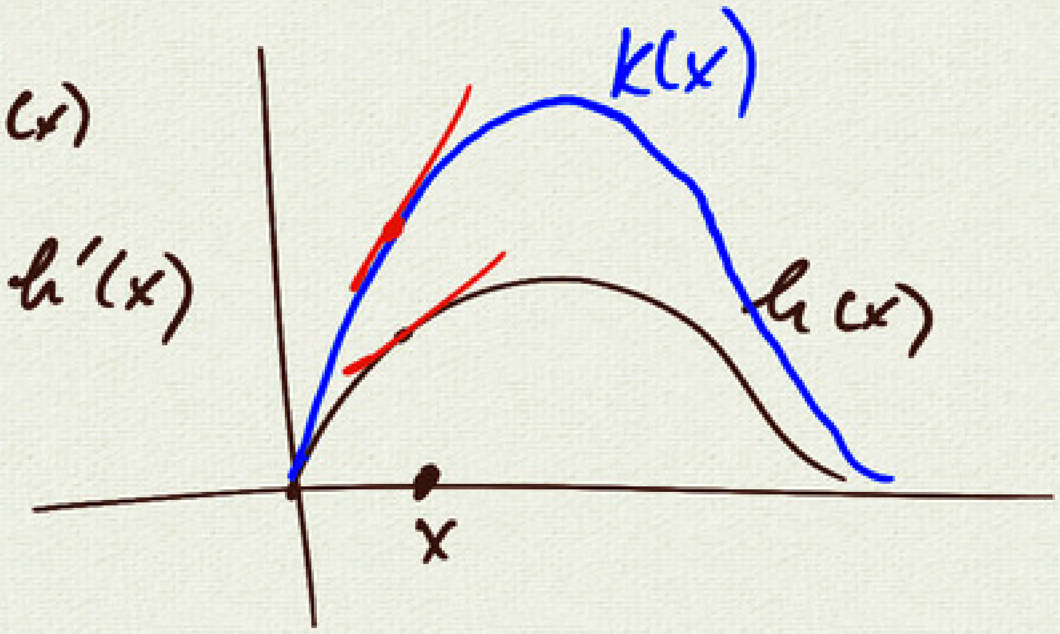


$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$
$$= \lim_{h\to 0} \frac{m(x+h) - mx}{h}$$
$$= \lim_{h\to 0} \frac{mh}{h}$$
$$= m$$

$h(x)$

$k(x) = 2h(x)$

$k'(x) = 2h'(x)$



$$\Rightarrow \boxed{\begin{array}{l} \text{if } \quad g(x) = cf(x) \\ \quad \text{then } \; g'(x) = cf'(x) \end{array}}$$

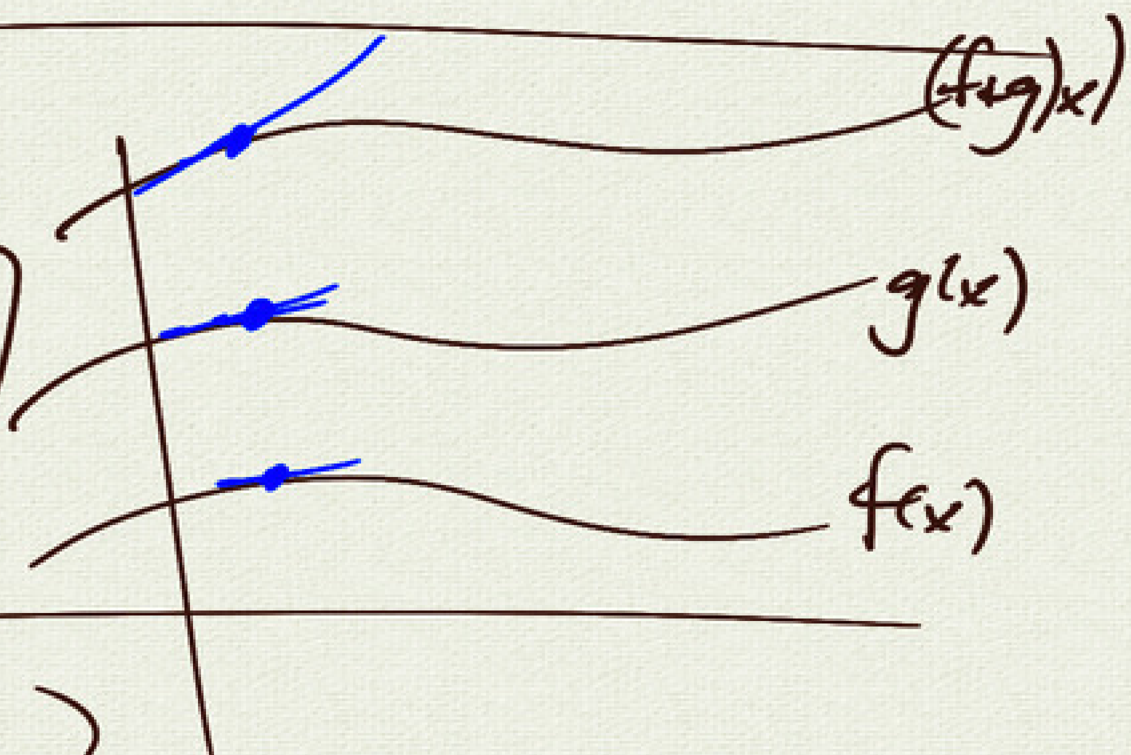
constant multiple rule

$f(x), g(x)$

$$\boxed{(f+g)'(x) = f'(x) + g'(x)}$$

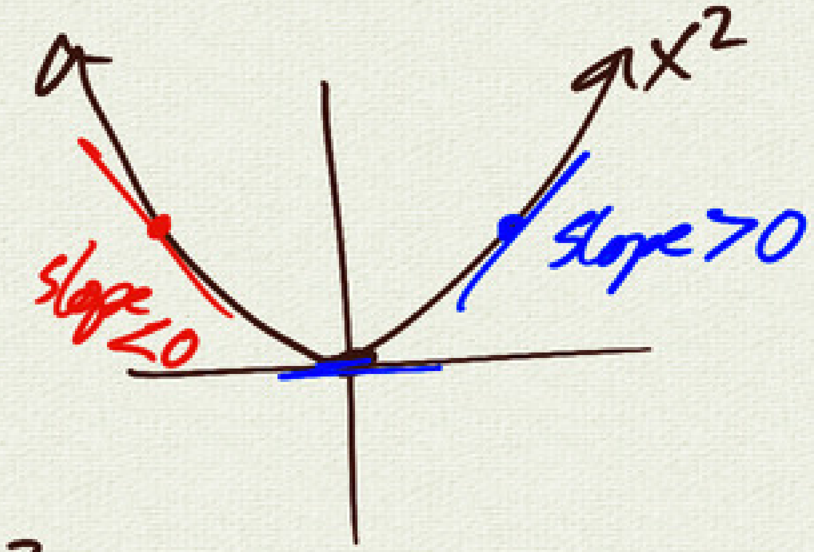
$$\frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx}$$

sum rule

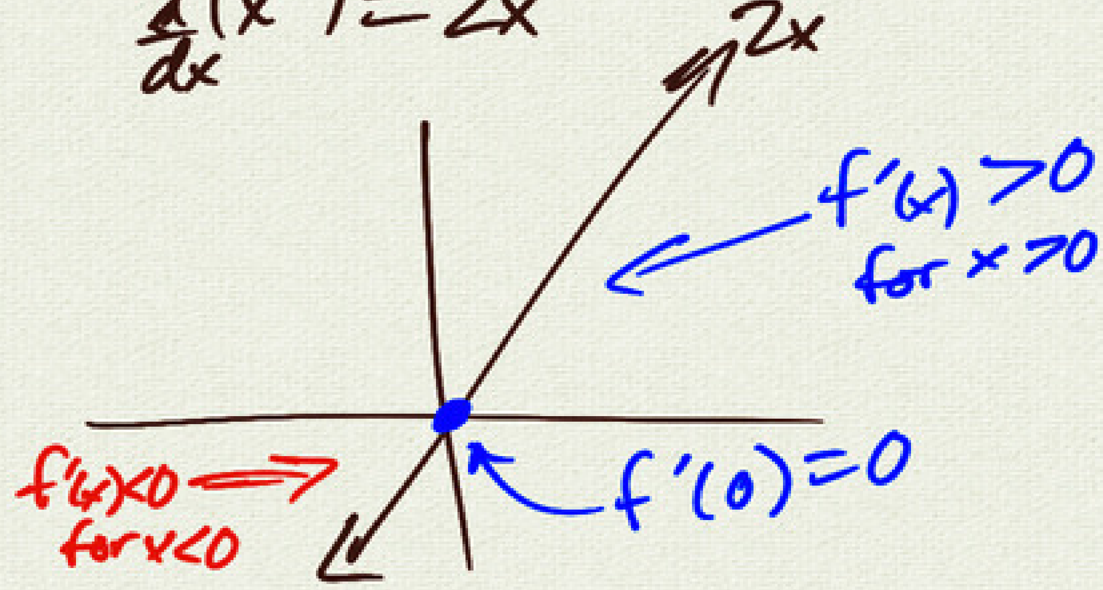


$f(x) = x^2$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$
$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$
$$= \lim_{h\to 0} \frac{2xh + h^2}{h}$$
$$= \lim_{h\to 0} 2x + h$$
$$= 2x$$



$$\frac{d}{dx}(x^2) = 2x$$



$g(x) = x^n$

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$$
$$= \lim_{h\to 0} \frac{(x^n + nx^{n-1}h + \boxed{}\, h^2 + \dots) - x^n}{h}$$
$$= \lim_{h\to 0} nx^{n-1} + \underbrace{\boxed{}\, h}_{\to 0 \text{ as } h\to 0}$$
$$= nx^{n-1}$$

$$(x+h)^n = x^n + \binom{n}{1} x^{n-1} h^1 + \boxed{\binom{n}{2} x^{n-2}} h^2 + \dots + h^n$$

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}}$$

power rule

example:

$$\frac{d}{dx}(x^2) = 2x$$
$$\frac{d}{dx}(x^3) = 3x^2$$
$$\frac{d}{dx}(x^{100}) = 100x^{99}$$

example:

$$p(x) = 7x^5 + 3x^4 + 2x + 5$$

find $p'(x)$

$$\boxed{p'(x) = 7(5x^4) + 3(4x^3) + 2}$$

$$= \boxed{35x^4 + 12x^3 + 2}$$

$$q(x) = 5x^3 + 3x^2 + 7x + 5$$

$$q'(x) = 15x^2 + 6x + 7$$

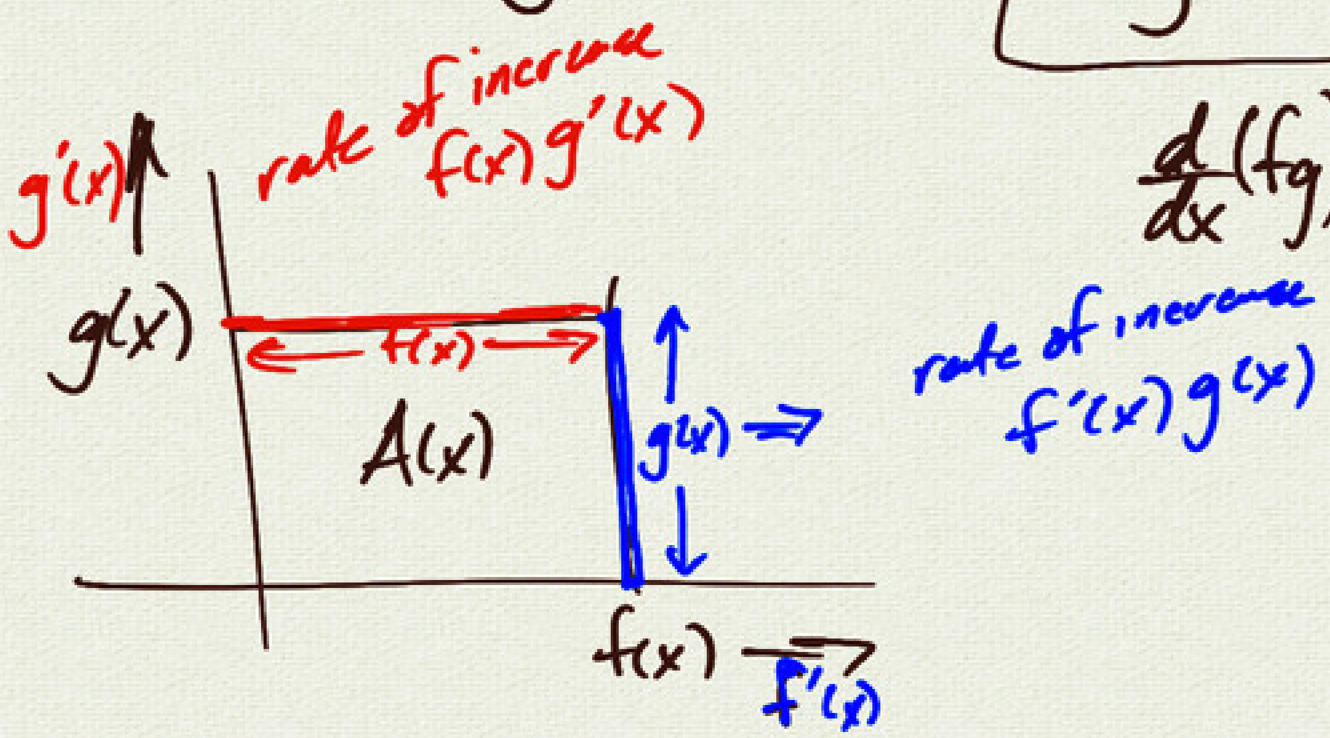
$$A(x) = (fg)(x) \Rightarrow A'(x) = f'(x)g(x) + f(x)g'(x)$$

$$= f(x)g(x)$$

$$\boxed{(fg)'(x) = f'(x)g(x) + f(x)g'(x)}$$

product rule

$$\frac{d}{dx}(fg) = \frac{df}{dx}g + f\frac{dg}{dx}$$



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quotient rule:

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

don't memorize

examples:

$$f(x) = (2x-1)(3x^2+x)$$
$$= 6x^3 - 3x^2 + 2x^2 - x$$
$$= 6x^3 - x^2 - x$$

① $f'(x) = 18x^2 - 2x - 1$

② product rule:
$$(fg)' = f'g + fg'$$

$$f'(x) = (2)(3x^2+x) + (2x-1)(6x+1)$$
$$= 6x^2 + 2x + (12x^2 + 2x - 6x - 1)$$
$$= 18x^2 - 2x - 1$$

---

example: $h(x) = \dfrac{1}{x}$ $f(x)$ $g(x)$

quotient rule:
$$(f/g)' = \frac{f'g - fg'}{g(x)^2}$$

$$h'(x) = \frac{0 - 1}{x^2}$$
$$h'(x) = -\frac{1}{x^2}$$

$h(x) = x^{-1}$

$\Rightarrow h'(x) = -x^{-2}$