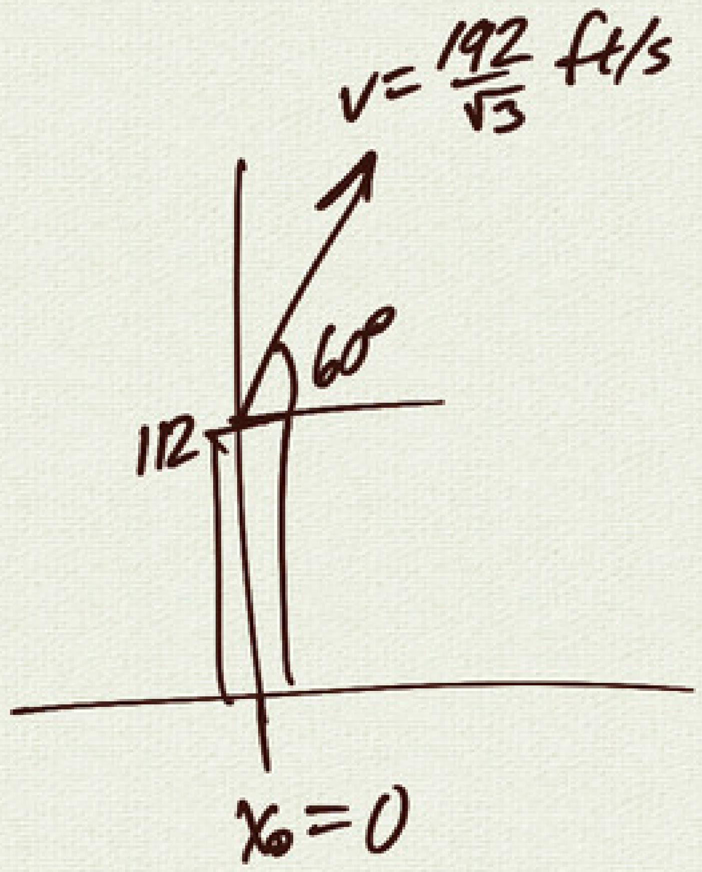
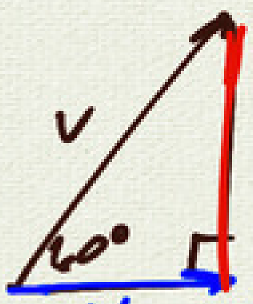
9.5 Applications

projectile motion



$$x(t)=x_0+\underline{v_x}t$$

$$y(t)=\underbrace{y_0}_{112}+\underline{v_y}t-16t^2$$



$$v_y=v\sin 60^\circ = \frac{192}{\sqrt{3}}\cdot\frac{\sqrt{3}}{2}=96$$

$$v_x=v\cos 60^\circ = \frac{192}{\sqrt{3}}\cdot\frac{1}{2} = \frac{96}{\sqrt{3}}$$

$$x(t)=\frac{96}{\sqrt{3}}t$$

$$y(t)=112+96t-16t^2$$

$$x'(t)=\frac{96}{\sqrt{3}}$$

velocity

$$y'(t)=96-32t$$

$$x''(t)=0$$

acceleration

$$y''(t)=-32$$

max height:

$$y'(t)=0$$

$$96-32t=0$$

$$t=3$$

$$y(3)=112+\underbrace{96\cdot 3}_{288}-\underbrace{16\cdot 9}_{144}$$

$$=256$$

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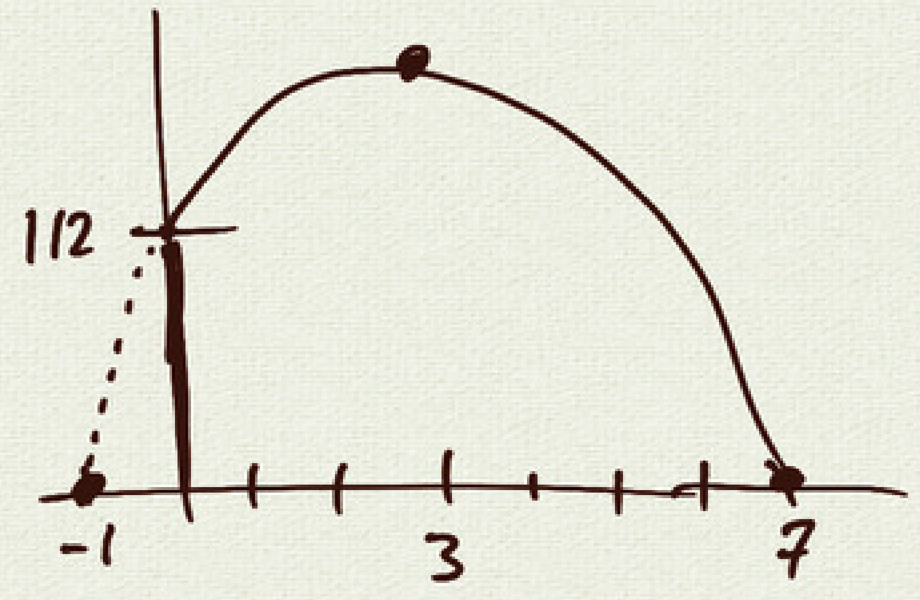
when does it hit ground?

$$y(t)=0 \implies y(t)=-16t^2+96t+112$$

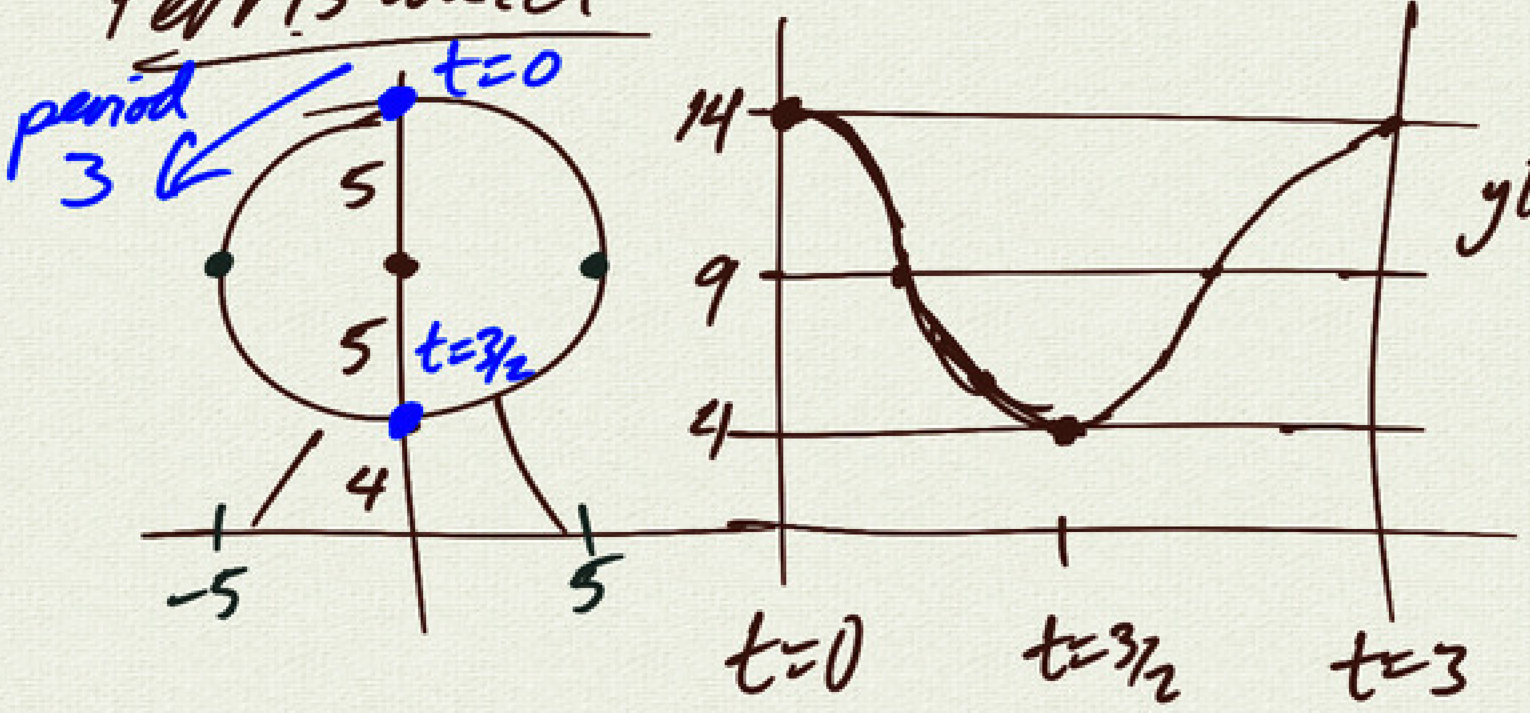
$$=-16(t^2-6t-7)$$

$$=-16(t-7)(t+1)$$

$$y(t)=0 \implies t=7 \text{ or } t=-1$$



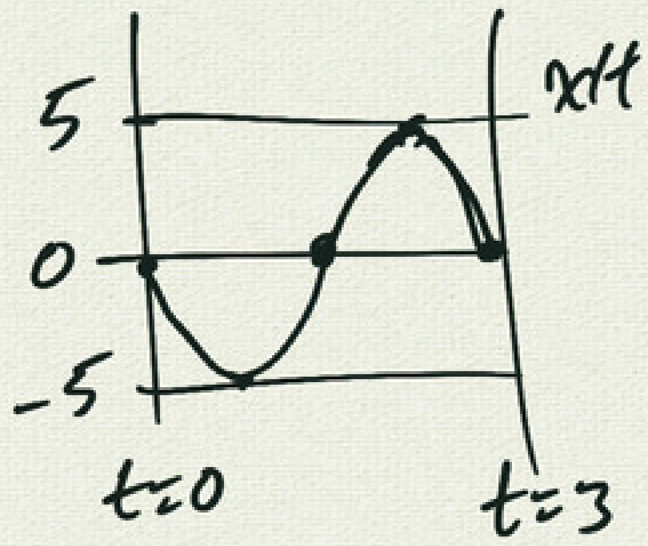
Ferris Wheel



$y(t) = 9 + 5\cos(\underset{b}{\frac{2\pi}{3}} t)$

period $= \frac{2\pi}{b} = 3$

$b = \frac{2\pi}{3}$



$x(t) = -5\sin(\frac{2\pi}{3}t)$

check $t = 3/2$: (bottom)

$x(\frac{3}{2}) = -5\sin(\pi)$
$= 0$

$y(\frac{3}{2}) = 9 + 5\cos(\pi)$
$= 9 - 5$
$= 4$ ✓

(0,4) bottom

$x(t) = -5\sin(\frac{2\pi}{3}t)$
$y(t) = 9 + 5\cos(\frac{2\pi}{3}t)$
position

$x'(t) = -5\cos(\underbrace{\frac{2\pi}{3}t}) \cdot \frac{2\pi}{3}$

$= -\frac{10\pi}{3}\cos(\frac{2\pi}{3}t)$

$y'(t) = -5\sin(\frac{2\pi}{3}t) \cdot \frac{2\pi}{3}$

$= -\frac{10\pi}{3}\sin(\frac{2\pi}{3}t)$

velocity

at $t = 3/2$:

$x'(\frac{3}{2}) = -\frac{10\pi}{3}\underbrace{\cos\pi}_{-1}$
$= 10\pi/3$

$\langle \frac{10\pi}{3}, 0 \rangle$

$y'(\frac{3}{2}) = -\frac{10\pi}{3}\underbrace{\sin\pi}_{0}$
$= 0$

$x''(t) = +\frac{10\pi}{3}\sin(\frac{2\pi}{3}t) \cdot \frac{2\pi}{3}$

$= \frac{20\pi^2}{9}\sin(\frac{2\pi}{3}t)$

$y''(t) = -\frac{10\pi}{3}\cos(\frac{2\pi}{3}t) \cdot \frac{2\pi}{3}$

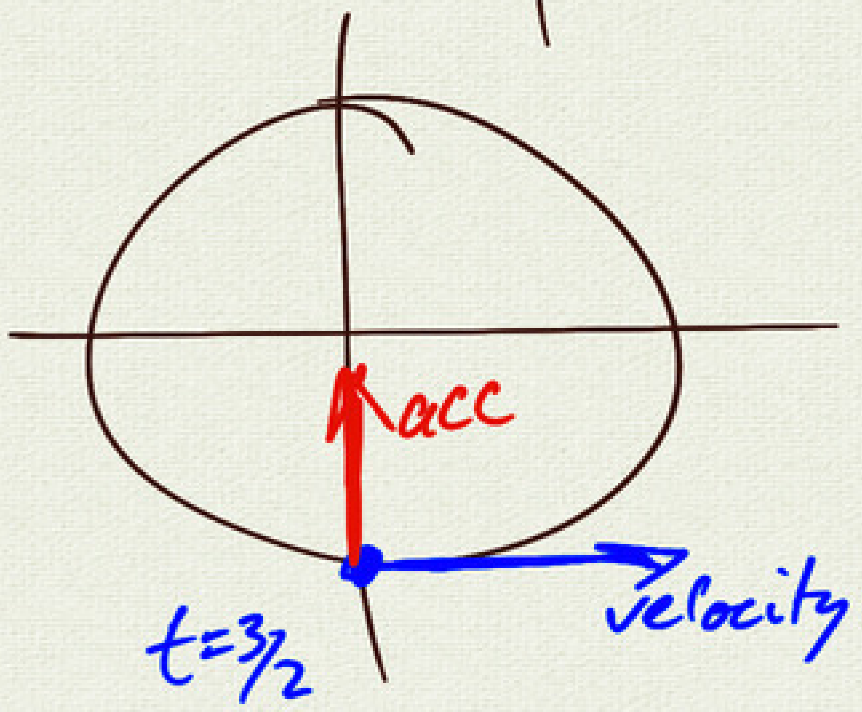
$= -\frac{20\pi^2}{9}\cos(\frac{2\pi}{3}t)$

acceleration

$x''(\frac{3}{2}) = \frac{20\pi^2}{9}\sin\pi = 0$

$y''(\frac{3}{2}) = -\frac{20\pi^2}{9}\cos\pi$
$= +\frac{20\pi^2}{9}$

$\langle 0, \frac{20\pi^2}{9} \rangle$



radioactive decay

half-life 4000 years
initial amount 1024
amount $A(t)$

t	$A(t)$	
0	1024	$\frac{1}{2}$
4000	$512 = 1024\cdot\frac{1}{2}$	$\frac{1}{2}$
8000	$256 = 1024\left(\frac{1}{2}\right)^2$	
12000	$128 = 1024\left(\frac{1}{2}\right)^3$	

$$A(t) = 1024\cdot\left(\frac{1}{2}\right)^{(t/4000)}$$

half-lives

$$A'(t) = 1024\left(\frac{1}{2}\right)^{\underline{t/9000}}\cdot\underbrace{\ln\frac{1}{2}}_{<0}\cdot\frac{1}{4000}$$

$$\frac{d}{dx}(a^x) = a^x\ln a$$

$$\frac{d}{dx}(2^x) = 2^x\ln 2$$

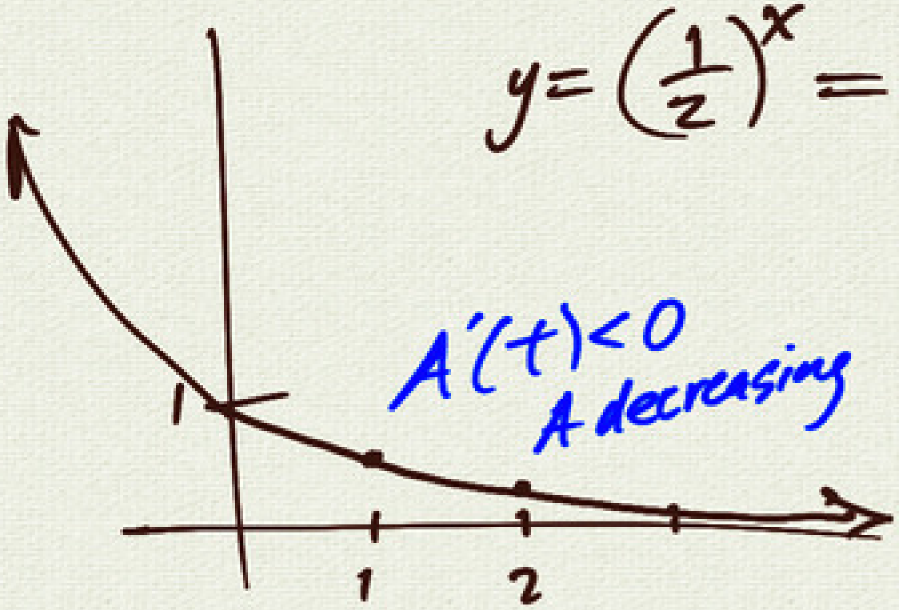
$$\frac{d}{dx}\left(\left(\frac{1}{2}\right)^x\right) = \left(\frac{1}{2}\right)^x\underbrace{\ln\frac{1}{2}}$$

$$\ln\frac{1}{2} = \ln 2^{-1} = -\ln 2$$

$$\ln\frac{1}{2} < 0$$

$$y = \left(\frac{1}{2}\right)^x = \frac{1}{2^x} = 2^{-x}$$

reflection of 2^x



$A'(t) < 0$
A decreasing