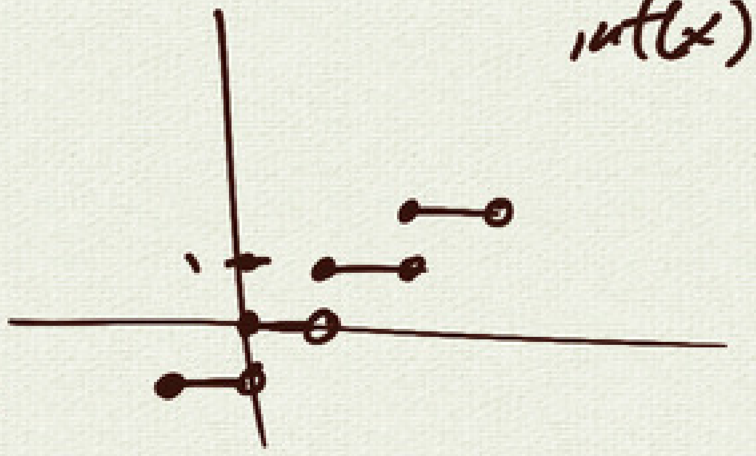
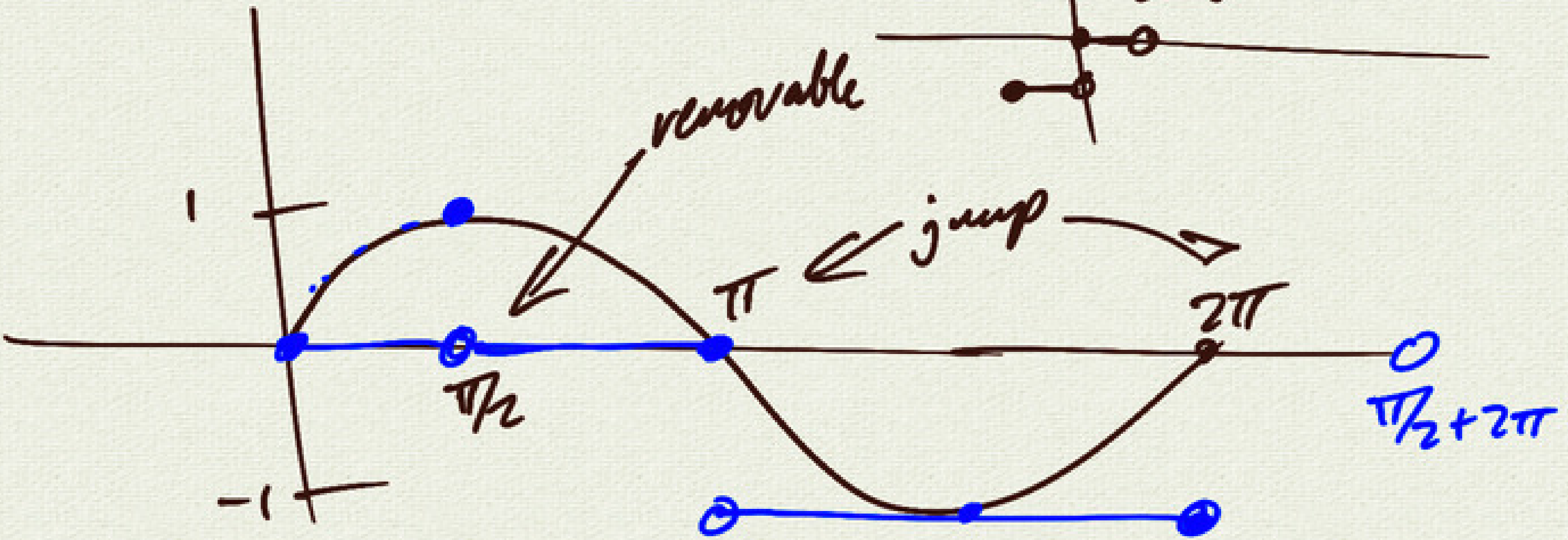
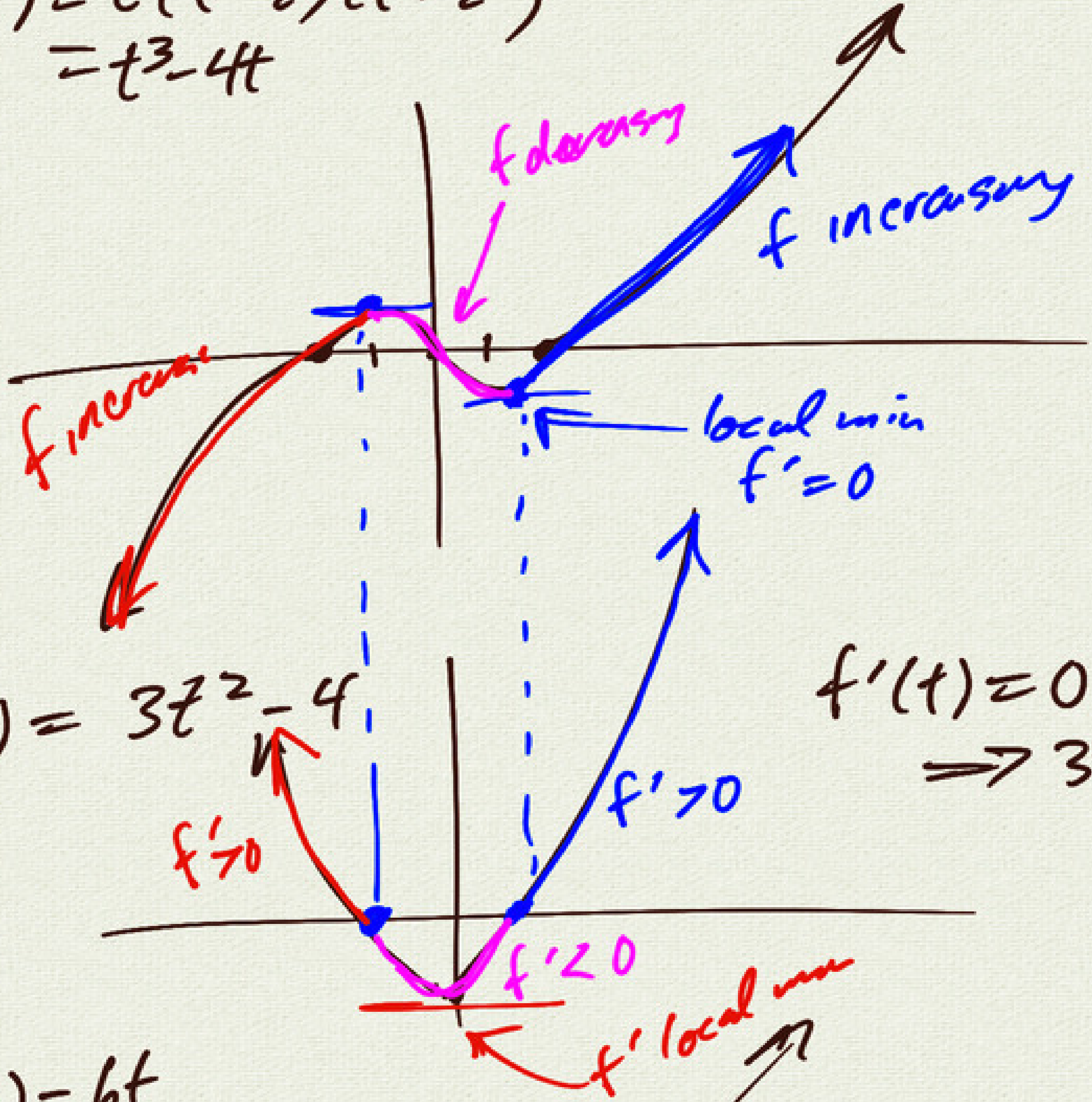
Challenge:

$\mathrm{int}(\sin(x))$





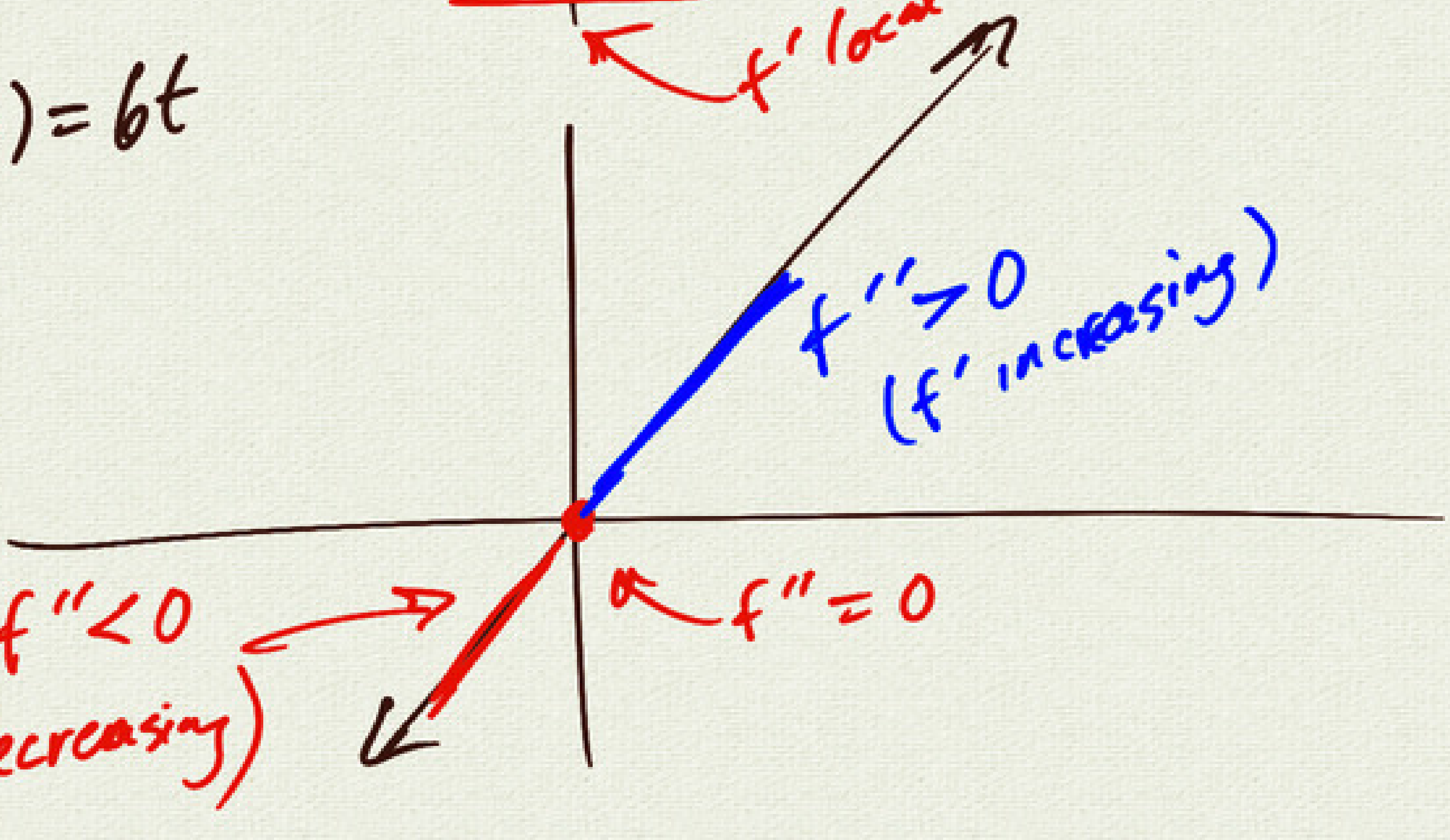
$$f(t) = t(t-2)(t+2)$$
$$= t^3 - 4t$$



$$f'(t) = 3t^2 - 4$$

$$f'(t) = 0$$
$$\Longrightarrow 3t^2 - 4 = 0$$
$$t = \pm\sqrt{4/3}$$

$$f''(t) = 6t$$



Summary

① limits: special limit $\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$ | $\lim_{x\to\infty} \frac{\sin x}{x} = 0$

$\sin x \approx x$
as $x \to 0$

② continuity: $f(x)$ is continuous at $x=a$ if

$$\lim_{x\to a} f(x) = f(a)$$

(and both sides exist)

③ limit definition of derivative:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad \text{(function)}$$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} \quad \text{(alternate)}$$

$f(x) = x^2 \longrightarrow f'(a) = 2a$

$f'(x) = 2x$

$f'(a) = 2x \Longleftarrow$ no!

④ rules:

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}\left(cf(x)\right) = c\frac{df}{dx} \quad \begin{pmatrix}\text{scalar}\\ \text{mult.}\end{pmatrix}$$

$$\frac{d}{dx}\left(5x^2\right) = 5\frac{d}{dx}(x^2)$$
$$= 5(2x)$$
$$= 10x$$

$$\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{df}{dx} + \frac{dg}{dx} \quad (\text{sum})$$

$$\frac{d}{dx}(fg) = \frac{df}{dx}\cdot g + f\frac{dg}{dx} \quad (\text{product})$$

$$(fg)' = f'g + fg'$$

$$\frac{d}{dx}(x^n) = nx^{n-1} \quad (\text{power rule})$$

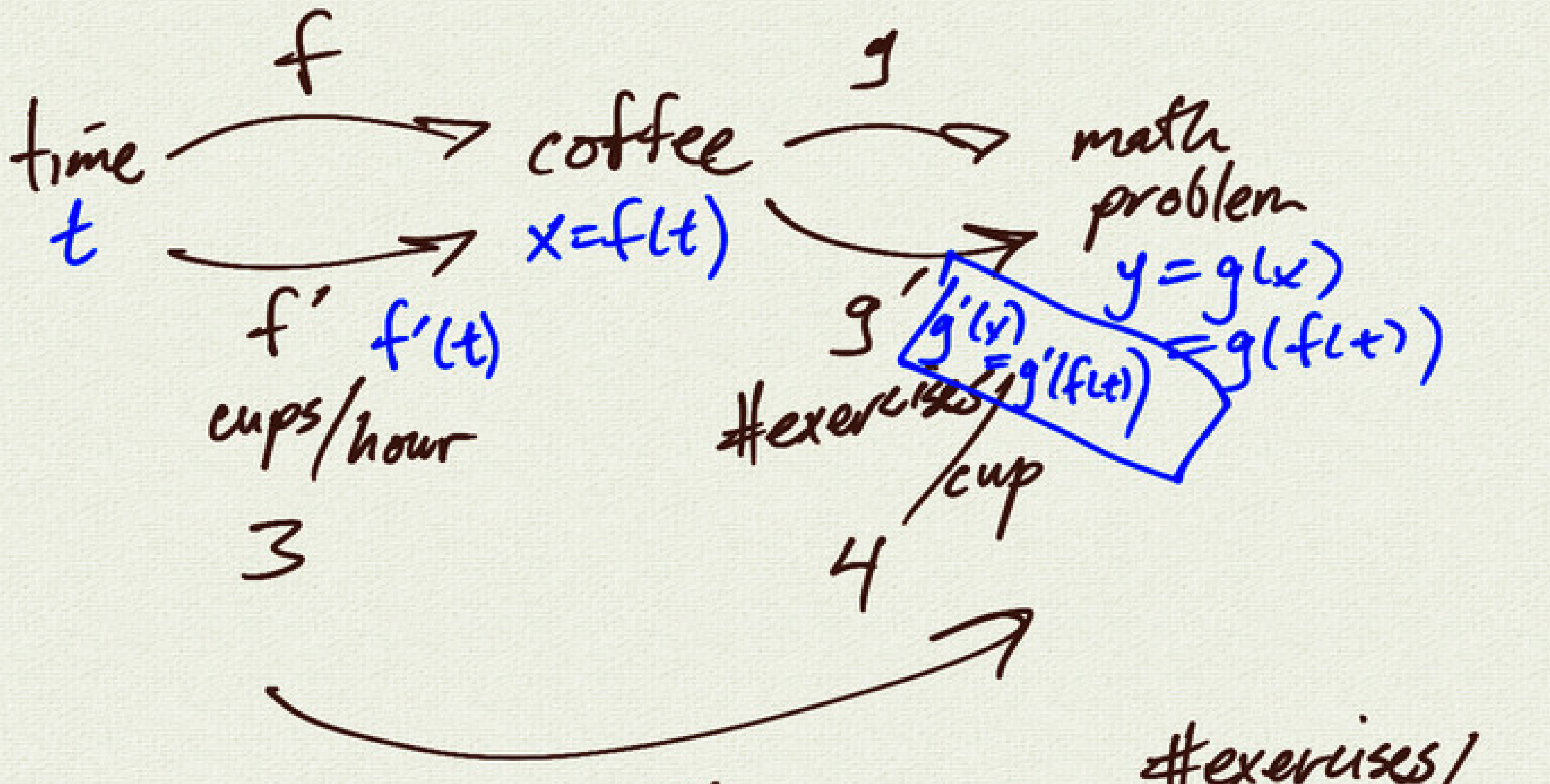
⑤ trig: $\frac{d}{dx}(\sin x) = \cos x$ $\qquad \frac{d}{dx}(\cos x) = -\sin x$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

9.1 Chain Rule

$h(x) = (f \circ g)(x)$ composition
$= f(g(x))$



$3 \cdot 4 =$ #exercises/hour

$f' \cdot g'$ $\quad (g \circ f)'$

moral:
rates multiply

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chain rule: $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

example:

$h(x) = \sin(x^2)$ $\quad | \; h = f \circ g$

$f(x) = \sin x$
$f'(x) = \cos x$

$g(x) = x^2$
$g'(x) = 2x$

$\Rightarrow h'(x) = f'(g(x)) \cdot g'(x)$
$= \cos(x^2) \cdot 2x$
$= 2x \cos x^2$

$h(x) = \sin(x^2) \Longrightarrow h'(x) = \cos(x^2) \cdot 2x$

$h(x) = \tan(x^3 + 2x + 1)$

$\Rightarrow h'(x) = \sec^2(x^3 + 2x + 1) \cdot (3x^2 + 2)$

$f(x) = (x-1)^5$

$$f'(x) = 5(x-1)^4 \cdot 1$$

$g(x) = (x^2+2x+1)^{100}$

$$\Rightarrow g'(x) = 100(x^2+2x+1)^{99} \cdot (2x+2)$$

(factor): $g(x) = ((x+1)^2)^{100}$

$= (x+1)^{200}$

same? (challenge - HW)

$$\Rightarrow g'(x) = 200(x+1)^{199} \cdot 1$$

$h(x) = \sec(x^4)$

$$\Rightarrow h'(x) = \sec(x^4)\tan(x^4) \cdot 4x^3$$

$f(x) = \sin^4(x^3+2x)$

$= [\sin(x^3+2x)]^4$

$$\Rightarrow f'(x) = 4[\sin(x^3+2x)]^3 \cdot \cos(x^3+2x) \cdot (3x^2+2)$$

$g(x) = \cot^5(3\sin x)$

$= [\cot(3\sin x)]^5$

$$g'(x) = 5[\cot(3\sin x)]^4 \left(-\csc^2(3\sin x)\right)(3\cos x)$$

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observation:

$h(x) = 3\sin x$ (product)

$f(x) = 3$ $\quad g(x) = \sin x$ $\Rightarrow h'(x) = f'g + fg'$

$f'(x) = 0$ $\quad g'(x) = \cos x$

$= 0 \cdot \sin x + 3 \cdot \cos x$

$= 3\cos x$