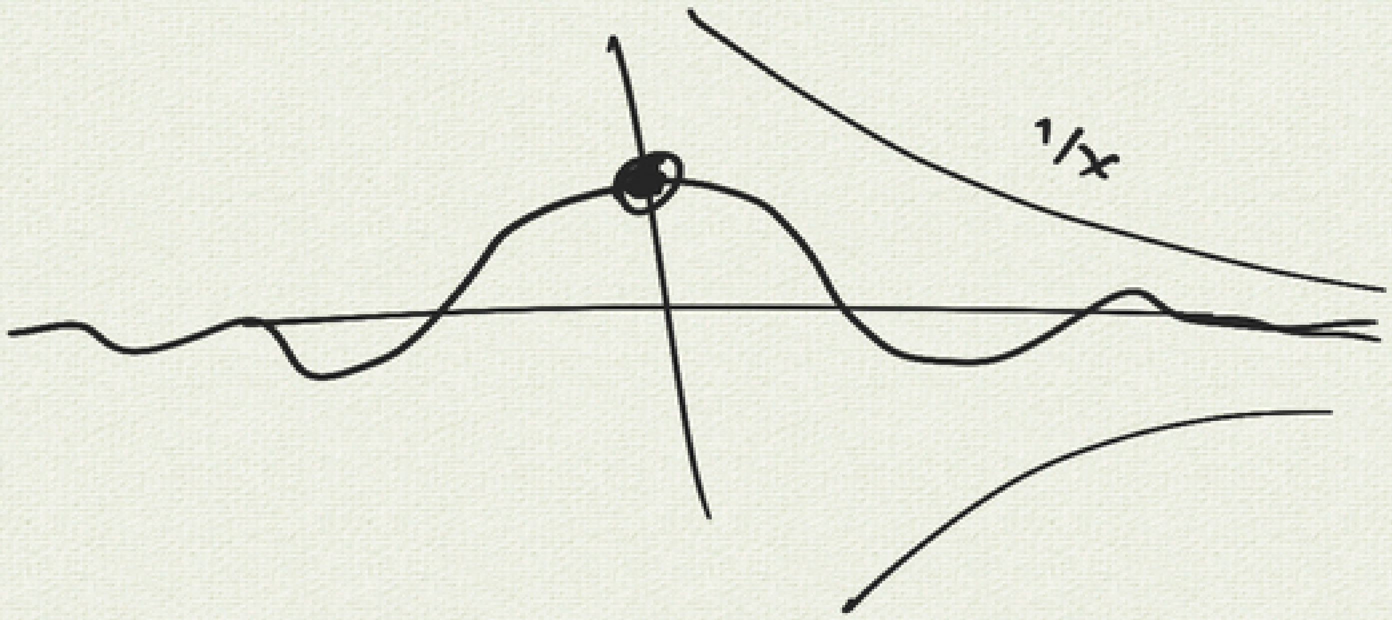
(1a)

$$\lim_{x\to0}\frac{\sin x}{x}=1$$



$$\lim_{x\to0}\frac{\sin 2x}{2x}=1$$

$$\Rightarrow \lim_{x\to0}\frac{\sin 2x}{x}\cdot\frac{2}{2}=2$$

(1a) $$\lim_{x\to0} x\csc\left(\frac{x}{3}\right) = \lim_{x\to0}\underbrace{\frac{x}{\sin(x/3)}\cdot\frac{1/3}{1/3}}_{\to 1}$$

$$=3$$

$$\lim_{x\to0}\frac{\sin(x/3)}{\left(\frac{x}{3}\right)}=1$$

$$\lim_{x\to0}\boxed{\frac{\left(\frac{x}{3}\right)}{\sin\left(\frac{x}{3}\right)}}=1$$

---

(4) $\boxed{x=10^y}\Rightarrow y=\log_{10}x$

$$1=10^y\ln 10\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{1}{10^y\ln 10}$$

$$=\frac{1}{x\ln 10}\ \checkmark$$

$$\frac{dy}{dx}=\frac{1}{x\ln 10}$$

$\boxed{x}^n$ power

$e^{\boxed{x}}$ exp

(2a) $$\lim_{h\to0}\frac{\cos 2h-1}{h}$$

$$=\lim_{h\to0}\underbrace{\frac{\cos 2h-1}{h}\cdot\frac{2}{2}}_{\to 0}$$

$$=0$$

$$\lim_{x\to0}\frac{\cos x-1}{x}=0$$

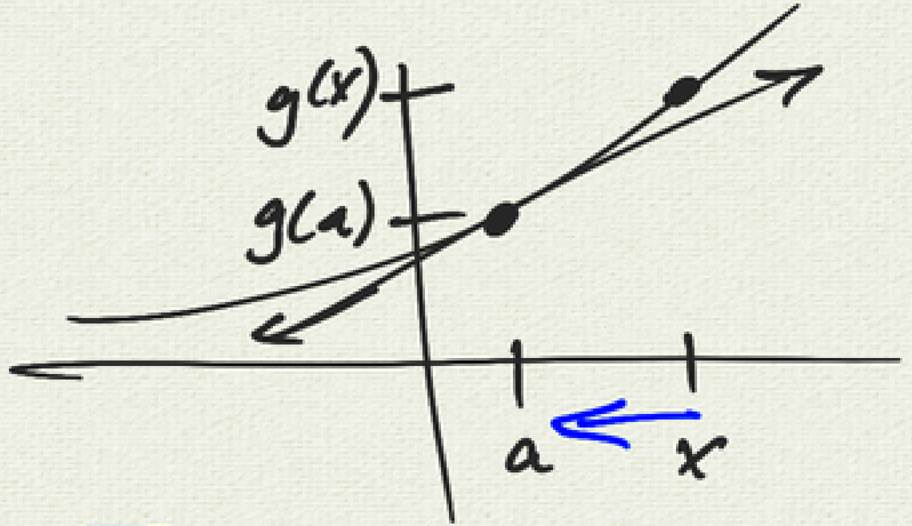
$$\Rightarrow \lim_{x\to0}\frac{(\cos 2x)-1}{2x}=0$$

$2x\to0$

(2b)

$$g(x) = \frac{1}{x}$$

$$g'(a) = \lim_{x \to a} \frac{g(x) - g(a)}{x - a}$$



$$= \lim_{x \to a} \frac{\frac{1}{x} - \frac{1}{a}}{x - a}$$

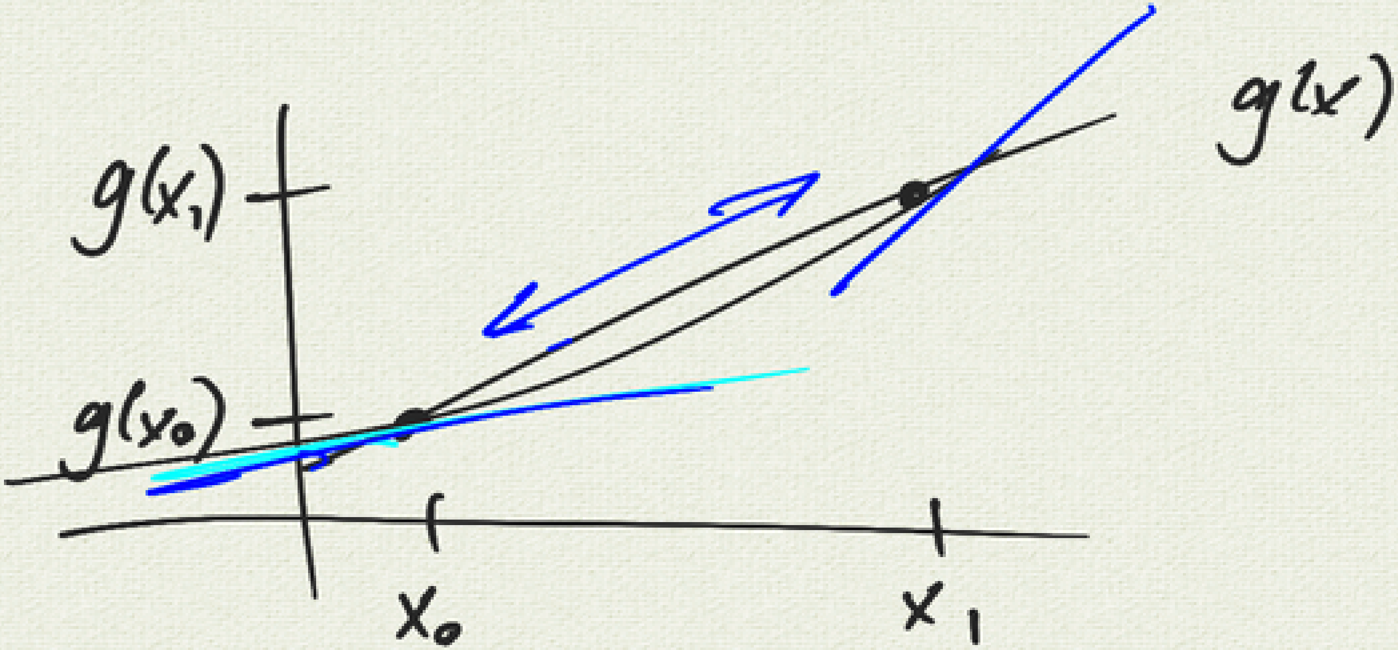
$$\boxed{\frac{1}{x} - \frac{1}{a} = \frac{a - x}{ax}}$$

$$= \lim_{x \to a} \left[\frac{a - x}{ax}\right] \frac{1}{x - a}$$

$$a - x = -(x - a)$$

$$= \lim_{x \to a} -\frac{1}{ax}$$

$$\boxed{g'(a) = -\frac{1}{a^2}} \Longrightarrow \boxed{g'(x) = -\frac{1}{x^2}}$$



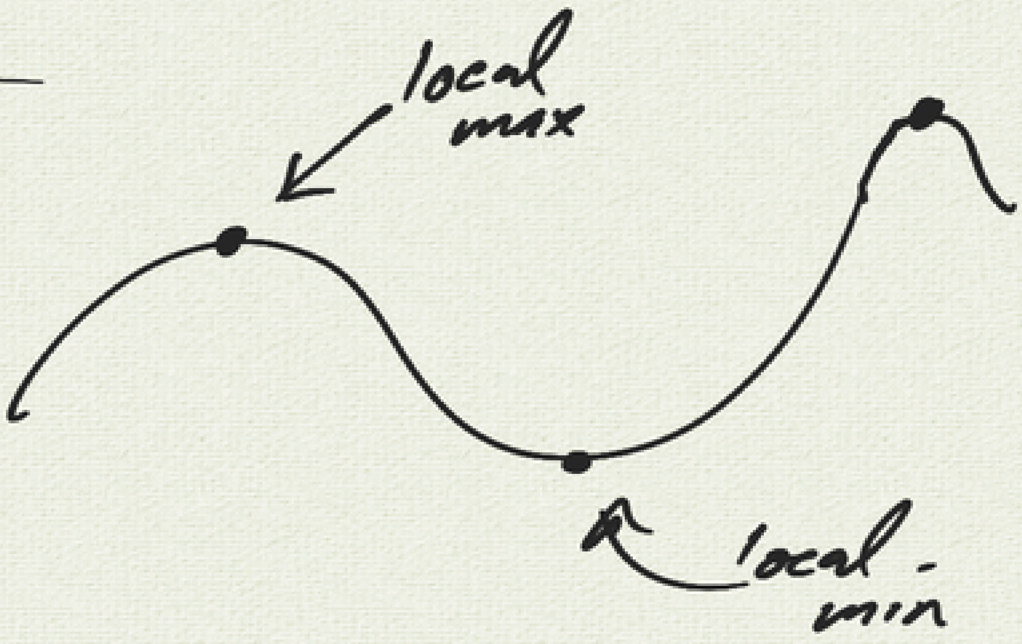
average rate of change = slope of secant $= \dfrac{\Delta y}{\Delta x}$

$$= \frac{g(x_1) - g(x_0)}{x_1 - x_0}$$

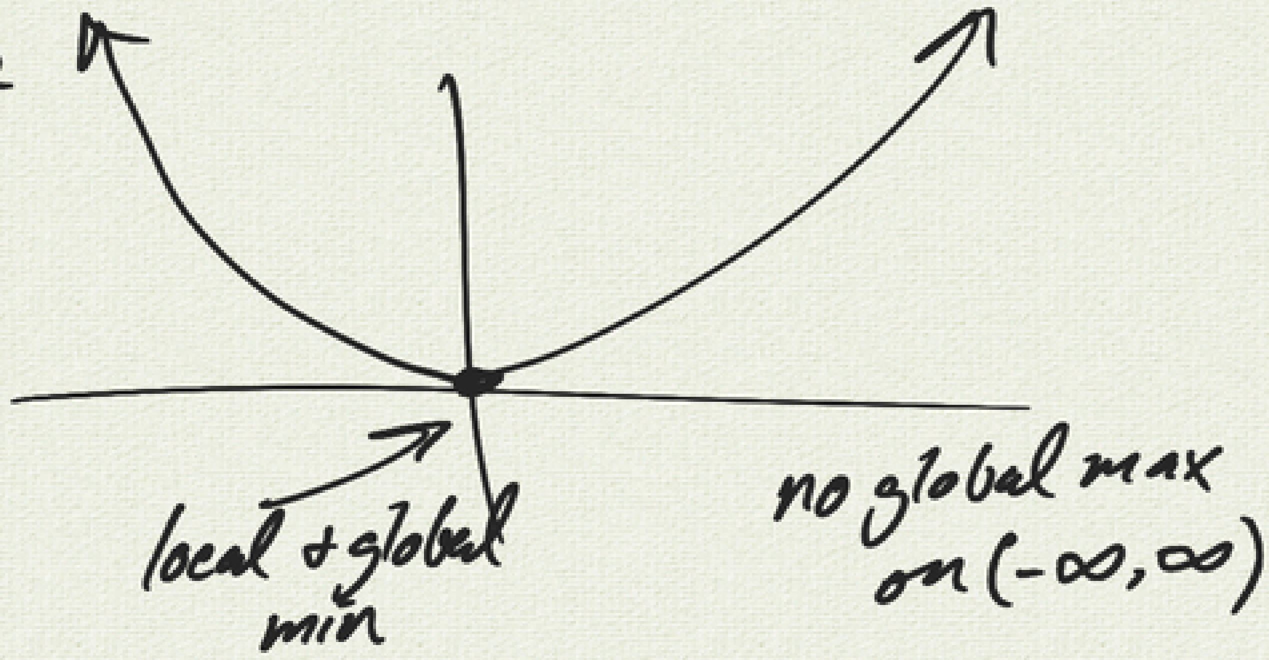
10.1 Extreme values

local min/max
(relative)

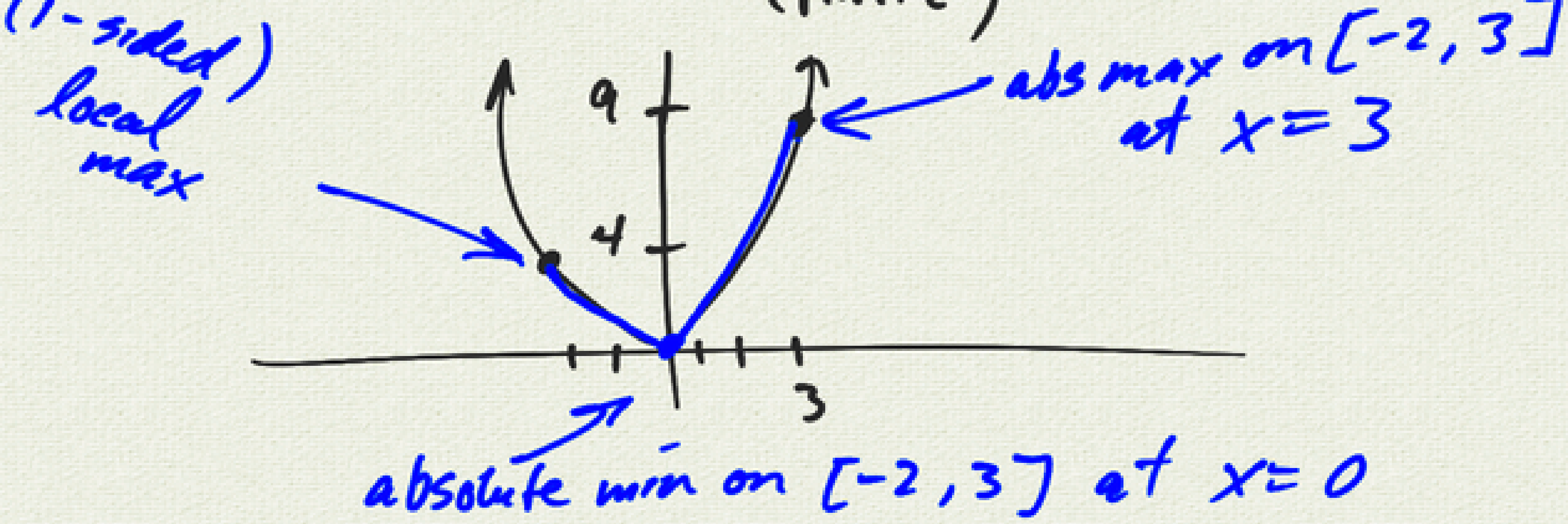
global min/max
(absolute)



example: $f(x) = x^2$

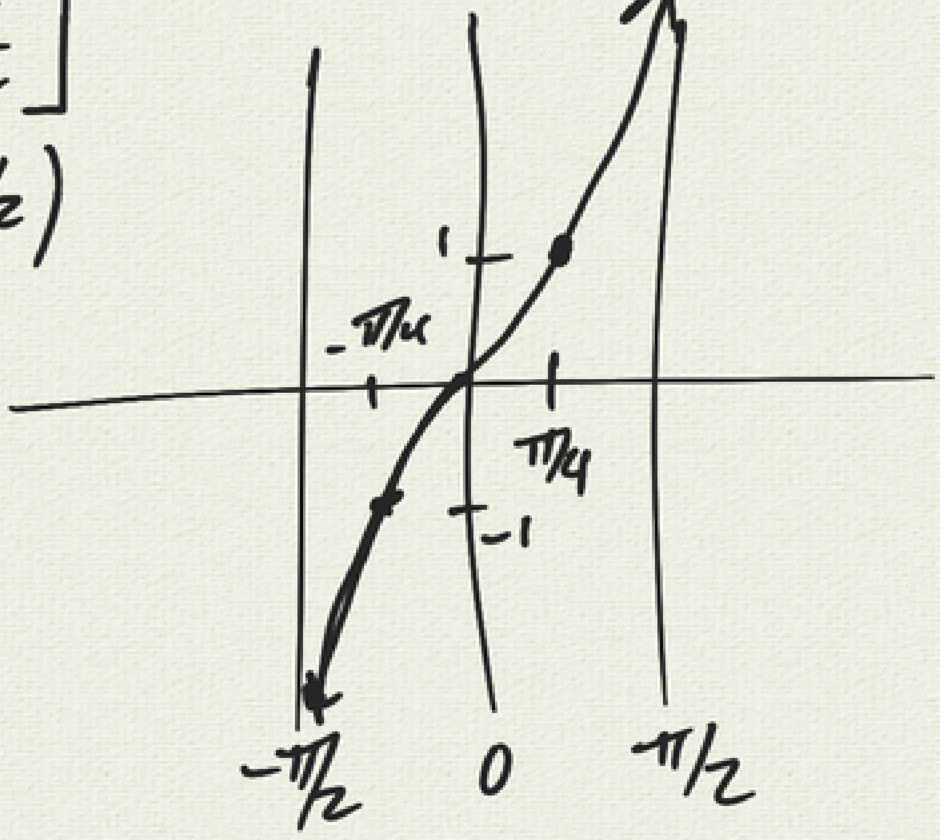


restrict to $[-2, 3]$ closed interval (finite)

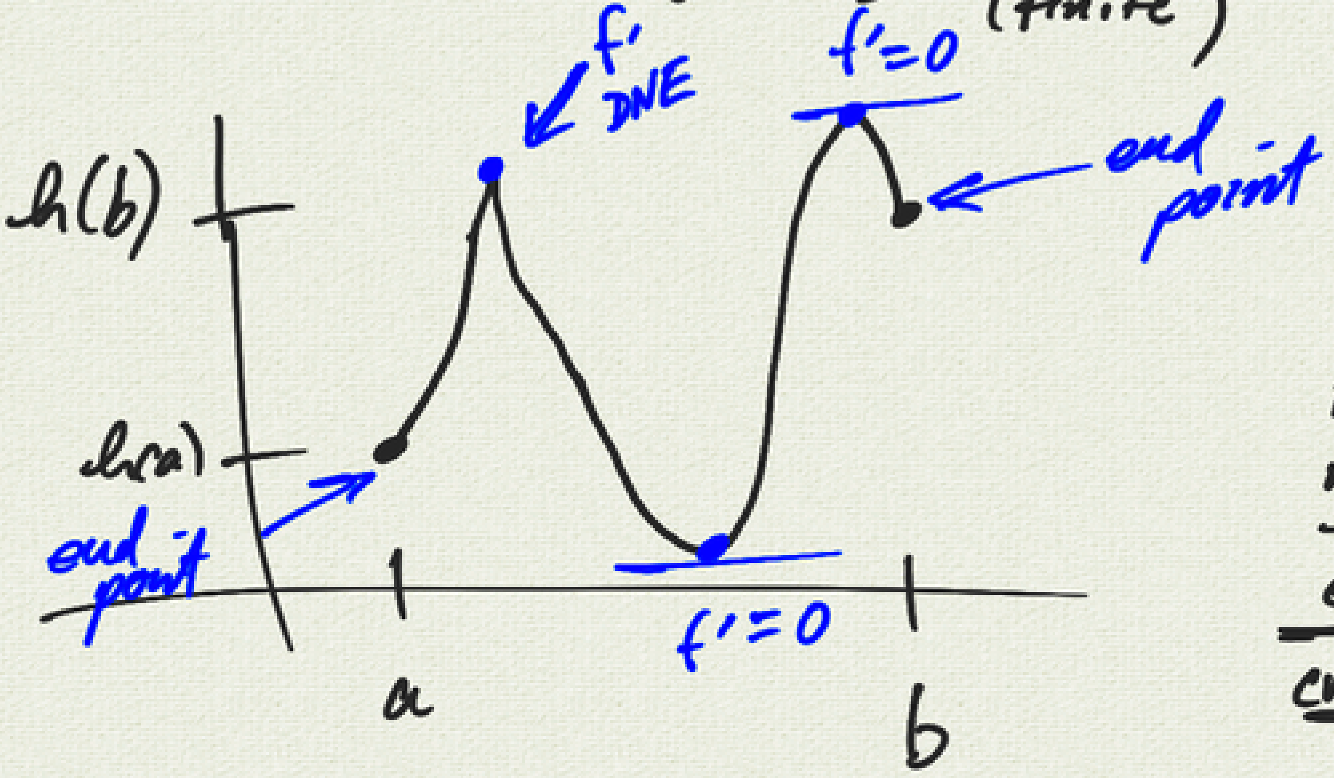


$g(x) = \tan x$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$
$(-\pi/2, \pi/2)$

no local/absolute min or max



$h(x)$ continuous on $[a, b]$ closed interval (finite)



minima
maxima

extrema | extremum

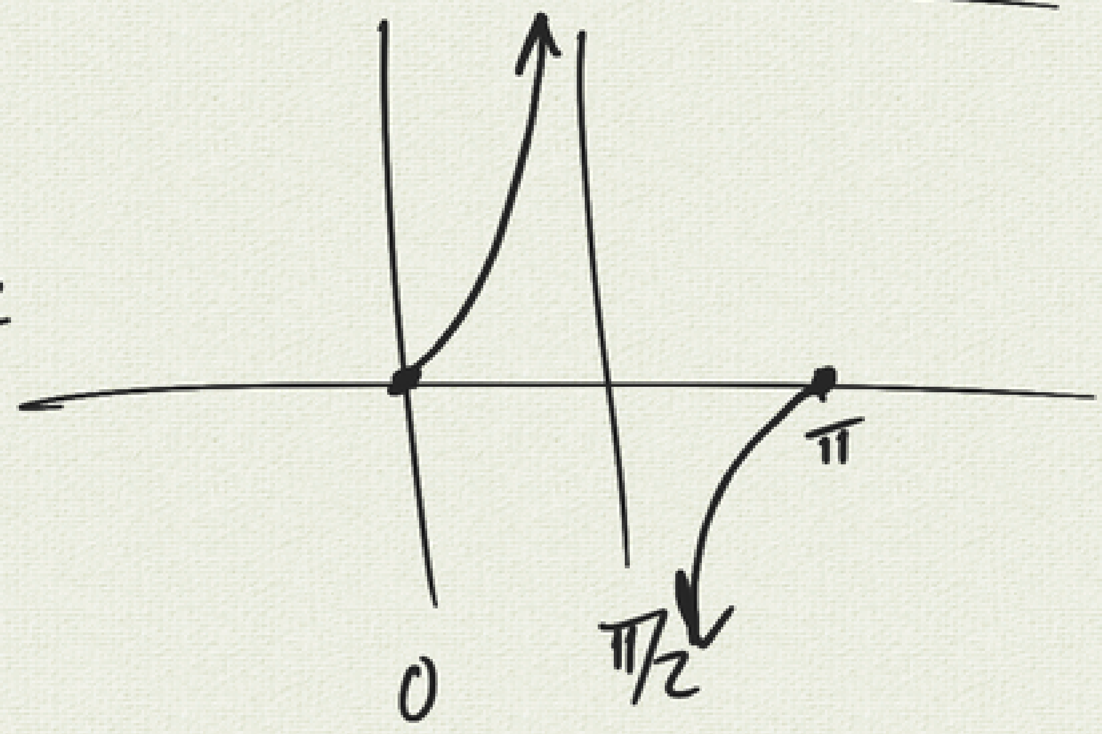
critical point $f'=0$ or f' DNE

Extreme value theorem: if a function $f(x)$ is continuous on finite closed interval $[a, b]$, then f has an absolute min and max on $[a, b]$.

$\tan x$ on $[0, \pi]$

⇑

not continuous at $\pi/2$



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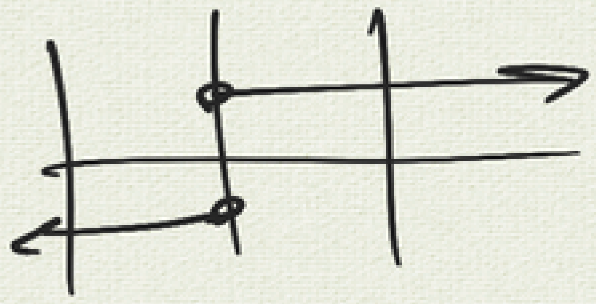
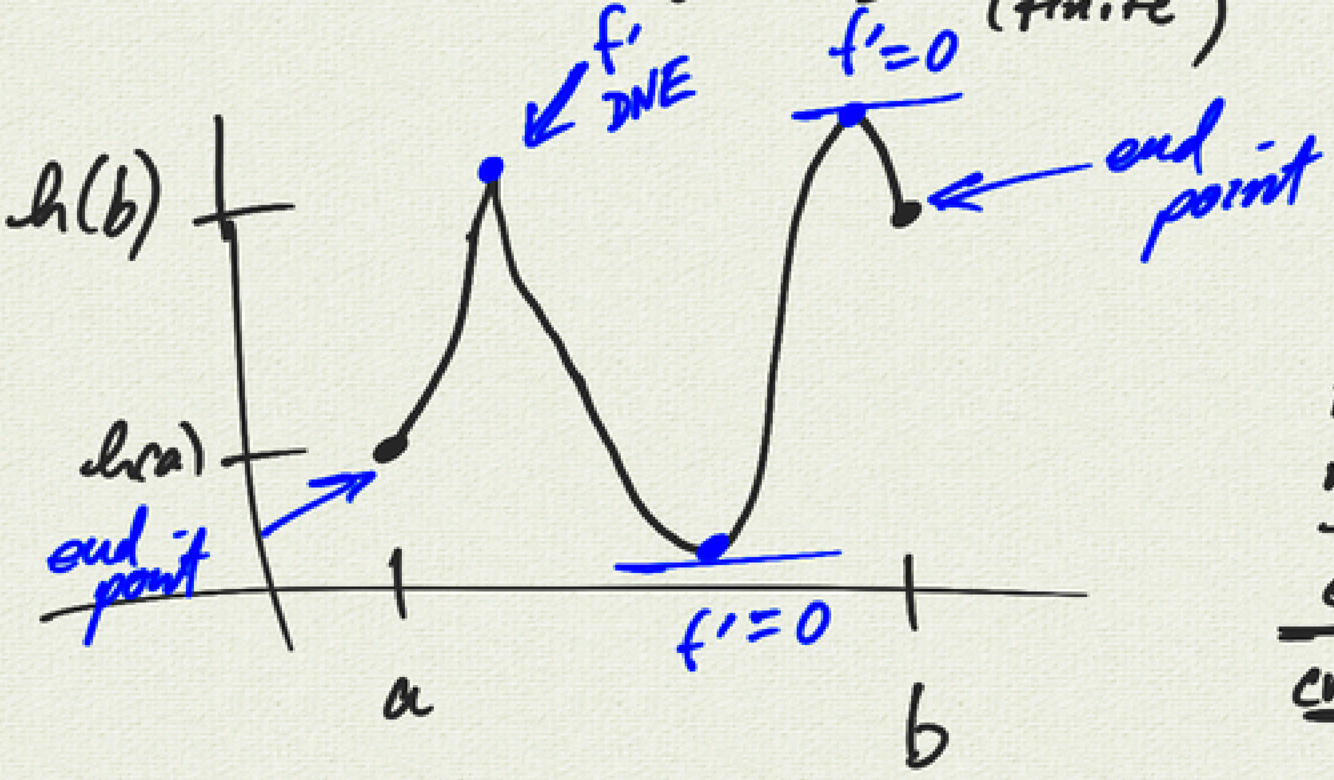
f continuous on closed interval $\Longrightarrow$ f has min & max

$\Longleftarrow$ converse not necessarily true

$f(x) = \frac{x}{|x|}$



$h(x)$ continuous on $[a, b]$ closed interval (finite)



minima
maxima

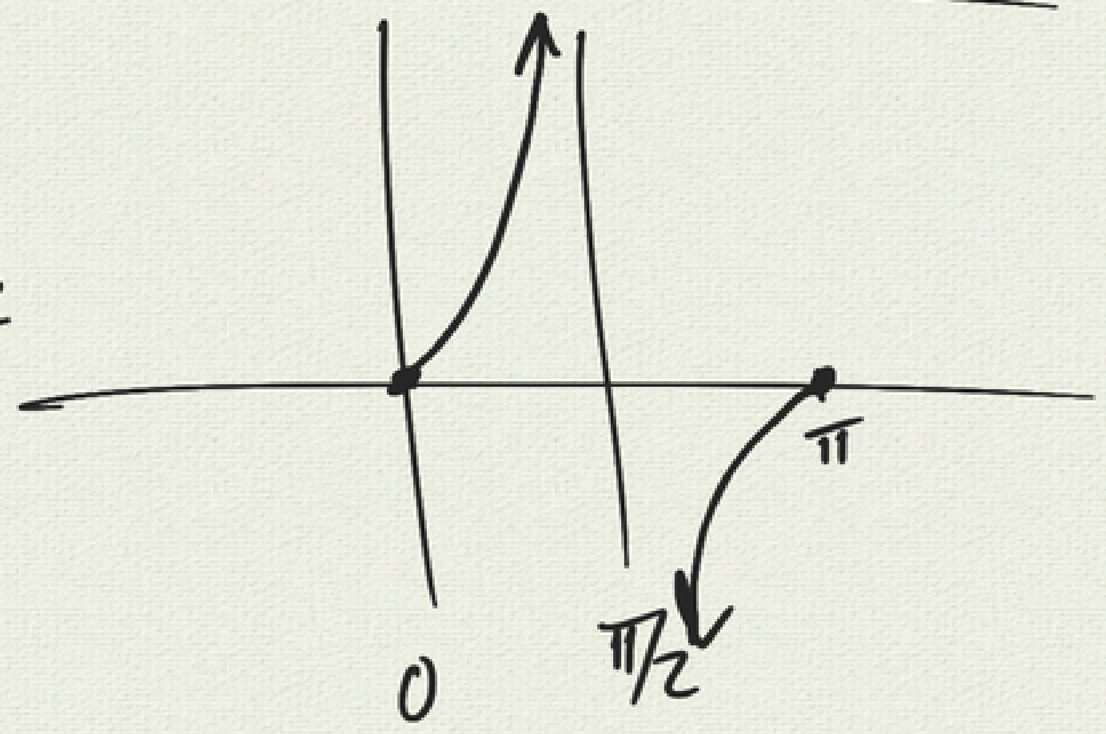
extrema | extremum

critical point $f'=0$ or f' DNE

Extreme value theorem: if a function $f(x)$ is continuous on finite closed interval $[a,b]$, then f has an absolute min and max on $[a,b]$.

$\tan x$ on $[0, \pi]$

not continuous at $\pi/2$



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f continuous on closed interval $\Longrightarrow$ f has min & max

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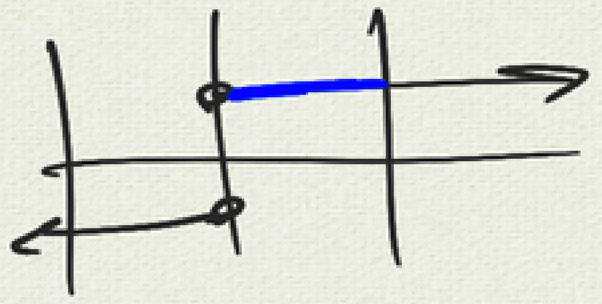
$f(x) = \frac{x}{|x|}$

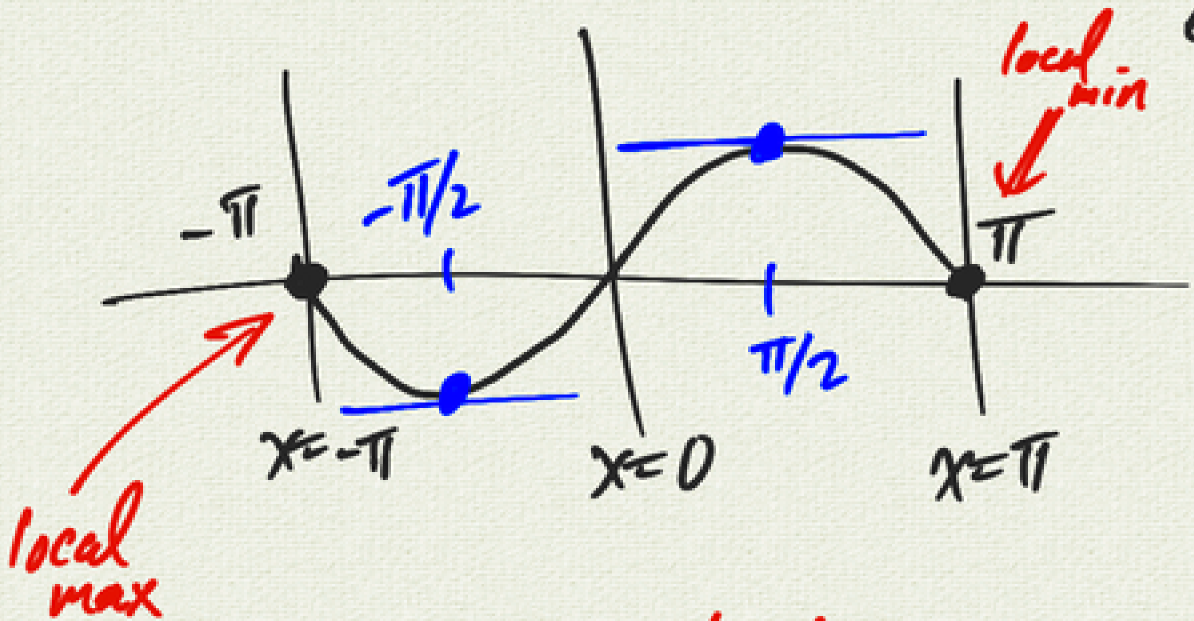


example

$g(x) = \sin x$ continuous

find all local/absolute min/max

on $[-\pi, \pi]$ finite closed interval



end points:

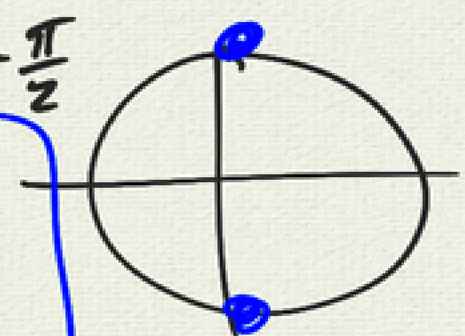
$$\boxed{\begin{aligned} \sin(-\pi) &= 0 \\ \sin(\pi) &= 0 \end{aligned}}$$

critical points:

$g'(x) = \cos x$

$g'(x) = 0 \Rightarrow \cos x = 0$

$x = \frac{\pi}{2}, -\frac{\pi}{2}$

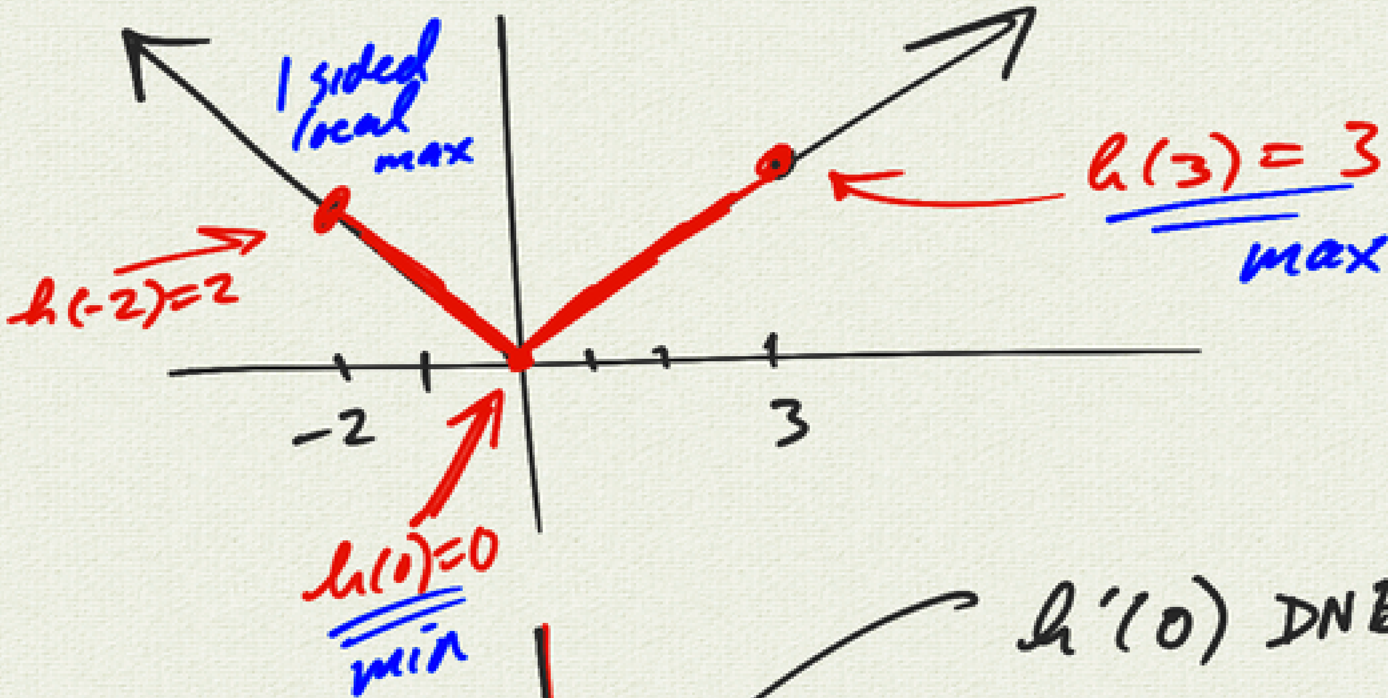


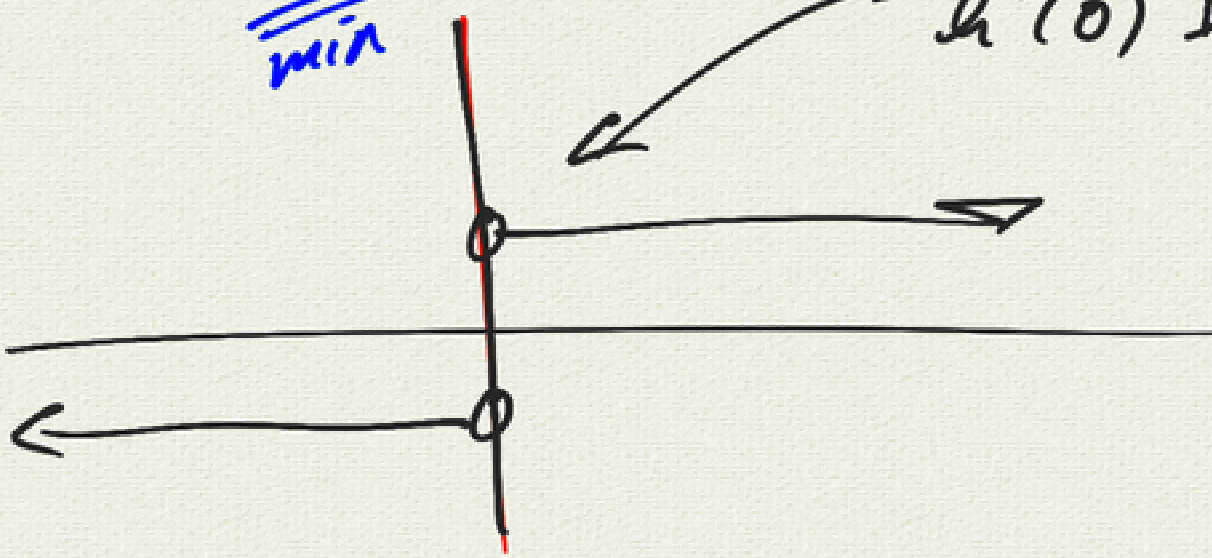
absolute max on $[-\pi, \pi]$

absolute min

$$\boxed{\begin{aligned} g\left(\tfrac{\pi}{2}\right) &= 1 \\ g\left(-\tfrac{\pi}{2}\right) &= -1 \end{aligned}}$$

example $h(x) = |x|$ on $[-2, 3]$





$h'(0)$ DNE

0 is a critical pt.

$h(0) = 0$