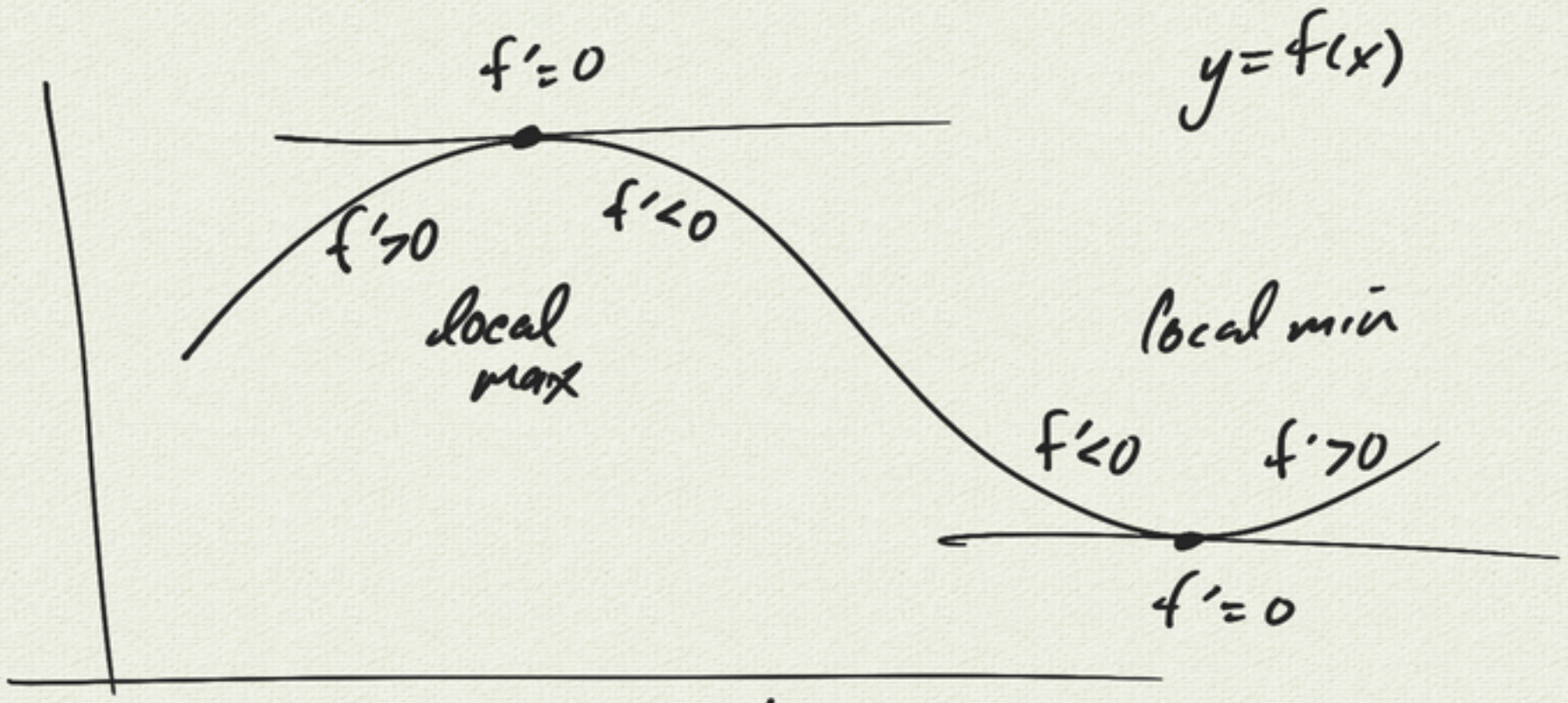
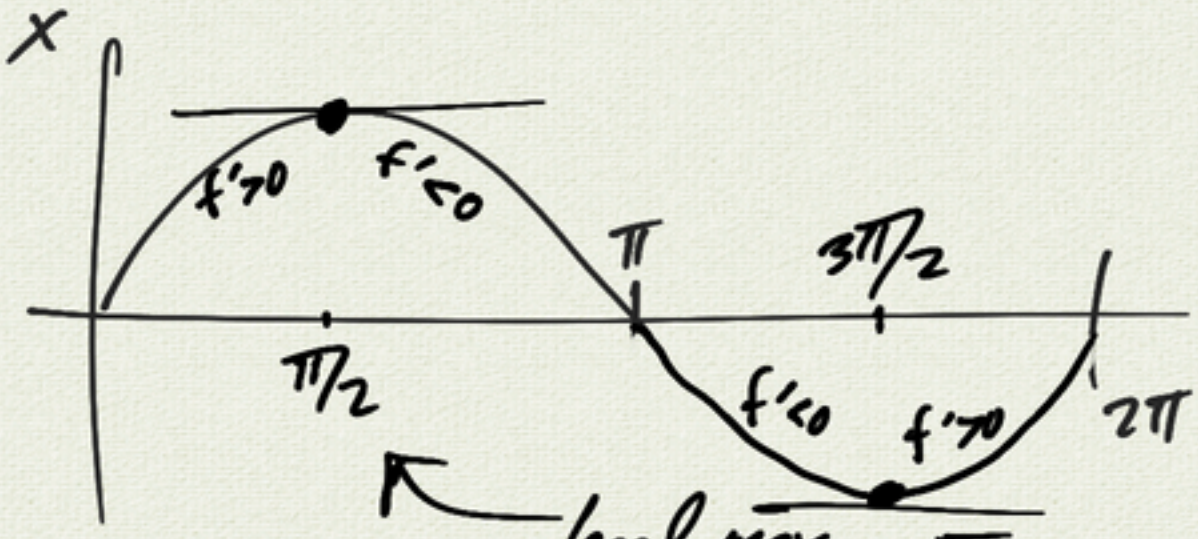
10.3 Extreme values



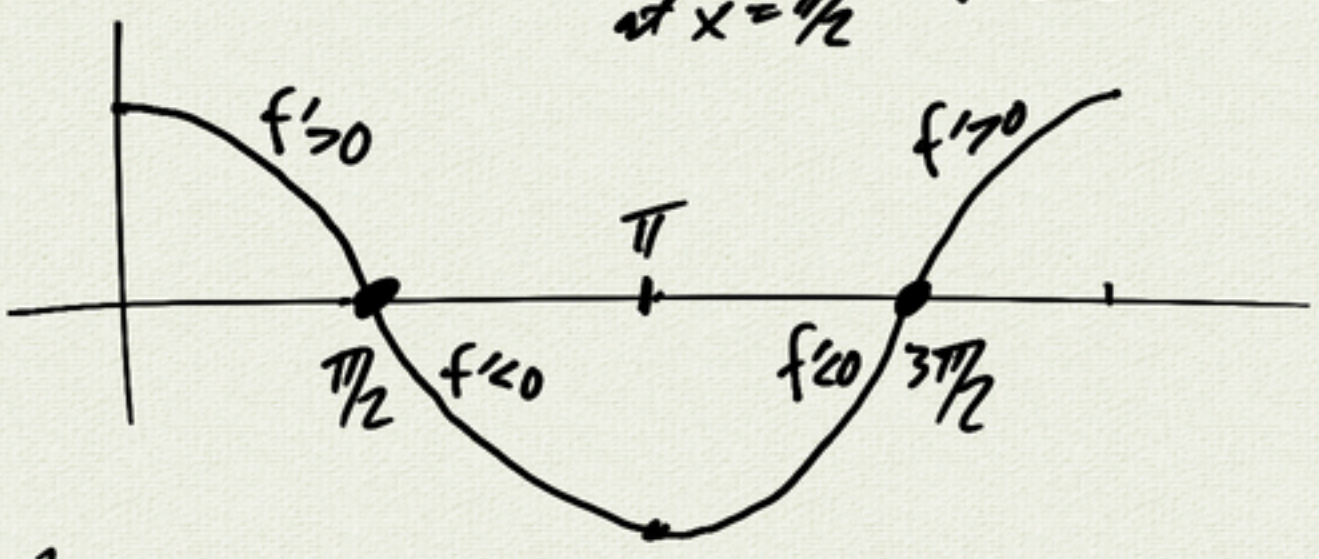
1st derivative test

Example:

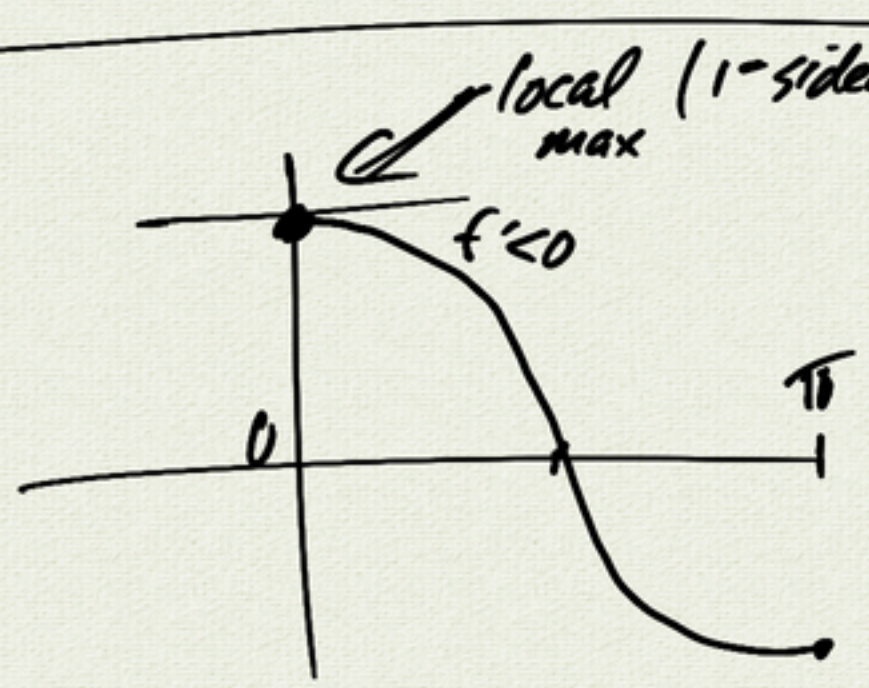
f(x) $y = \sin x$



$f'(x) = \cos x$



critical pts: $\cos x = 0$



$g(x) = \cos x$

$g'(x) = -\sin x$

$g'(0) = 0$
$g'(\pi) = 0$
} critical pts

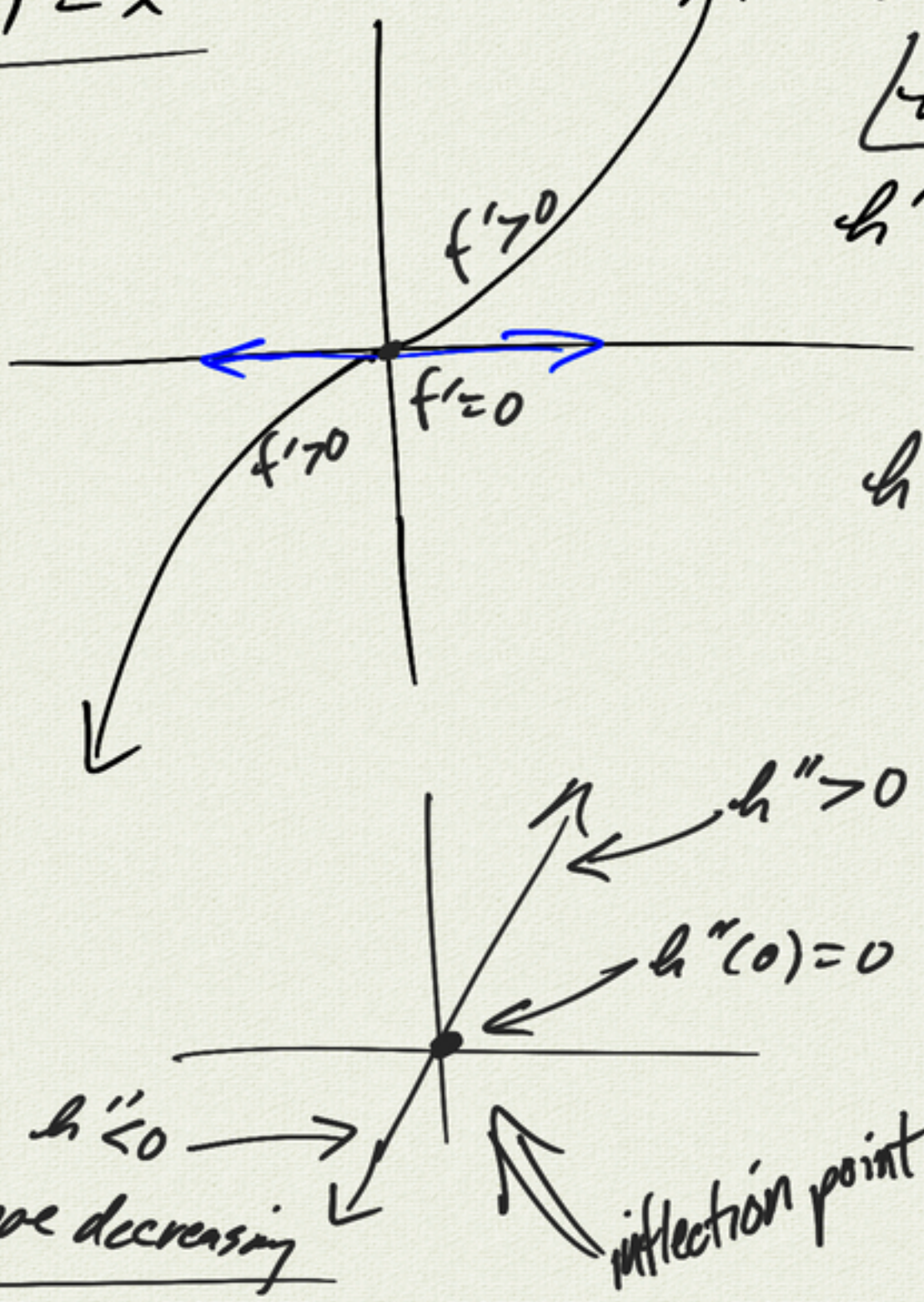
$h(x) = x^3$

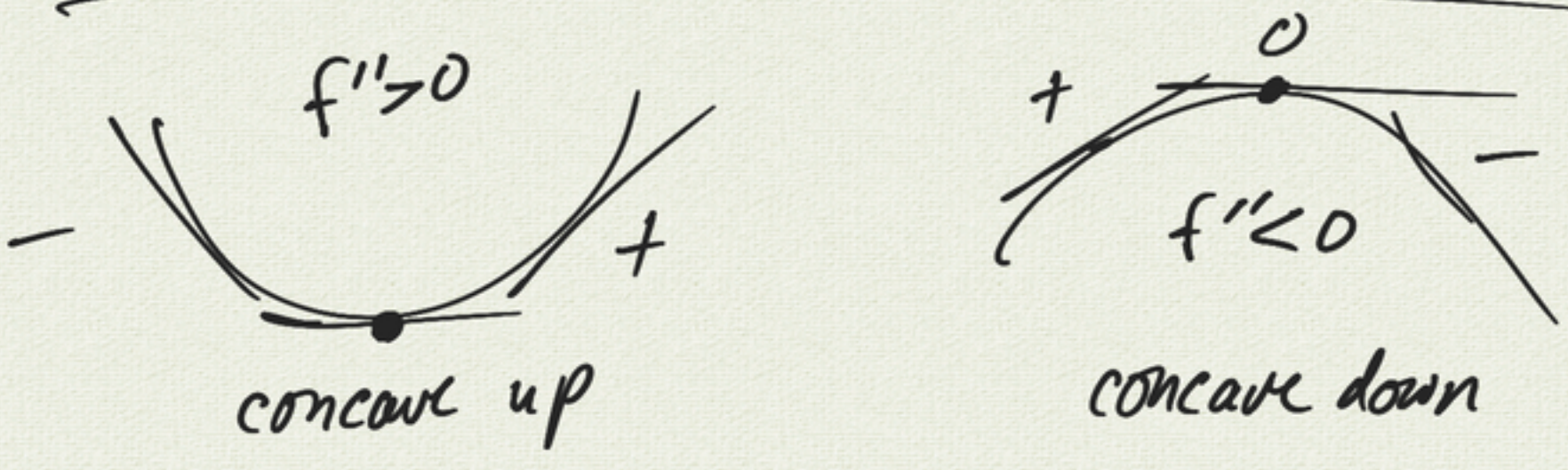
critical pts:

$h'(x) = 3x^2$

$h'(x) = 0 \Rightarrow x = 0$

$h''(x) = 6x$





2nd derivative test

Suppose $f'(x) = 0$

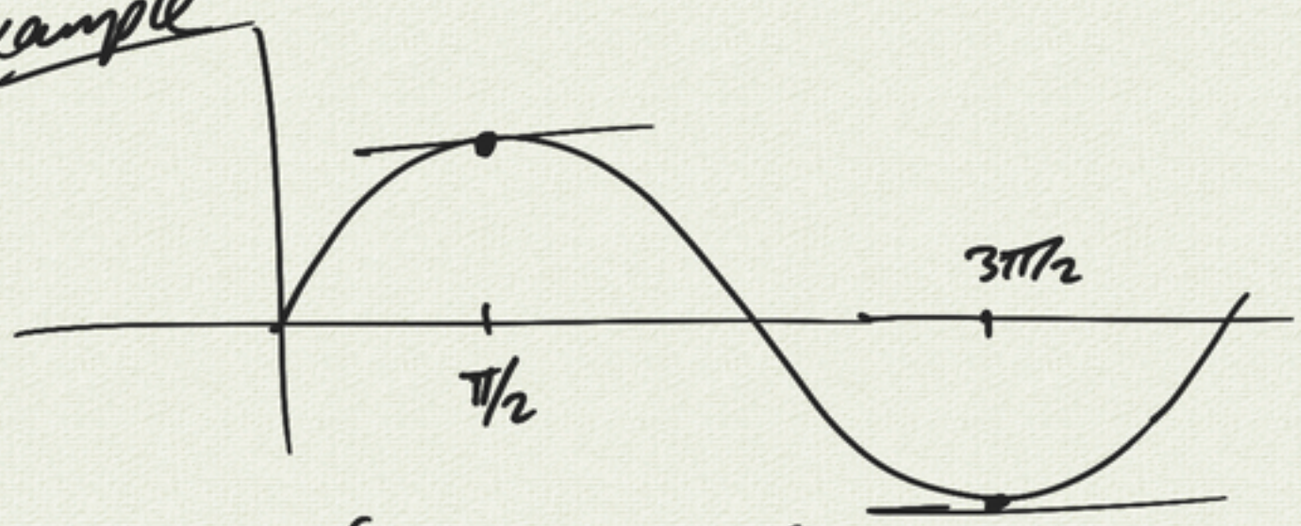
then if $f''(x) > 0 \Rightarrow$ local min

$f''(x) < 0 \Rightarrow$ local max

$f''(x) = 0 \Rightarrow$?

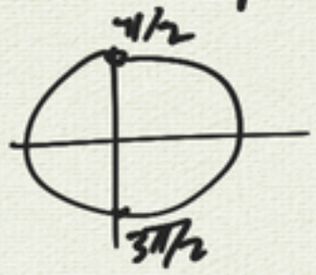
$f(x) = \sin x$

example



critical pts: $f'(x) = \cos x = 0$

$x = \pi/2, 3\pi/2$



$f''(x) = -\sin x$

$f''(\pi/2) = -1 < 0$ local max

$f''(3\pi/2) = -\sin\frac{3\pi}{2} = 1 > 0$ local min

example:

$g(x) = x(x-1)(x+1)$

$= x(x^2-1)$

$= x^3 - x$

find local min/max

critical pts:

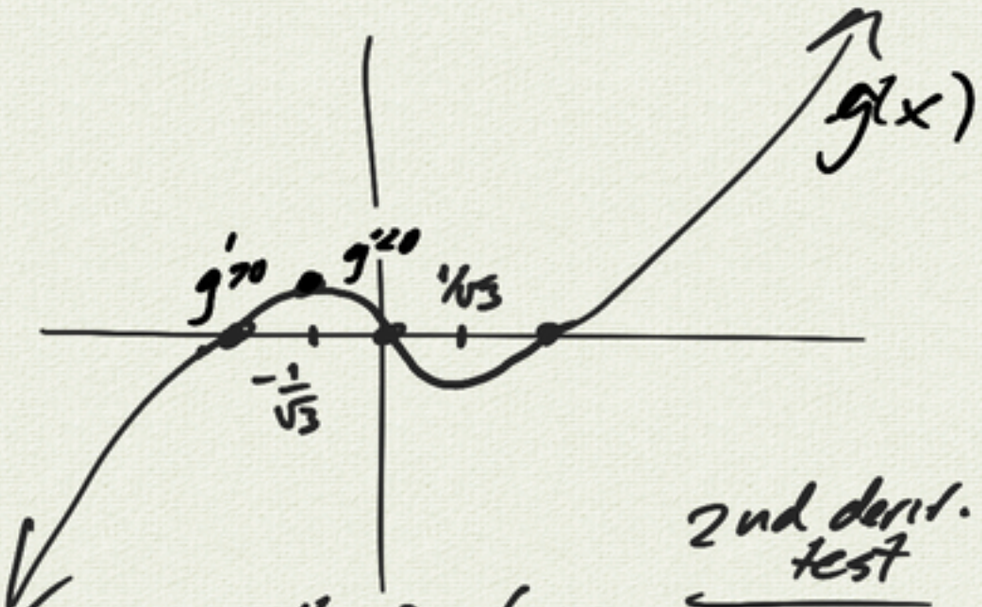
$g'(x) = 3x^2 - 1$

$g'(x) = 0 \longrightarrow 3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\frac{1}{\sqrt{3}}$

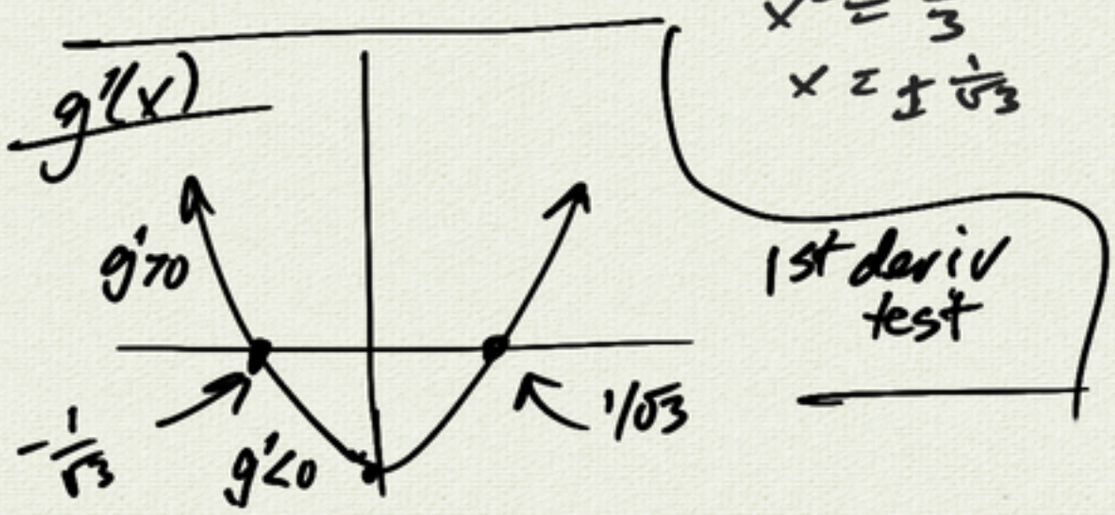


2nd deriv. test

$g''(x) = 6x$

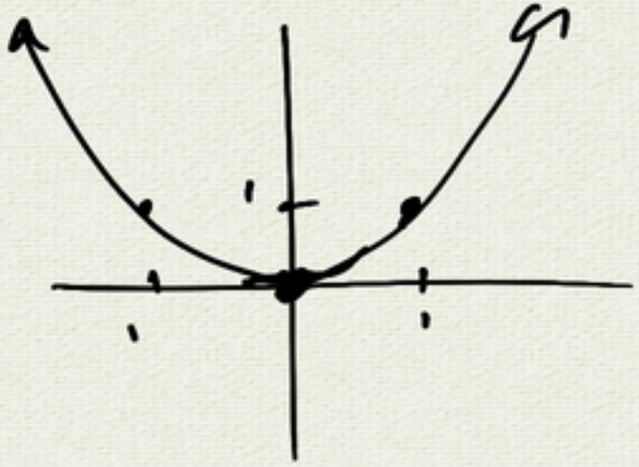
$g''(\frac{1}{\sqrt{3}}) > 0$ local min

$g''(\frac{-1}{\sqrt{3}}) < 0$ local max



1st deriv test

$f(x) = x^4$



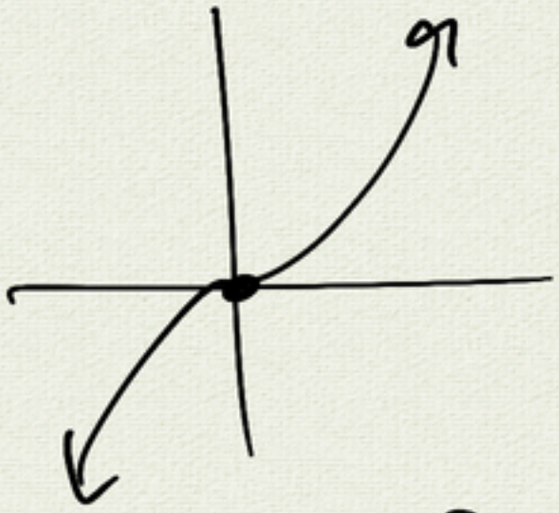
$f'(x) = 4x^3$

$f'(0) = 0$

$f''(x) = 12x^2$

$f''(0) = 0$

$g(x) = x^3$



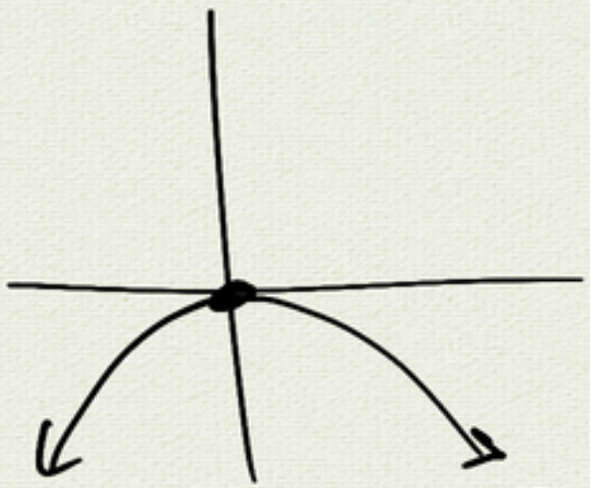
$g'(x) = 3x^2$

$g'(0) = 0$

$g''(x) = 6x$

$g''(0) = 0$

$h(x) = -x^4$



$h'(x) = -4x^3$

$h'(0) = 0$

$h''(x) = -12x^2$

$h''(0) = 0$

2nd deriv test inconclusive

but 1st deriv test will work