8.5 Rates of Change
8.6 Trig Functions

$$f(x) = 7x^2 + 5x + 6$$

$$\Rightarrow f'(x) = 14x + 5$$

$$g(t) = 5t^3 + 3t^2 + 2t + 3$$

$$g'(t) = 15t^2 + 6t + 2$$

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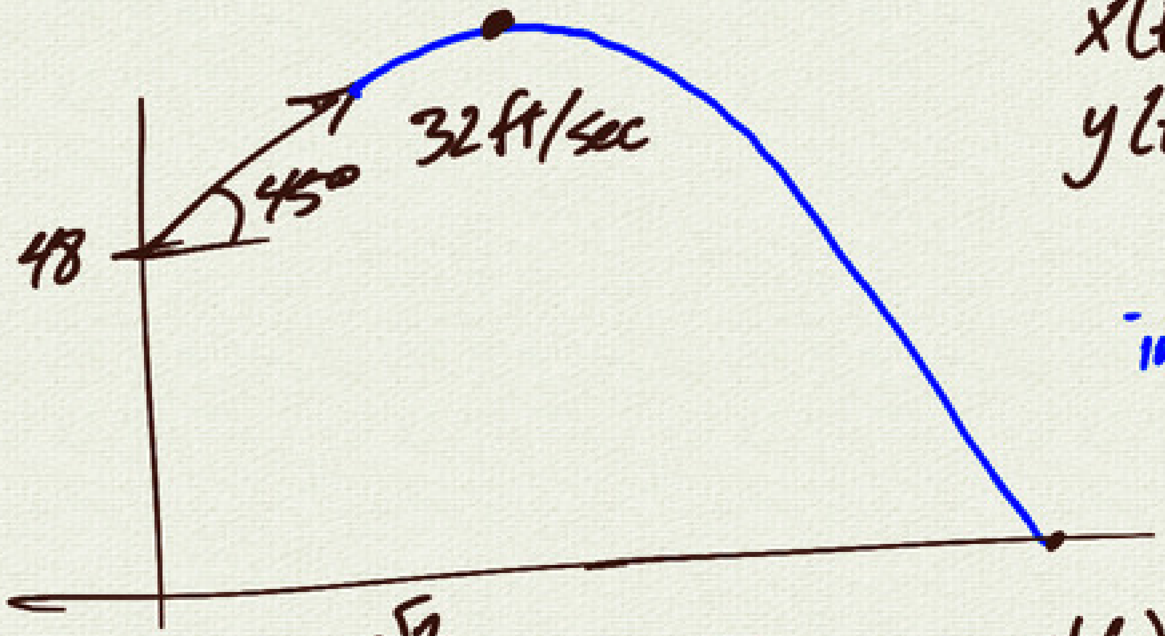
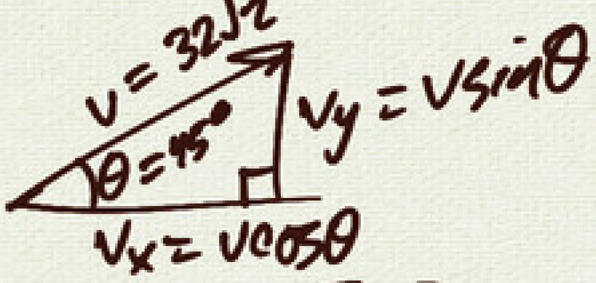
example: projectile motion



$$x(t) = x_0 + v_x t$$

$$y(t) = y_0 + v_y t - 16t^2$$

(y_0: initial height; v_y: initial y-velocity)



$= 32\sqrt{2} \cdot \frac{\sqrt{2}}{2}$

$= 32$

$x(t) = 32t$ $y(t) = 48 + 32t - 16t^2$	position
$x'(t) = 32$ $y'(t) = 32 - 32t$	velocity
$x''(t) = 0$ $y''(t) = -32$	acceleration

what is max height?

before: found vertex at $t = \frac{-b}{2a}$ (of $y(t)$)

today: max height is when $y'(t) = 0$

$$y'(t) = 32 - 32t = 0$$

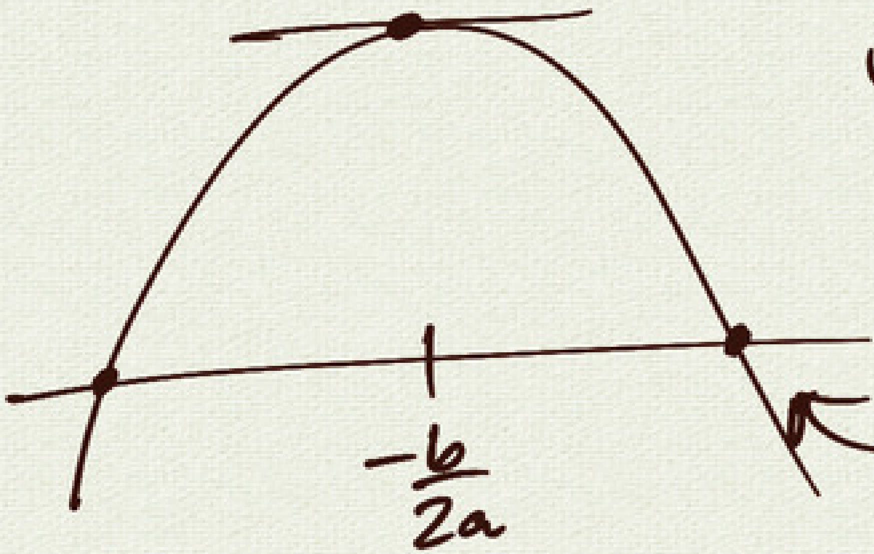
$$t = 1$$

$$y(1) = 48 + 32 - 16 = 64$$

$$y(t) = -16t^2 + 32t + 48$$

$$\frac{-b}{2a} = \frac{-32}{-32} = 1$$

$y = ax^2 + bx + c$



roots $\frac{-b \pm \square}{2a}$

$$\frac{dy}{dx} = 2ax + b$$

$$\frac{dy}{dx} = 0 \Rightarrow 2ax + b = 0$$

$$x = \frac{-b}{2a}$$

2nd derivatives

$f(x)$		function	position
$f'(x)$	$\frac{df}{dx}$	rate of change	velocity
$f''(x)$	$\frac{d^2f}{dx^2}$	rate of change of $f'(x)$	acceleration

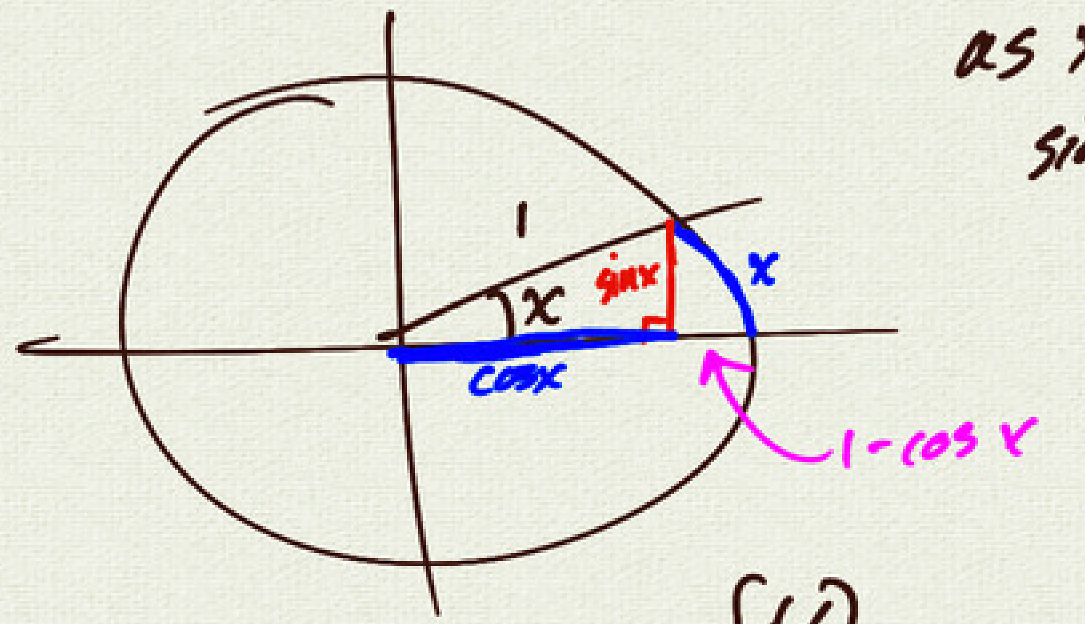
jerk

snap, crackle, pop

Trig functions

special limit:

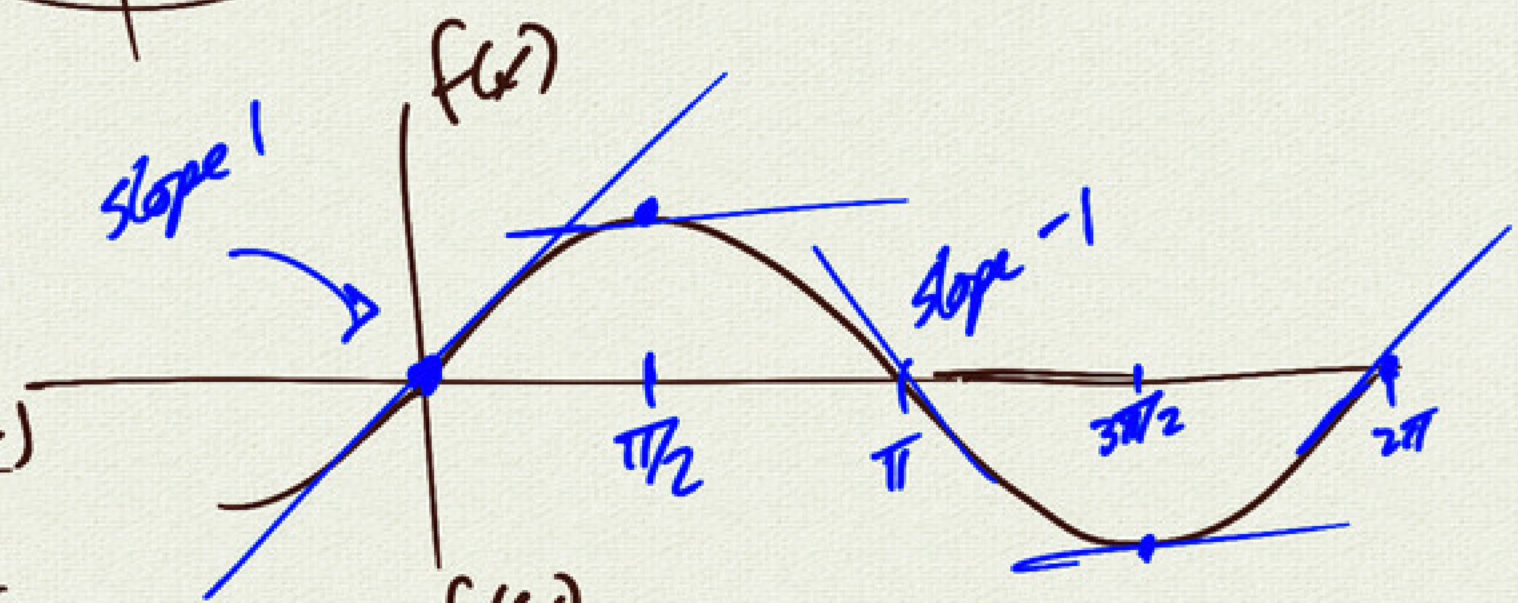
$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{\cos x-1}{x}=0$$



as $x\to 0$:
$\sin x \approx x$

$f(x)=\sin x$

$\Rightarrow f'(x)=?$



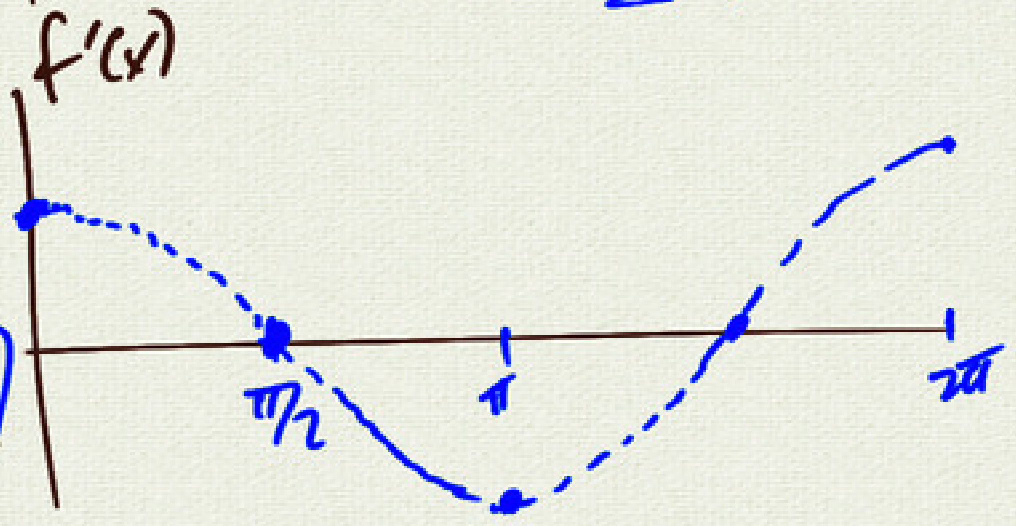
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$$

$$=\lim_{h\to 0}\frac{(\sin x\cosh+\cos x\sinh)-\sin x}{h}$$

$$=\lim_{h\to 0}\sin x\frac{(\cos h-1)}{h}+\cos x\frac{\sin h}{h}$$

($\frac{\cos h-1}{h}\to 0$, $\frac{\sin h}{h}\to 1$)



$$=\cos x$$

$$\frac{d(\sin x)}{dx}=\cos x \qquad \frac{d(\cos x)}{dx}=-\sin x$$

$$\frac{d}{dx}(\tan x)=\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

quotient rule: $(f/g)'=\frac{f'g-fg'}{g^2}$

$$=\frac{\overset{f'}{(\cos x)}\overset{g}{(\cos x)}-\overset{f}{\sin x}\overset{g'}{(-\sin x)}}{\cos^2 x}$$

$$=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}$$

$$=\frac{1}{\cos^2 x}$$

$$=\sec^2 x$$

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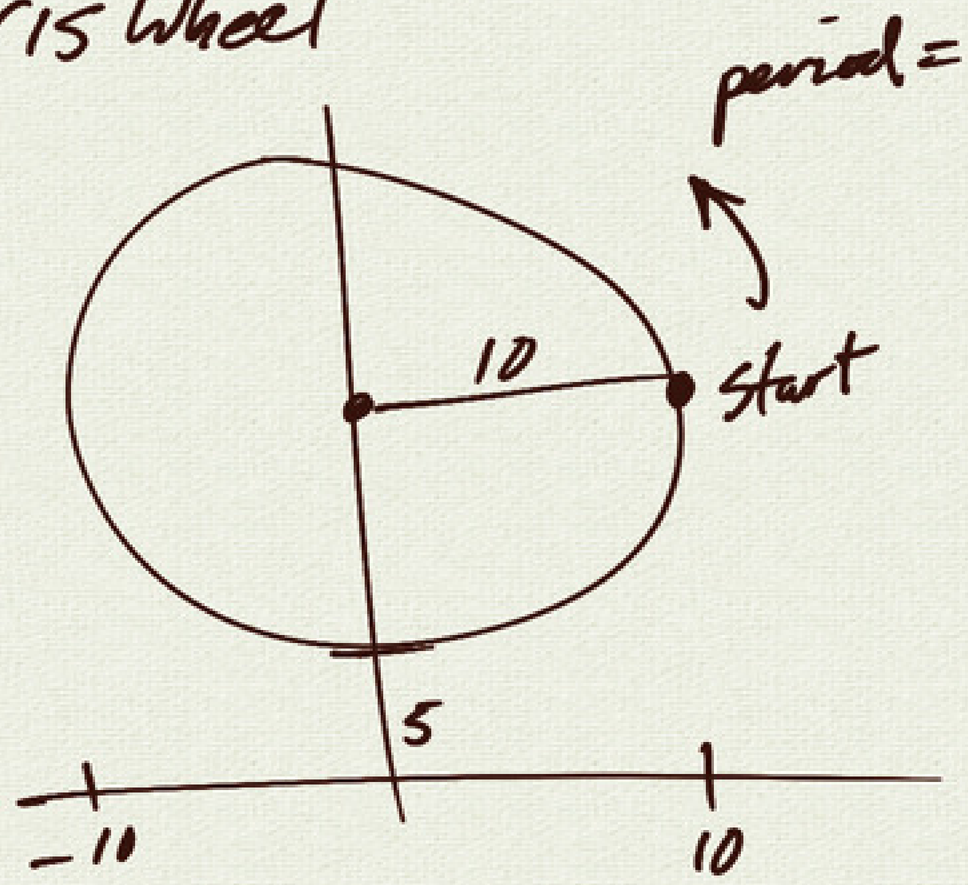
summary:

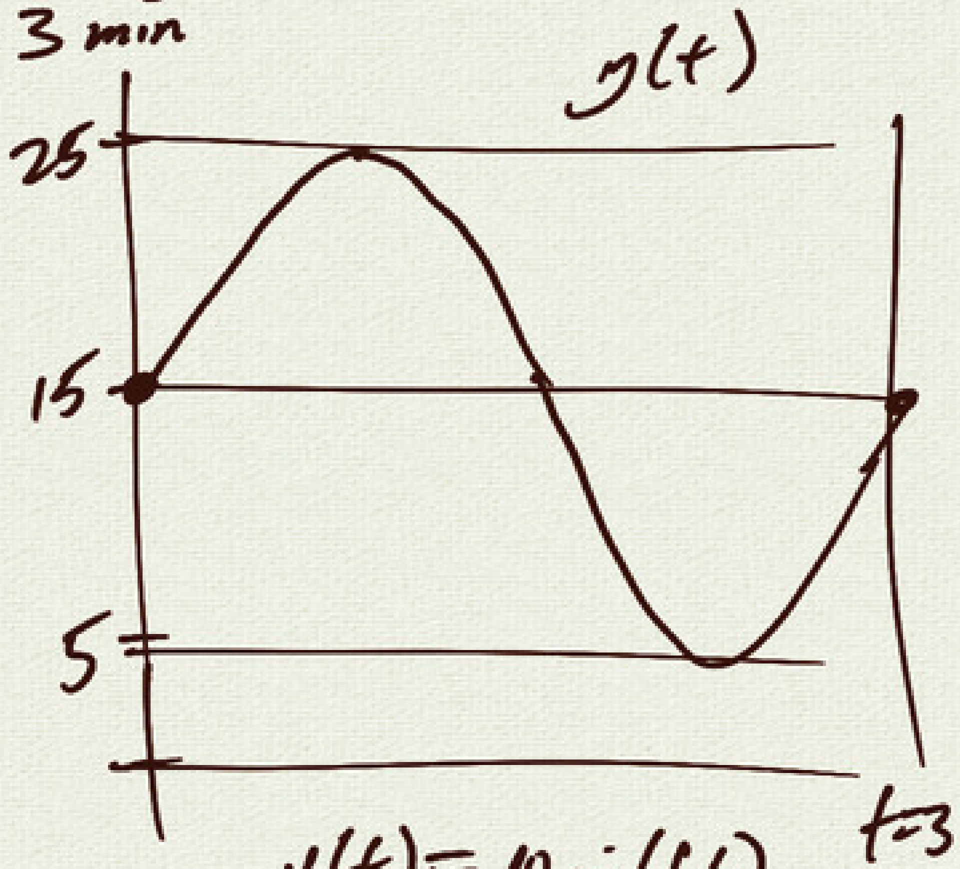
$$\frac{d}{dx}(\sin x)=\cos x \qquad \frac{d}{dx}(\cos x)=-\sin x$$

$$\frac{d}{dx}(\tan x)=\sec^2 x \qquad \frac{d}{dx}(\cot x)=-\csc^2 x$$

$$\frac{d}{dx}(\sec x)=\sec x\tan x \qquad \frac{d}{dx}(\csc x)=-\csc x\cot x$$

Ferris Wheel



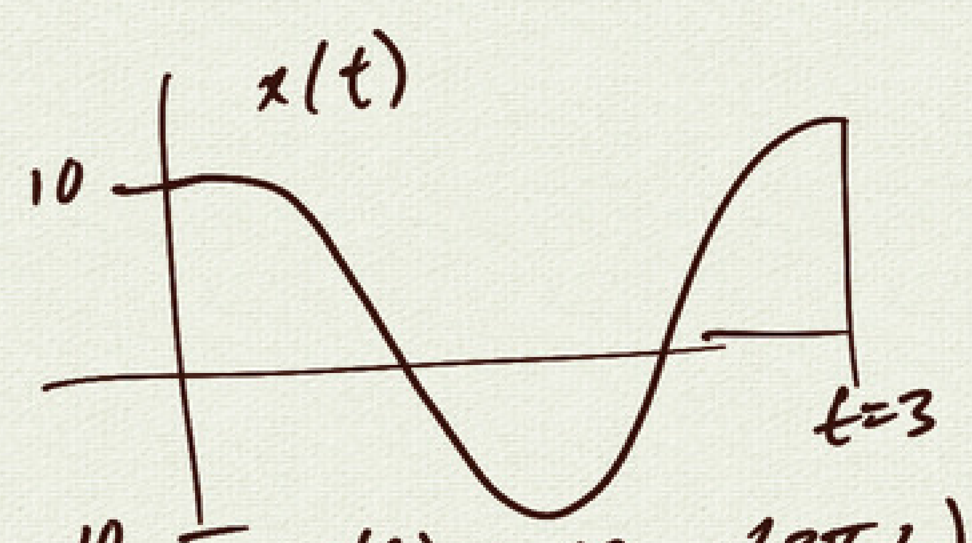


$y(t) = 10\sin(bt) + 15$

period $\frac{2\pi}{b} = 3$

$b = \frac{2\pi}{3}$

$y(t) = 10\sin\left(\frac{2\pi}{3}t\right) + 15$



$x(t) = 10\cos\left(\frac{2\pi}{3}t\right)$

$x(t) = 10\cos\left(\frac{2\pi}{3}t\right)$

$y(t) = 10\sin\left(\frac{2\pi}{3}t\right) + 15$

position

hold this thought

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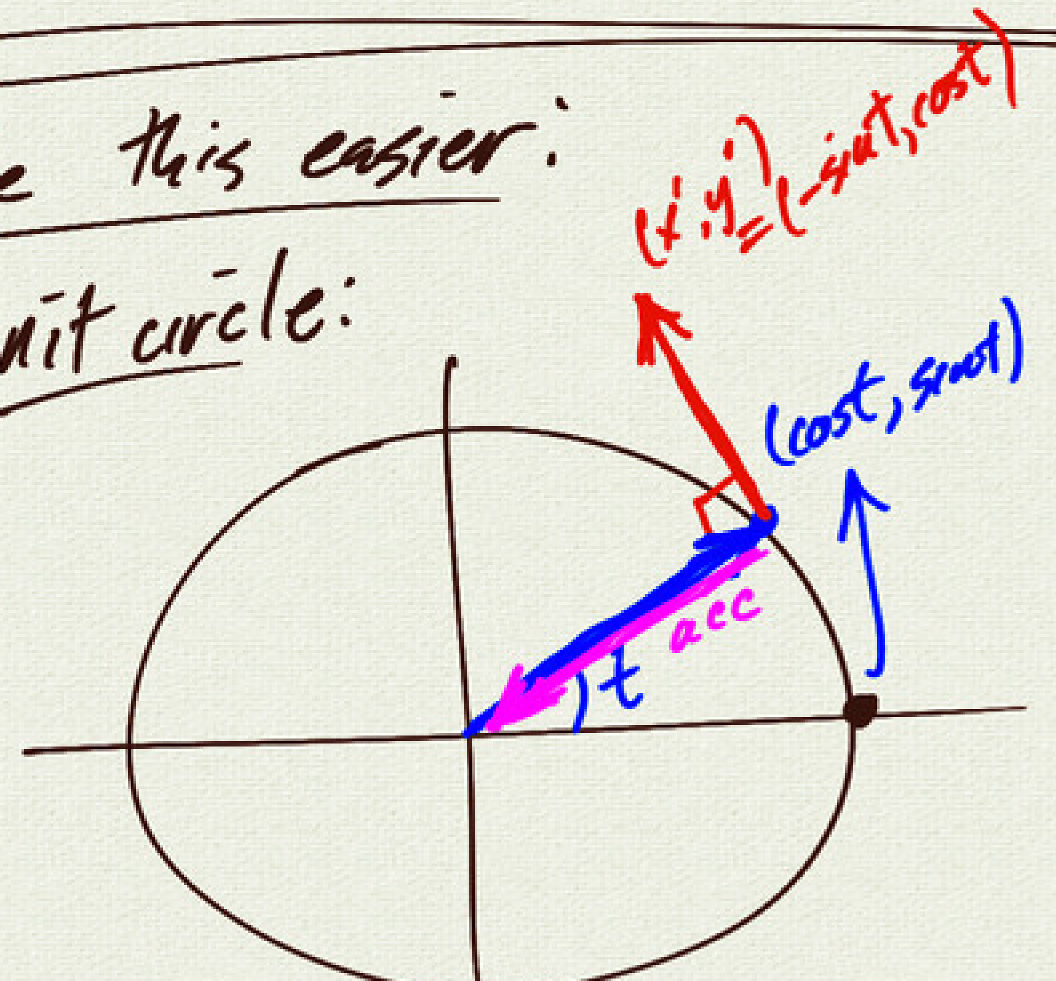
make this easier:

unit circle:



$x(t) = \cos t$
$y(t) = \sin t$ position

$x'(t) = -\sin t$
$y'(t) = \cos t$ velocity

$x''(t) = -\cos t$
$y''(t) = -\sin t$ acceleration

$\bar{v}, \bar{w}$ orthogonal $\longleftrightarrow$ $\bar{v} \cdot \bar{w} = 0$

$|\bar{v}||\bar{w}|\cos\theta = 0$

$\cos\theta = 0 \Longleftrightarrow \theta = \pi/2$

$\langle x, y\rangle \cdot \langle x', y'\rangle = xx' + yy'$

$= (\cos t)(-\sin t) + (\sin t)(\cos t)$

$= 0$