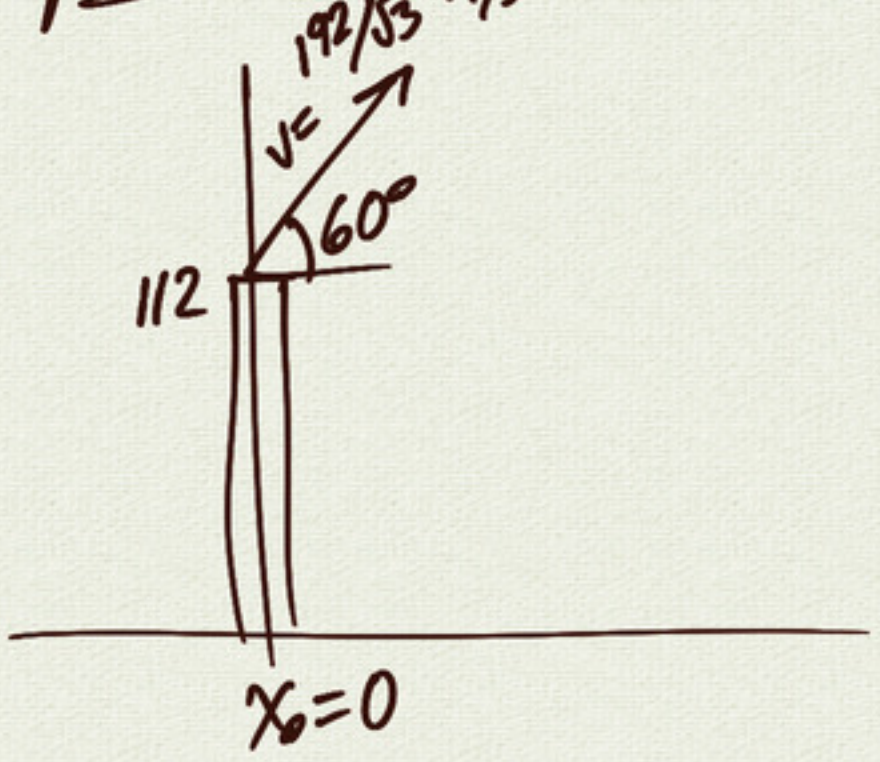
9.5 Applications

$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

(283) $y = (1+\tan^{-1}x)^3$

$\Rightarrow \frac{dy}{dx} = 3(1+\tan^{-1}x)^2 \cdot \frac{1}{1+x^2}$

projectile motion



$x(t) = x_0 + v_x t$

$y(t) = y_0 + v_y t - 16t^2$

$y_0 = 112$

$v = 192/\sqrt{3}$

$v_y = v\sin 60° = \frac{192}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} = 96$

$v_x = v\cos 60° = \frac{192}{\sqrt{3}} \cdot \frac{1}{2} = 96/\sqrt{3}$

$x(t) = \frac{96}{\sqrt{3}}t$

$y(t) = 112 + 96t - 16t^2$

$x'(t) = \frac{96}{\sqrt{3}}$ speed in x-direction (constant)

$y'(t) = 96 - 32t$ speed in y-dir (decreasing)

$x''(t) = 0$ constant acceleration $\downarrow -32$ ft/s²

$y''(t) = -32$

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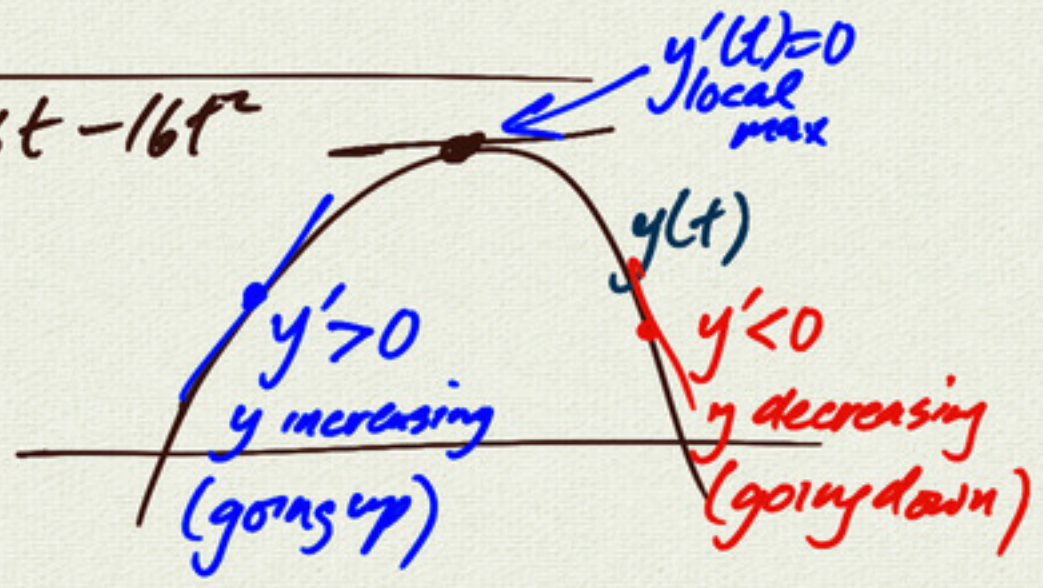
max height

$y(t) = 112 + 96t - 16t^2$

$y'(t) = 96 - 32t$

$y'(t) = 0 \Rightarrow 96 - 32t = 0$

$t = 3$



max height (max y-value) $= y(3) = 112 + \underbrace{96\cdot 3}_{288} - \underbrace{16\cdot 9}_{144}$

$= 256$

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when does it hit ground? (+ how far)

$y(t) = 0$

$y(t) = 112 + 96t - 16t^2$

$= -16(t^2 - 6t - 7)$

$= -16(t-7)(t+1)$

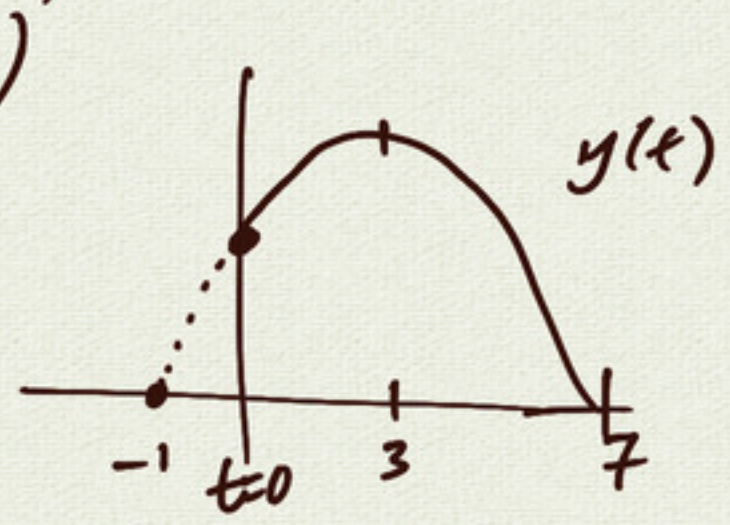
$t = 7$

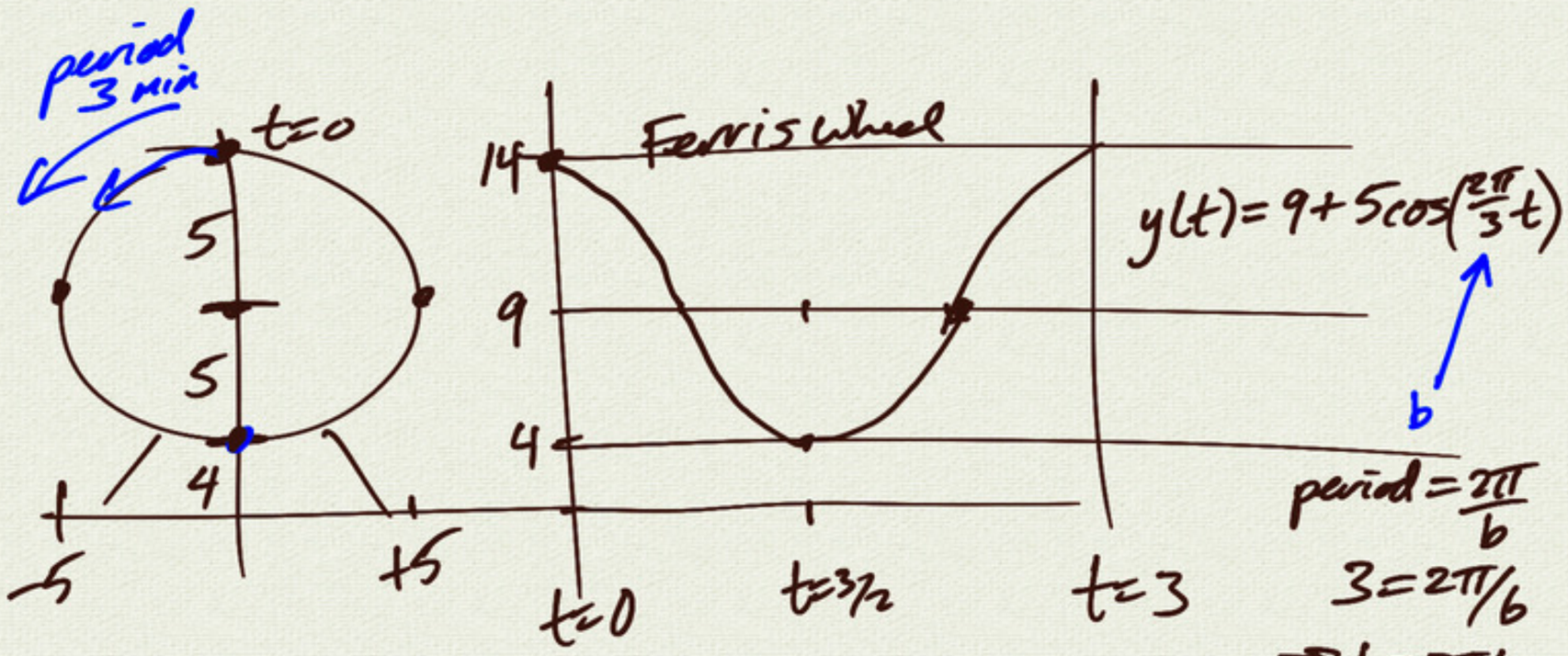
$\frac{112}{4} = 28$

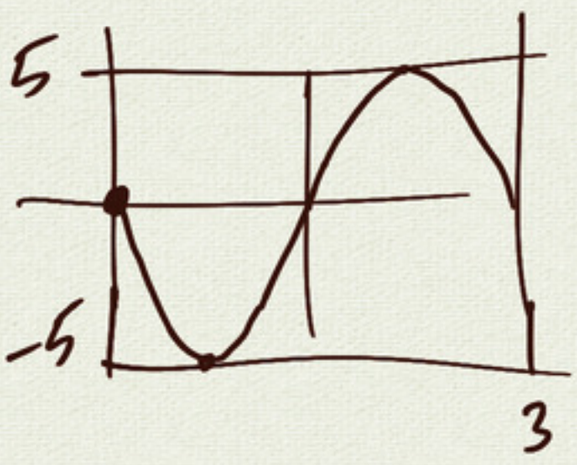
$\frac{112}{16} = 7$

how far?

$x(7) = \frac{96}{\sqrt{3}} \cdot 7$







$x(t) = -5\sin\left(\frac{2\pi}{3}t\right)$

$y(t) = 9 + 5\cos\left(\frac{2\pi}{3}t\right)$

$x'(t) = -5\cos\left(\frac{2\pi}{3}t\right)\cdot\left(\frac{2\pi}{3}\right)$

$= -\frac{10\pi}{3}\cos\left(\frac{2\pi}{3}t\right)$

$y'(t) = -5\sin\left(\frac{2\pi}{3}t\right)\cdot\frac{2\pi}{3}$

$= -\frac{10\pi}{3}\sin\left(\frac{2\pi}{3}t\right)$

$x''(t) = -\frac{10\pi}{3}\left(-\sin\left(\frac{2\pi}{3}t\right)\cdot\frac{2\pi}{3}\right)$

$= +\frac{20\pi^2}{9}\sin\left(\frac{2\pi}{3}t\right)$

$y''(t) = -\frac{10\pi}{3}\cos\left(\frac{2\pi}{3}t\right)\cdot\frac{2\pi}{3}$

$= -\frac{20\pi^2}{9}\cos\left(\frac{2\pi}{3}t\right)$

$x(t) = -5\sin\left(\frac{2\pi}{3}t\right)$

Check: $t = \frac{3}{2}$ should be bottom

$x\left(\frac{3}{2}\right) = -5\sin(\pi) = 0$

$y\left(\frac{3}{2}\right) = 9 + 5\cos(\pi) = 4$

$(0,4)$ bottom ✓

$\frac{d}{dx}(cf) = c\frac{df}{dx}$

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velocity at $t = 3/2$ (bottom)

$x'\left(\frac{3}{2}\right) = -\frac{10\pi}{3}\cos\pi$

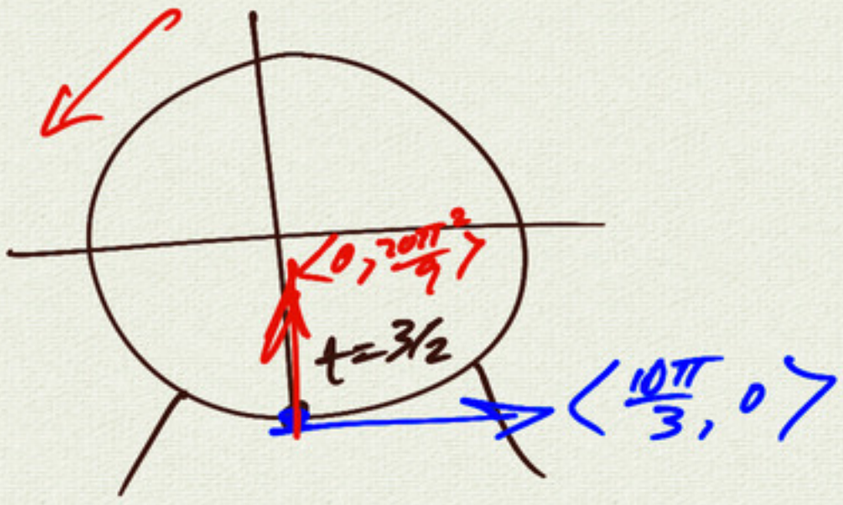
$= +\frac{10\pi}{3}$

$y'\left(\frac{3}{2}\right) = -\frac{10\pi}{3}\sin\pi = 0$

$\left\langle \frac{10\pi}{3}, 0 \right\rangle$

$x''\left(\frac{3}{2}\right) = 0$

acceleration $y''\left(\frac{3}{2}\right) = +\frac{20\pi^2}{9}$ $\left\langle 0, \frac{20\pi^2}{9} \right\rangle$



radioactive decay

half life 4000 years

starting amount 1024

$A(t)$ amount

$A_0 = A(0)$

t	$A(t)$
0	1024
4000	512 $= 1024 \cdot \frac{1}{2}$
8000	256 $= 1024\left(\frac{1}{2}\right)^2$
12000	128 $= 1024\left(\frac{1}{2}\right)^3$

$$A(t) = 1024 \cdot \left(\frac{1}{2}\right)^{t/4000}$$

$t/4000$ ← # half lives

in general: $A(t) = A_0 \left(\frac{1}{2}\right)^{t/4000}$

$$A(t) = 1024 \cdot \left(\frac{1}{2}\right)^{t/4000}$$

$$A'(t) = 1024 \cdot \left(\frac{1}{2}\right)^{t/4000} \left(\ln \tfrac{1}{2}\right) \cdot \frac{1}{4000}$$

$\ln \frac{1}{2} < 0$

$\Rightarrow A'(t) < 0$

A decreasing

$\frac{d}{dx}(e^x) = e^x$

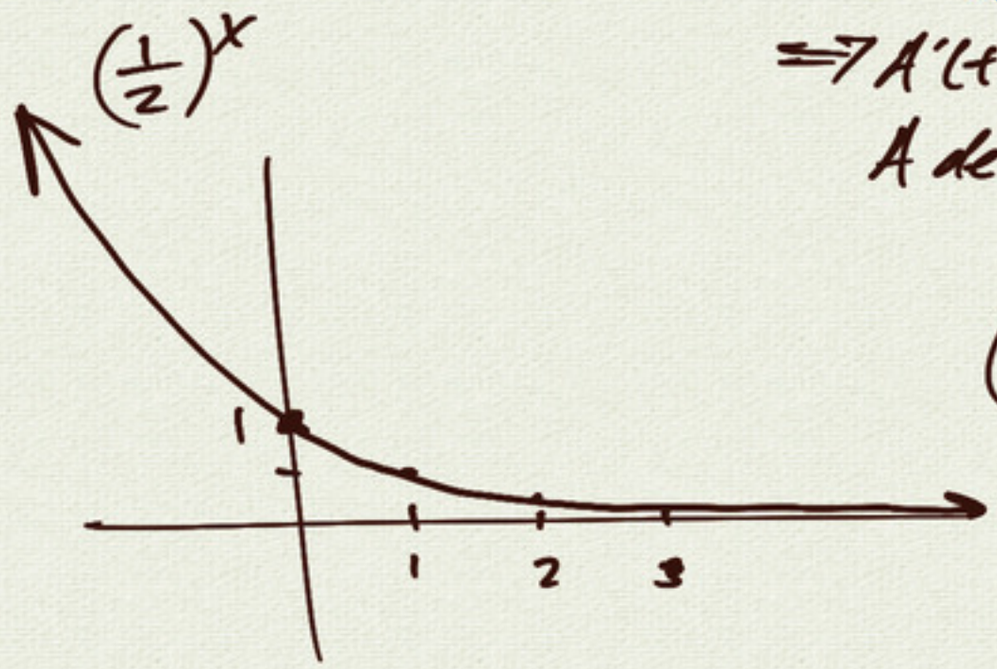
$\frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(2^x) = 2^x \ln 2$

$\frac{d}{dx}\left[\left(\frac{1}{2}\right)^x\right] = \left(\frac{1}{2}\right)^x \ln \frac{1}{2}$

$\ln \frac{1}{2} = \ln 2^{-1}$

$= -\ln 2 < 0$



$$\left(\frac{1}{2}\right)^x = \frac{1}{2^x} = 2^{-x}$$

reflection of 2^x across y-axis

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example: 8 g left of substance $A(t) = 1024\left(\frac{1}{2}\right)^{t/4000}$

$\Rightarrow$ how old is substance?

$A(t) = 8 \Rightarrow$ find t

$$8 = 1024\left(\frac{1}{2}\right)^{t/4000}$$

2^3 → 2^{10}

$$\frac{1}{2^7} = \left(\frac{1}{2}\right)^{t/4000}$$

$$7 = t/4000$$

$$t = 28000$$