9.2 Implicit Differentiation

$$f(x) = \sin(x^2)$$

y

$$\Longrightarrow f'(x) = \cos(x^2)\cdot(2x)$$

$$\boxed{y = x^2} \longrightarrow \frac{dy}{dx} = 2x$$

$$\Longrightarrow f(x) = \sin(y)$$

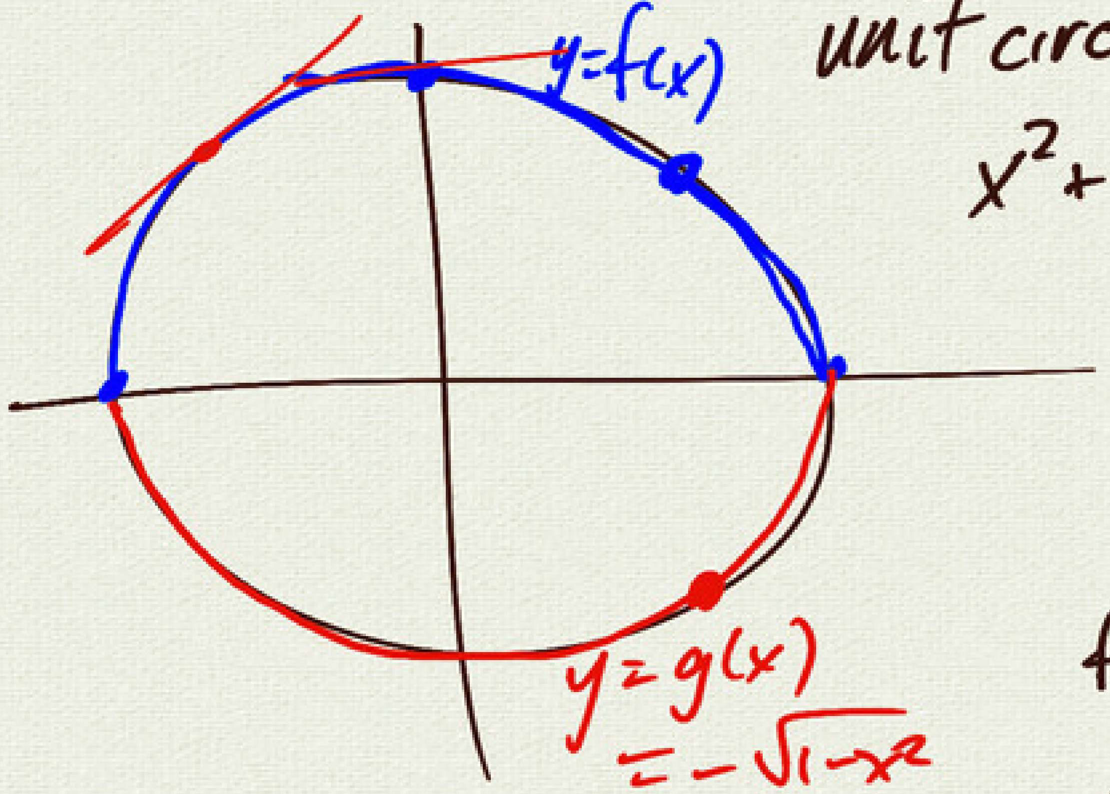
$$\frac{df}{dx} = \underbrace{\cos(y)\cdot\frac{dy}{dx}}_{\cos(x^2)\cdot(2x)}$$

$$\frac{d}{dx}(\sin y) = (\cos y)\frac{dy}{dx}$$

$$\frac{d}{dx}(y^3 + \cos y) = 3y^2\cdot\frac{dy}{dx} + (-\sin y)\frac{dy}{dx}$$

$$\frac{d}{dx}(y^4 + 3x^2) = 4y^3\cdot\frac{dy}{dx} + 6x\left(\underbrace{\frac{dx}{dx}}_{1}\right)$$

implicit differentiation



unit circle

$$x^2+y^2=1$$

$$\Rightarrow y^2=1-x^2$$

$$y=\pm\sqrt{1-x^2}$$

$$f(x)=\sqrt{1-x^2}$$

$$g(x)=-\sqrt{1-x^2}$$

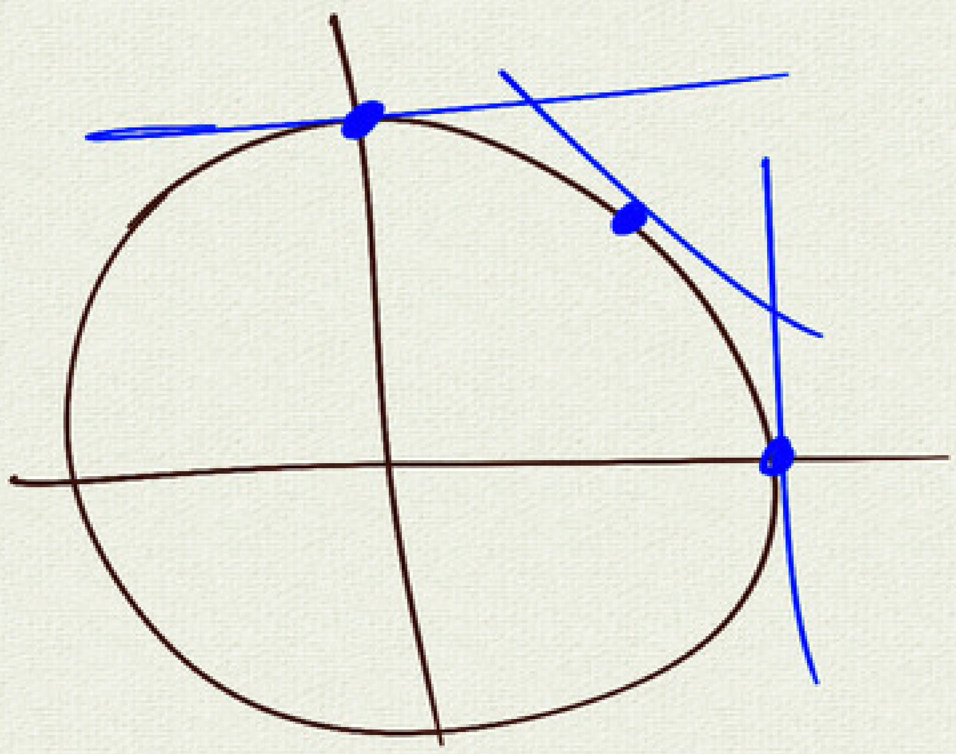
$$x=\frac{\sqrt{2}}{2} \Rightarrow f\left(\frac{\sqrt{2}}{2}\right)=\sqrt{1-\frac{1}{2}}$$

$$=\sqrt{\tfrac{1}{2}}=\frac{\sqrt{2}}{2}$$

$$f(x)=\sqrt{1-x^2} \Rightarrow f'(x)=?$$

$$=(1-x^2)^{1/2}$$

$$x^2+y^2=1 \Rightarrow 2x+2y\frac{dy}{dx}=0$$



$$x+y\frac{dy}{dx}=0$$

$$\boxed{\frac{dy}{dx}=\frac{-x}{y}}$$

$$\frac{dy}{dx}\begin{pmatrix}x=0\\y=1\end{pmatrix}=\frac{0}{1}=0$$

$$\frac{dy}{dx}\begin{pmatrix}x=\sqrt{2}/2\\y=\sqrt{2}/2\end{pmatrix}=-1$$

$$\frac{dy}{dx}\begin{pmatrix}x=1\\y=0\end{pmatrix}=\text{undef}$$

$f(x) = \sqrt{x} = x^{1/2}$

$y = x^{1/2}$

$y^2 = x$

$\Longrightarrow \quad 2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y} = \frac{1}{2x^{1/2}} = \frac{1}{2}x^{-1/2}$

power rule works for $n = \frac{1}{2}$

Challenge: (1) Show for $\sqrt[n]{x} = x^{1/n}$

(2) $y = x^{p/q}$ $\quad$ p/q rational

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power rule works for all rationals

(and all reals)

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}} \quad (n \in \mathbb{R})$$

example $xy = 1$

1. solve for y: $y = \frac{1}{x} = x^{-1}$

$$\Rightarrow \frac{dy}{dx} = -1x^{-2}$$

$$= -\frac{1}{x^2}$$

$x^2 + y^2 = 1$

functions (y)

defined implicitly

$y = \boxed{}$

explicit

2. implicit diff:

$$xy = 1$$

$$\Rightarrow 1 \cdot y + x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-y}{x} = -\frac{1}{x^2}$$

(a) diff both sides

(b) solve for dy/dx

HW: #303

$$3x^3 + 9xy^2 = 5x^3$$

find dy/dx

ⓐ diff both sides

ⓑ solve for dy/dx

simplify: $9xy^2 = 2x^3$

differentiate $\Longrightarrow$

$$9\left(1 \cdot y^2 + x \cdot 2y \frac{dy}{dx}\right) = 6x^2$$

$$3\left(y^2 + 2xy \frac{dy}{dx}\right) = 2x^2$$

$$3y^2 + 6xy \frac{dy}{dx} = 2x^2$$

$$\frac{dy}{dx} = \frac{2x^2 - 3y^2}{6xy}$$

$$= \frac{x}{3y} - \frac{y}{2x}$$

$$\left(f/g\right)'(x)$$

$$= \frac{d}{dx}\left(f(x)\cdot g(x)^{-1}\right)$$

$$= f'(x)\, g(x)^{-1} + f(x)\left[-1\, g(x)^{-2}\cdot g'(x)\right]$$

$$= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$