# 10.4 Optimization

projectile

$v_y = 96$ ft/s

$$y(t) = y_0 + v_y t - 16t^2$$

$y_0$: initial height; $v_y$: initial y speed

$$y(t) = 96t - 16t^2$$

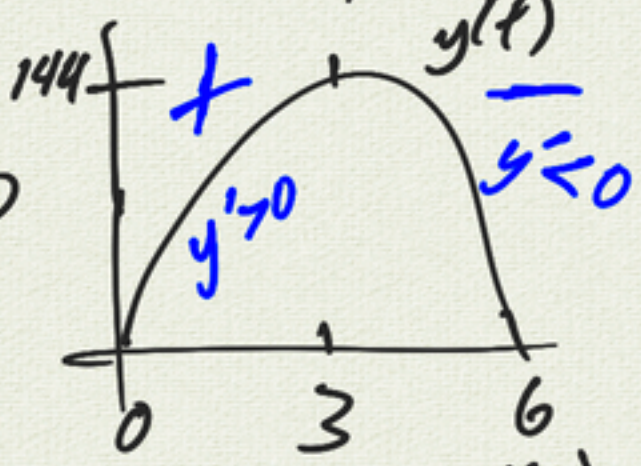
what is max height?

$$y'(t) = 96 - 32t$$

critical pts: $y'(t) = 0 \Rightarrow 96 = 32t$

$t = 3$

$$y(3) = 96 \cdot 3 - 16 \cdot 9 = 144$$




2nd deriv test: $y''(t) = -32 < 0$

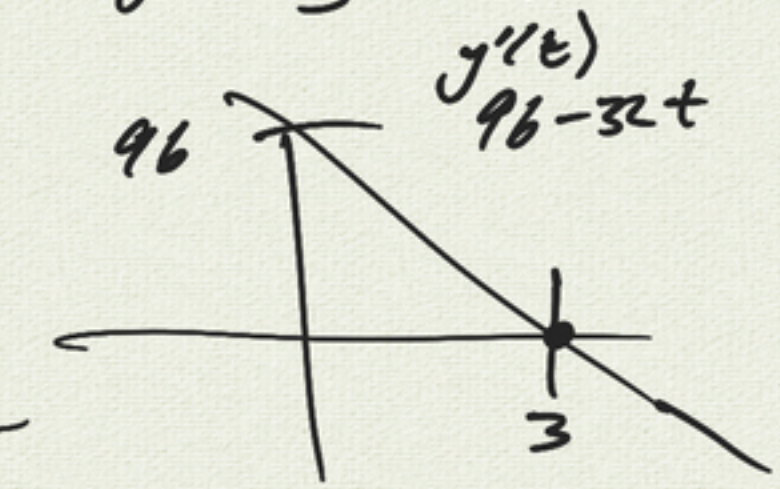
local max at $t = 3$

1st deriv test:

$t < 3$: $y'(t) = 96 - 32t > 0$

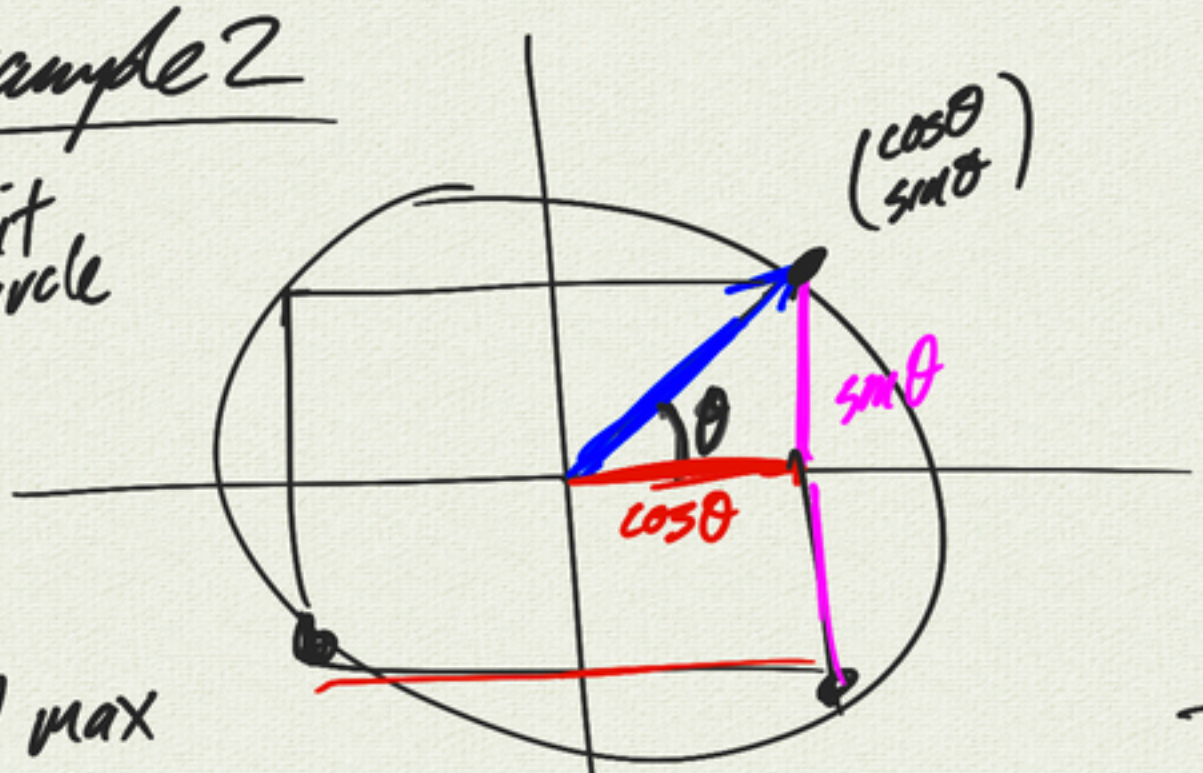
$t > 3$ $y' < 0$

local max at $t = 3$



## example 2

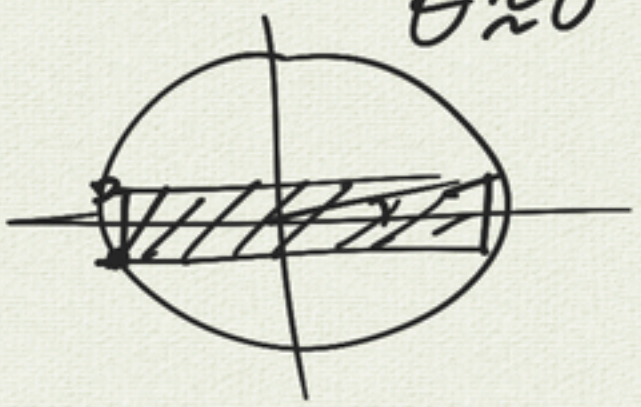
unit circle

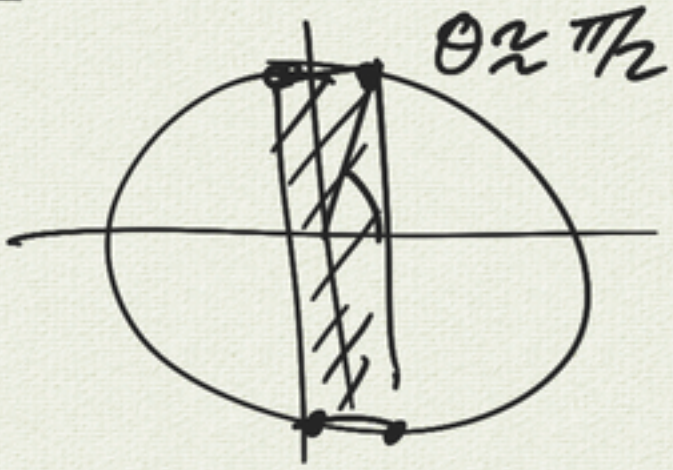



find max area

$$A(\theta) = (2\cos\theta)(2\sin\theta)$$
$$= 4\sin\theta\cos\theta$$
$$= 2\sin 2\theta$$

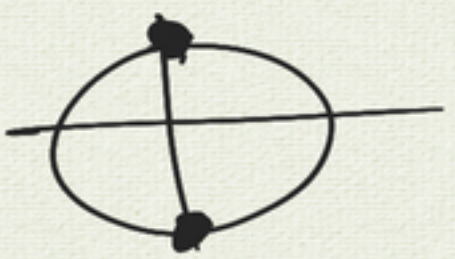
$\Leftarrow \sin 2\theta = 2\sin\theta\cos\theta$




$$A'(\theta) = (2\cos 2\theta)\cdot 2$$
$$= 4\cos 2\theta$$

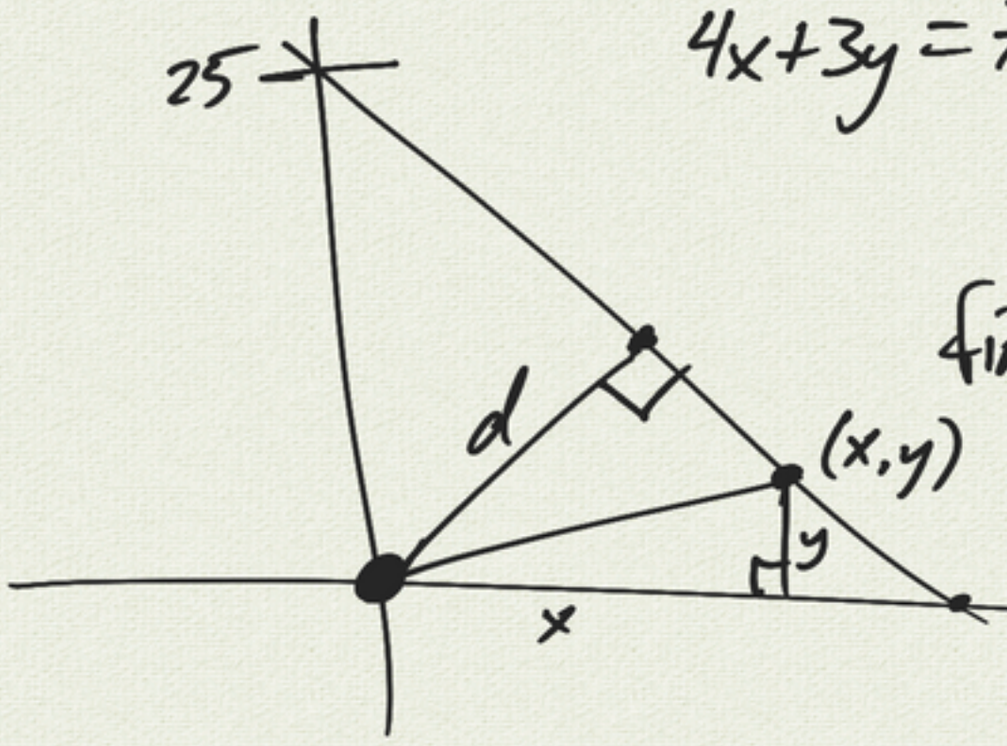
$\frac{d}{d\theta}(2\theta) = 2$



$$A'(\theta) = 0 \Rightarrow \cos 2\theta = 0$$
$$2\theta = \frac{\pi}{2} + k\pi$$
$$\theta = \frac{\pi}{4} + \frac{k\pi}{2}$$

$$A''(\theta) = 4(-\sin 2\theta)(2)$$
$$= -8\sin 2\theta$$
$$A''\left(\frac{\pi}{4}\right) = -8\sin\frac{\pi}{2} = -8 < 0$$ local max

example 3




$4x+3y=75$ line

$y=25-\frac{4}{3}x$

find d (distance from line to origin)

equivalent: find closest point

minimize distance to origin

$dist=\sqrt{x^2+y^2}$

want: minimize distance, where (x,y) constrained to the line

$f(x)=x^2+y^2$

$f(x)=x^2+(25-\frac{4}{3}x)^2$ ⟸ square of distance (minimize)

$y=25-\frac{4}{3}x$

find max/min of f(x).

$f'(x)=2x+2(25-\frac{4}{3}x)(-\frac{4}{3})$

$=2x-\frac{8}{3}\cdot 25+\frac{32}{9}x$

$=\frac{50}{9}x-\frac{200}{3}$

$f''(x)=\frac{50}{9}>0$ local min

critical pts: $f'(x)=0$

$\frac{50}{9}x-\frac{200}{3}=0$

$50x=600$

$x=12$

$y=25-\frac{4}{3}(12)=9$

$y=25-\frac{4}{3}x$

$4x+3y=75$

$4(12)+3(9)$

$=48+27=75$ ✓