9.2 Implicit Differentiation

$$f(x) = \sin(\overbrace{x^2}^{y})$$

$$\Rightarrow f'(x) = \cos(x^2) \cdot 2x$$

$$y = x^2 \qquad \frac{dy}{dx} = 2x$$

$$\Rightarrow f(x) = \sin(y)$$

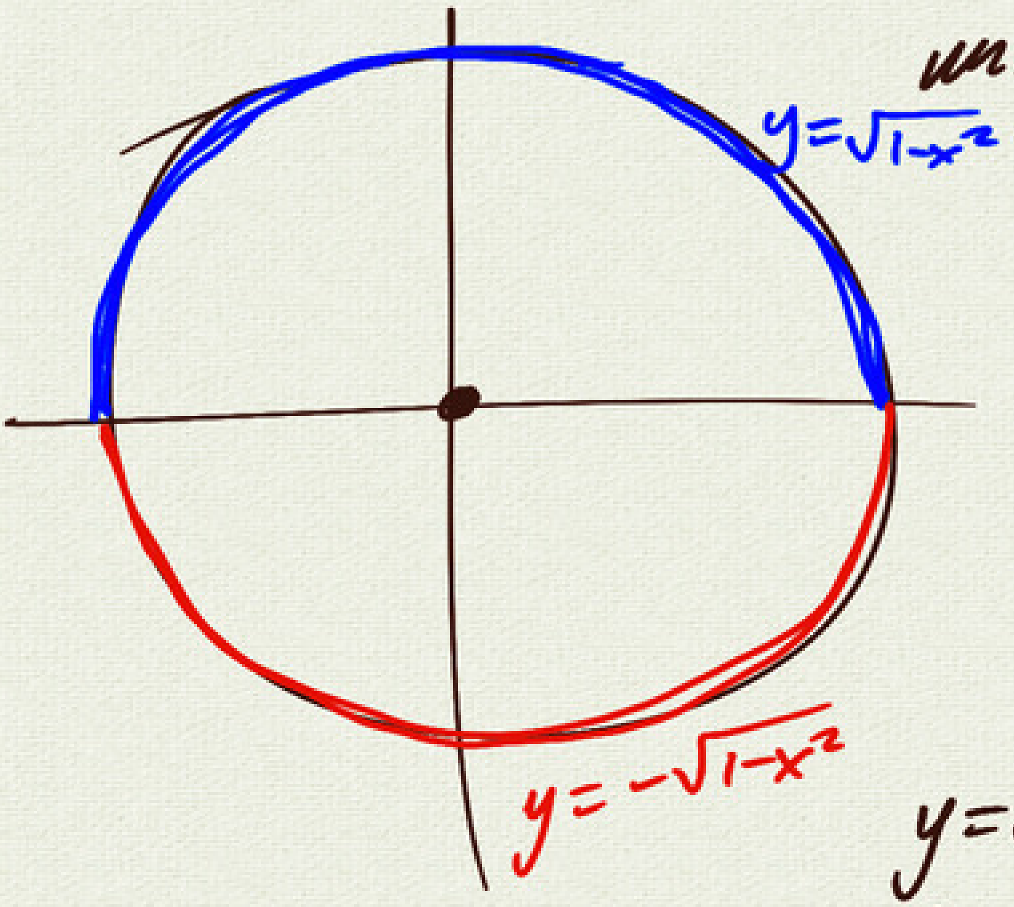
$$f'(x) = \cos(y)\,\frac{dy}{dx}$$

$$\cos(x^2) \cdot 2x$$

$$\frac{d}{dx}(\sin y) = \cos y \cdot \frac{dy}{dx}$$

$$\frac{d}{dx}\left(y^3 + \cos y\right) = 3y^2 \cdot \frac{dy}{dx} - \sin y \cdot \frac{dy}{dx}$$

$$\frac{d}{dx}\left(y^4 + 3x^2\right) = 4y^3 \cdot \frac{dy}{dx} + 6x\underbrace{\left(\frac{dx}{dx}\right)}_{1}$$



unit circle

implicitly defined

$$x^2+y^2=1$$

$$y^2=1-x^2$$

$$y=\pm\sqrt{1-x^2}$$

$$y=\sqrt{1-x^2}$$

$$f(x)=\sqrt{1-x^2}=(1-x^2)^{1/2}$$

$$\Rightarrow f'(x)=?$$

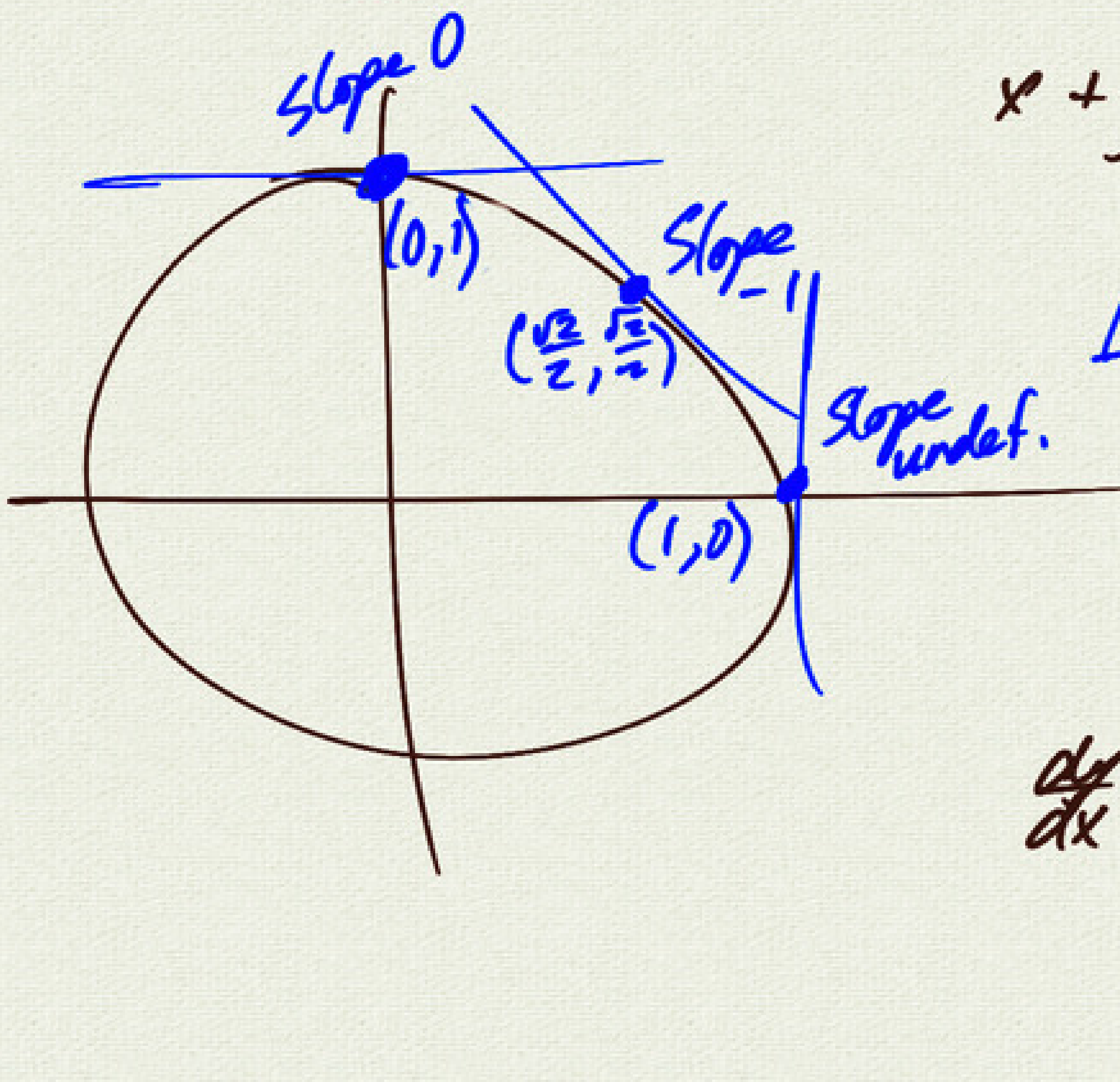
$x^2+y^2=1$

Find $\frac{dy}{dx}$.

$$\to 2x+2y\cdot\frac{dy}{dx}=0$$

$$x+y\frac{dy}{dx}=0$$

$$\boxed{\frac{dy}{dx}=\frac{-x}{y}}$$



$$\frac{dy}{dx}\overset{x\ \ y}{(0,1)}=0$$

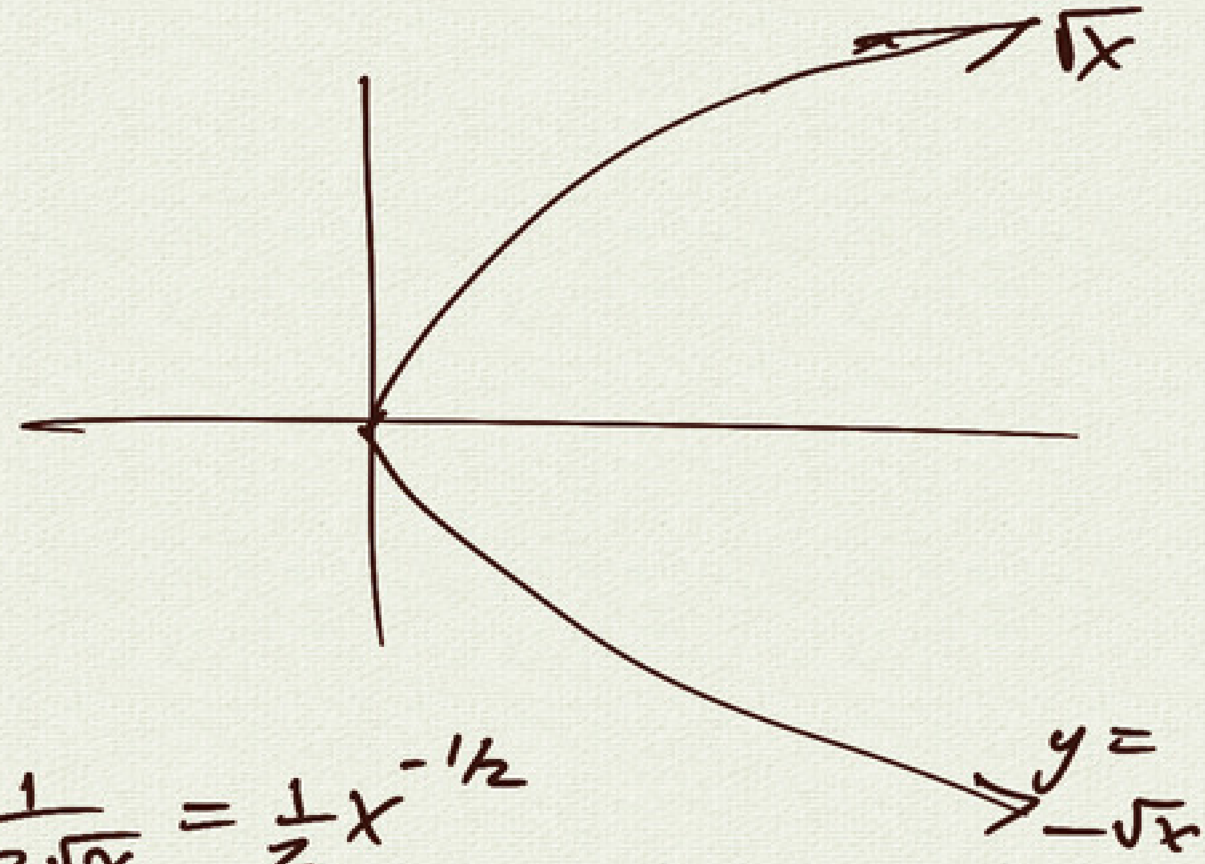
$$\frac{dy}{dx}\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)=-1$$

$$\frac{dy}{dx}(1,0)=\text{undef.}$$

$f(x) = \sqrt{x} = x^{1/2}$

$y = x^{1/2}$

$\boxed{y^2 = x}$



$2y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y} = \frac{1}{2\sqrt{x}} = \frac{1}{2} x^{-1/2}$

power rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

challenge: show for $x^{1/n}$

$x^{p/q}$

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power rule works for rationals

reals :

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}} \quad n \in \mathbb{R}$$

$$\left(f/g\right)'(x) = \frac{d}{dx}\left(f(x) \cdot g(x)^{-1}\right)$$

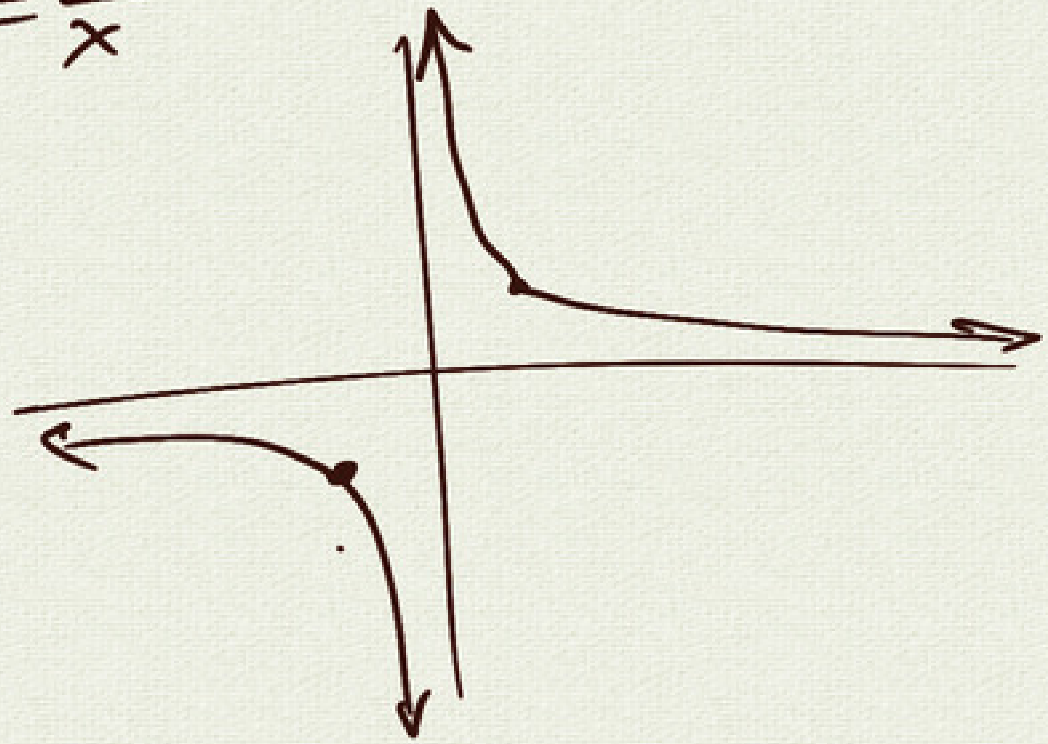
$$= f'(x) g(x)^{-1} + f(x)(-1) g(x)^{-2} g'(x)$$

$$= \frac{f'(x) g(x) - f(x) g'(x)}{g(x)^2}$$

quotient rule

$xy = 1 \implies y = \frac{1}{x}$

find $\frac{dy}{dx}$



① explicit $y = \frac{1}{x} = x^{-1}$

$$\frac{dy}{dx} = -1 \cdot x^{-2}$$

$$= -\frac{1}{x^2}$$

② implicit $xy = 1$

$$1 \cdot y + x\frac{dy}{dx} = 0$$

(product rule)

$$\frac{dy}{dx} = \frac{-y}{x} = \frac{-1}{x^2} \checkmark$$

① differentiate both sides

② solve for dy/dx

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Find $\frac{dy}{dx}$

① differentiate both sides

② solve for $\frac{dy}{dx}$

$$9xy^2 = 2x^3$$

$$\Rightarrow 9\left(1 \cdot y^2 + x \cdot 2y\frac{dy}{dx}\right) = 6x^2$$

$$9y^2 + 18xy\frac{dy}{dx} = 6x^2$$

$$3y^2 + 6xy\frac{dy}{dx} = 2x^2$$

$$\frac{dy}{dx} = \frac{2x^2 - 3y^2}{6xy}$$

$$= \frac{x}{3y} - \frac{y}{2x}$$