8.5 Rates of Change
8.6 Trig Functions

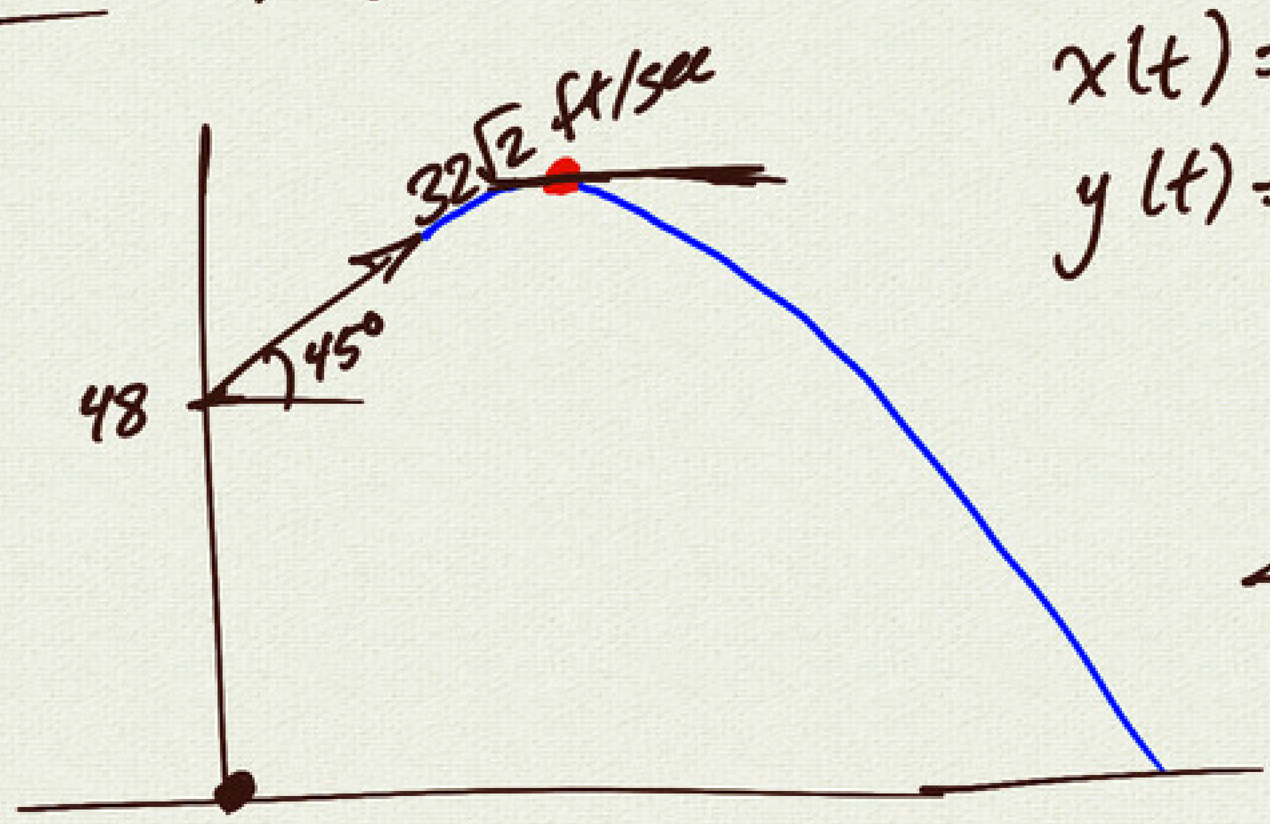
$$f(x) = 5x^2 + 3x + 5$$

$$\Rightarrow f'(x) = 10x + 3$$

$$g(t) = 17t^2 + 10t + 7$$

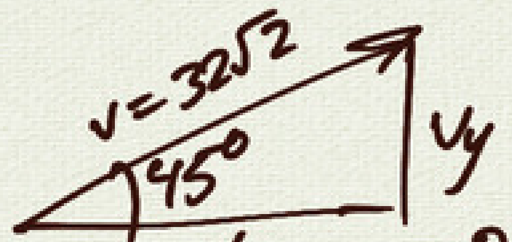
$$\Rightarrow g'(t) = 34t + 10$$

<u>example</u>: projectile motion



$$x(t) = x_0 + \underline{v_x} t$$

$$y(t) = y_0 + \underline{v_y} t - 16t^2$$



$$v_x = v\cos\theta$$

$$= 32\sqrt{2}\,\frac{\sqrt{2}}{2} = 32$$

$$v_y = 32$$

$$\left.\begin{aligned} x(t) &= 32t \\ y(t) &= 48 + 32t \underline{\underline{-16t^2}} \end{aligned}\right]\ \text{position}$$

$$\left.\begin{aligned} x'(t) &= 32 \\ y'(t) &= 32 - 32t \end{aligned}\right]\ \text{velocity}$$

$$\left.\begin{aligned} x''(t) &= 0 \\ y''(t) &= -\underline{\underline{32}} \end{aligned}\right]\ \text{acceleration}$$

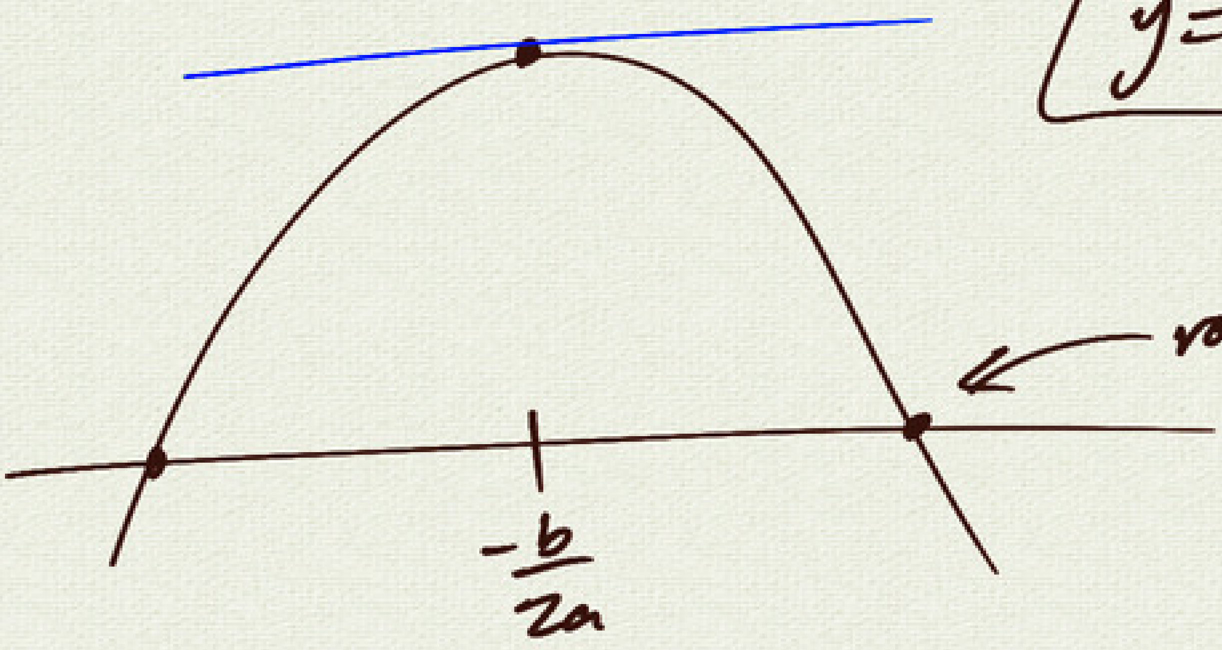
max height:
before: vertex
now:
$y'(t) = 0$
$32 - 32t = 0$
$t = 1$
max height
$y(1) = 64$

2^{nd} derivatives:

$f(x)$		position
$f'(x)$	$\frac{df}{dx}$	velocity
$f''(x)$	$\frac{d^2f}{dx^2}$	acceleration
$f^{(3)}(x)$	$\frac{d^3f}{dx^3}$	jerk

4, 5, 6 → snap, crackle, pop

$$\boxed{y = ax^2 + bx + c}$$



roots $x = \frac{-b \pm \sqrt{}}{2a}$

$y' = 0 \Longrightarrow \frac{dy}{dx} = 2ax + b$

$\frac{dy}{dx} = 0 \rightarrow 2ax + b = 0$

$x = \frac{-b}{2a}$

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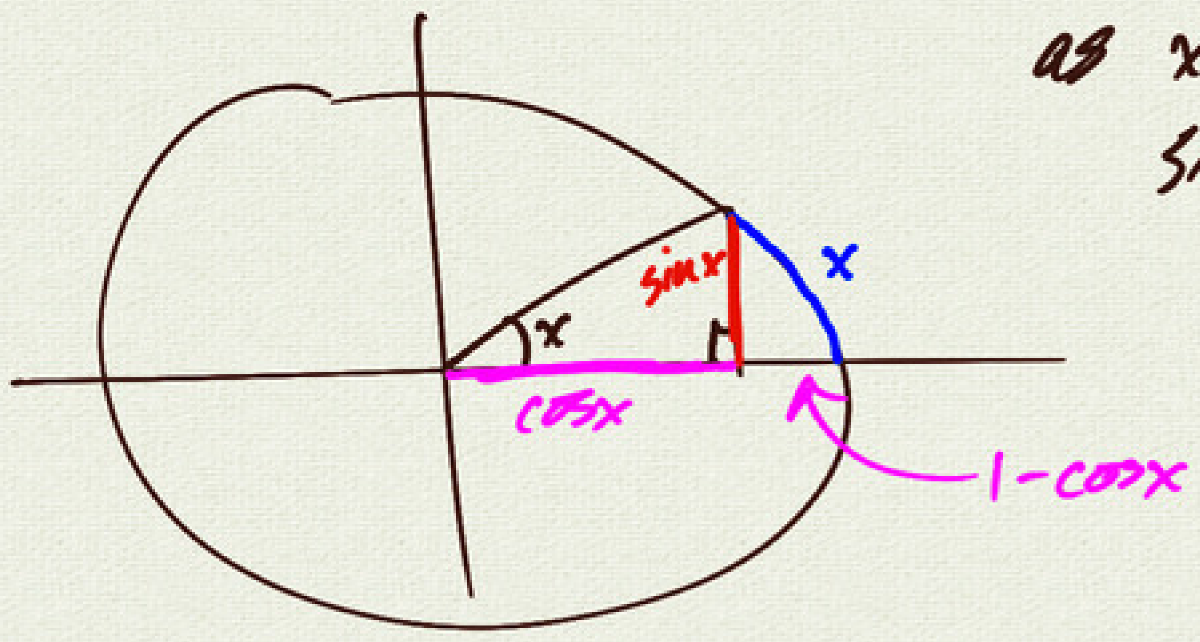
special limit:

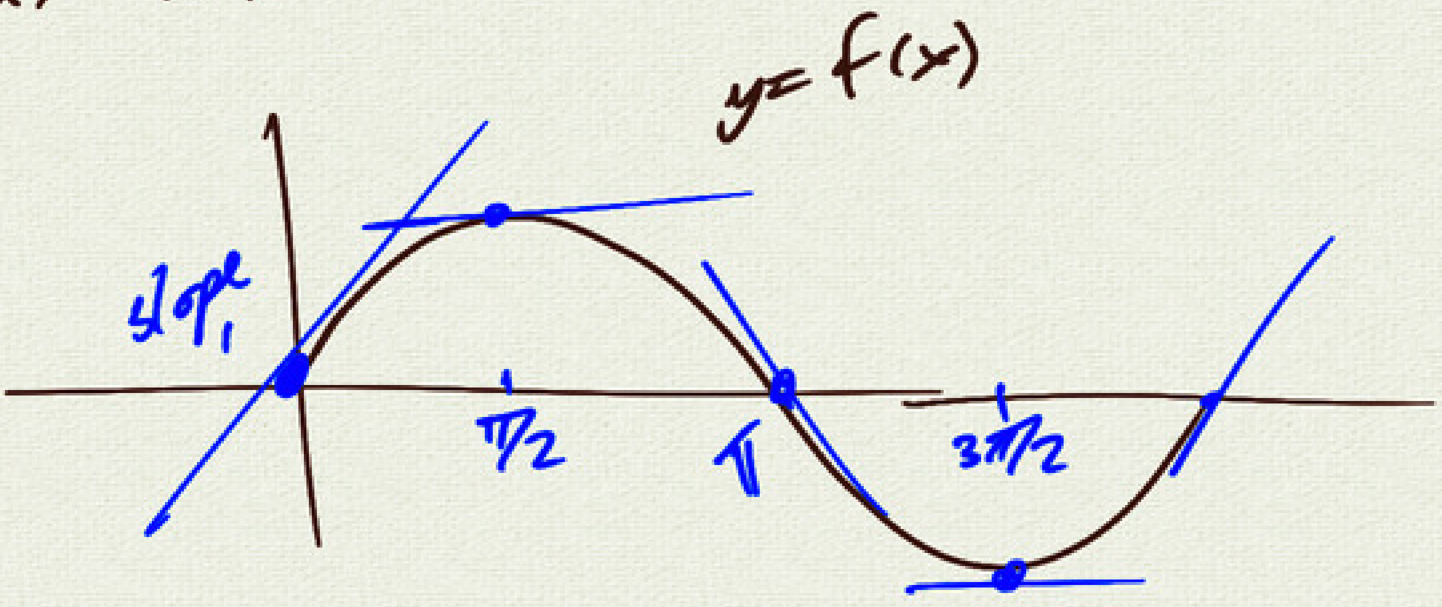
$\lim_{x \to 0} \frac{\sin x}{x} = 1$ $\qquad$ $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

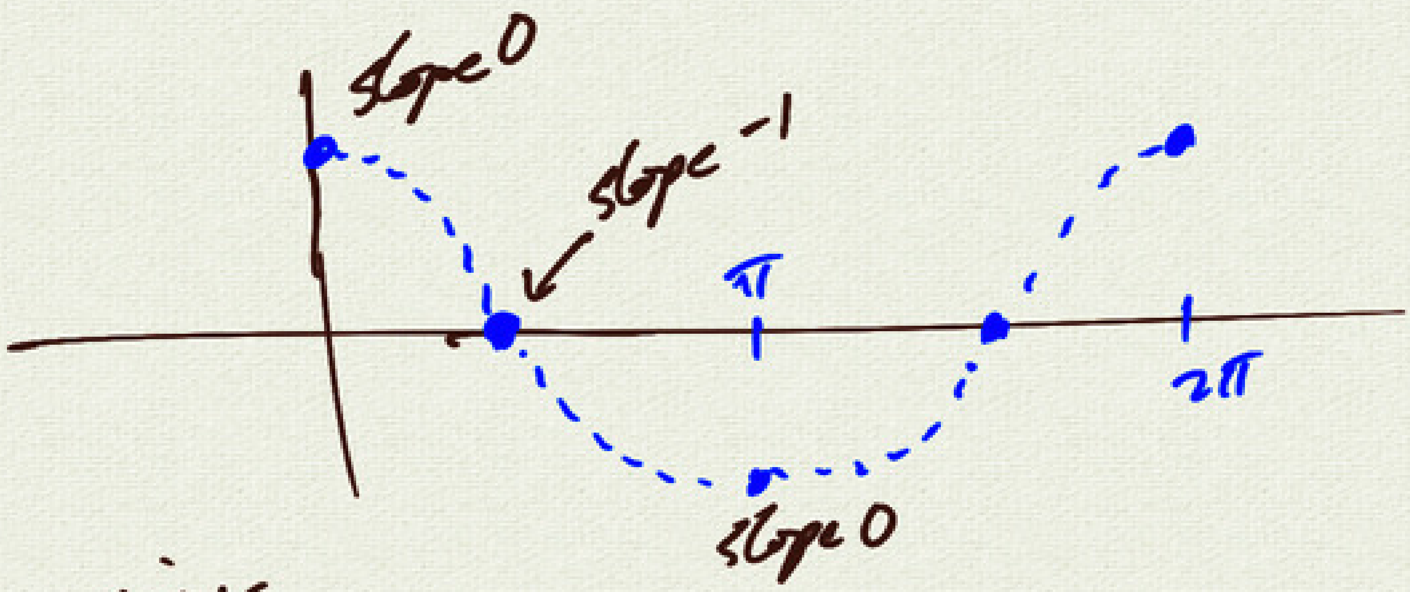
as $x \to 0$

$\sin x \approx x$



$f(x) = \sin x$





$f(x) = \sin x$

$\rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$

$= \lim_{h \to 0} \frac{(\sin x \cos h + \cos x \sin h) - \sin x}{h}$

$= \lim_{h \to 0} \sin x \underbrace{\left(\frac{\cos h - 1}{h}\right)}_{\to 0} + \cos x \underbrace{\frac{\sin h}{h}}_{\to 1}$

$= \cos x$

$\frac{d}{dx}(\sin x) = \cos x$ $\qquad$ $\frac{d}{dx}(\cos x) = -\sin x$

example:

$f(x) = 5\sin x + x^5$

$\Longrightarrow f'(x) = 5\cos x + 5x^4$

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

f

g

$$= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

quotient rule:

$$(f/g)' = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \ldots$$

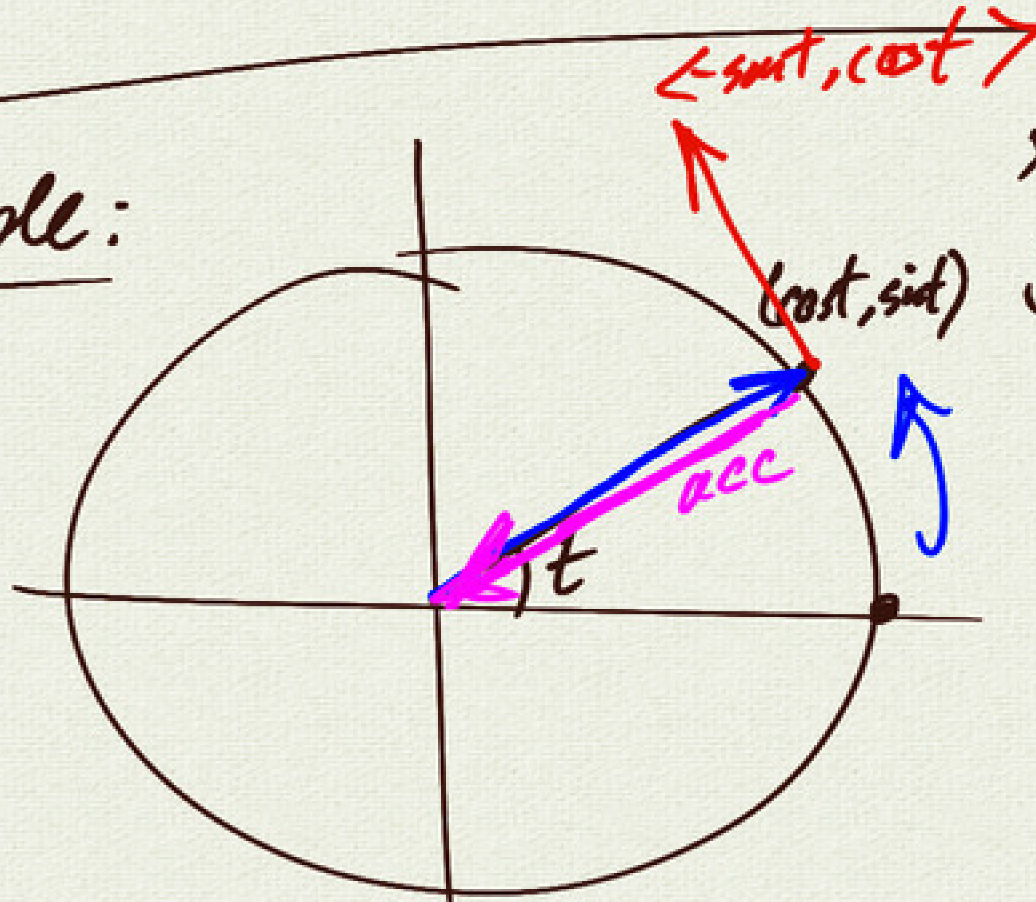
$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

<u>example:</u>

unit circle



$x(t) = \cos t$
$y(t) = \sin t$

$x'(t) = -\sin t$
$y'(t) = \cos t$

$$\left.\begin{aligned} x''(t) &= -\cos t \\ y''(t) &= -\sin t \end{aligned}\right] \text{acceleration toward origin}$$

$\bar{u}, \bar{v}$ orthogonal $\Longleftrightarrow$ $\bar{u} \cdot \bar{v} = 0$

$|\bar{u}||\bar{v}|\cos\theta$

$\cos\theta = 0 \Rightarrow \theta = \pi/2$

$$\langle x, y \rangle \cdot \langle x', y' \rangle = xx' + yy'$$
$$= (\cos t)(-\sin t) + (\sin t)(\cos t)$$
$$= 0 \Longrightarrow \text{position} \perp \text{velocity}$$

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$$f(x) = \begin{cases} 1 & x < 1 \\ x & x \geq 1 \end{cases}$$