Unit 8

① limits

$\frac{0}{0} \Rightarrow ?$

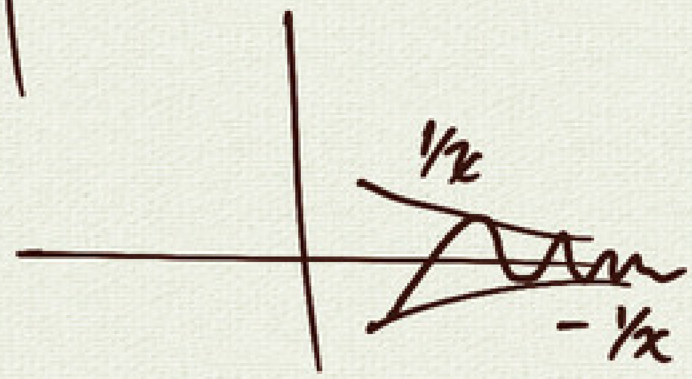
$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

as $x \to \infty$:

bounded

$$\frac{\sin x}{x} \to 0$$

big



② continuity

f <u>continuous</u> at $x = a$

if

$$\lim_{x \to a} f(x) = f(a)$$

exists

exists

③ limit definition of derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(at a point $x = a$) $\quad f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

(alternate) $\quad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

④ rules:

$$\frac{dc}{dx} = 0$$

$$\frac{d(cf)}{dx} = c\frac{df}{dx} \quad \text{(scalar mult.)}$$

$$\frac{d(f+g)}{dx} = \frac{df}{dx} + \frac{dg}{dx} \quad \text{(sum)}$$

$$\frac{d(fg)}{dx} = \frac{df}{dx}g + f\frac{dg}{dx} \quad \text{(product)}$$

$$(fg)' = f'g + fg'$$

$$\frac{d(x^n)}{dx} = nx^{n-1} \quad \text{(power)}$$

⑤ trig

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

9.1 Chain Rule

$$\begin{tikzcd}[column sep=huge]
\text{time } t \arrow[r, bend left, "f"] \arrow[r, bend right, "f'(t)", "f'"'] & \text{coffee } x=f(t) \arrow[r, bend left, "g"] \arrow[r, bend right, "g'(x)=g'(f(t))", "g'"'] & \text{math problems } y=g(x)=g(f(t))
\end{tikzcd}$$

cups/hour $\qquad\qquad$ #problems/cup

$$3 \quad \cdot \quad 4 \quad = 12$$

rates multiply

chain rule: $\boxed{(g\circ f)'(x) = g'(f(x))\cdot f'(x)}$

example: $h(x) = \sin(x^2)$

$g(x)=\sin x$, $g'(x)=\cos x$

$f(x)=x^2$, $f'(x)=2x$

$$h'(x) = g'(f(x))\cdot f'(x)$$

$$(g\circ f)'(x) = \cos(x^2)\cdot 2x$$

$$= 2x\cos x^2$$

$$h(x) = \sin x^2$$

$$h'(x) = \cos(x^2)\ (2x)$$

$f(x) = \cos(x^3 + 2x)$

$\Longrightarrow f'(x) = -\sin(x^3 + 2x)\,(3x^2 + 2)$

$g(x) = \tan^4(x)$

$= (\tan x)^4$

$\Longrightarrow g'(x) = 4(\tan x)^3 \cdot \sec^2 x$

$$h(x) = \sec(x^5 + x^2) \qquad \frac{d(\sec x)}{dx} = \sec x \tan x$$

$$\Longrightarrow \quad h'(x) = \sec(x^5+x^2)\tan(x^5+x^2)\cdot(5x^4+2x)$$

$$k(x) = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$$

$$\frac{d(x^{-1})}{dx} = -x^{-2}$$

(power rule)

$$\Longrightarrow \quad k'(x) = -(\cos x)^{-2}(-\sin x)$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \sec x \tan x$$

$$m(x) = \sin^4(x^6+7x)$$

$$= [\sin(x^6+7x)]^4$$

$$\Rightarrow m'(x) = 4[\sin(x^6+7x)]^3 \cdot \cos(x^6+7x) \cdot (6x^5+7)$$

$$n(x) = \tan^5(x^4+x^3)$$

$$n'(x) = 5[\tan(x^4+x^3)]^4 \cdot \sec^2(x^4+x^3) \cdot (4x^3+3x^2)$$