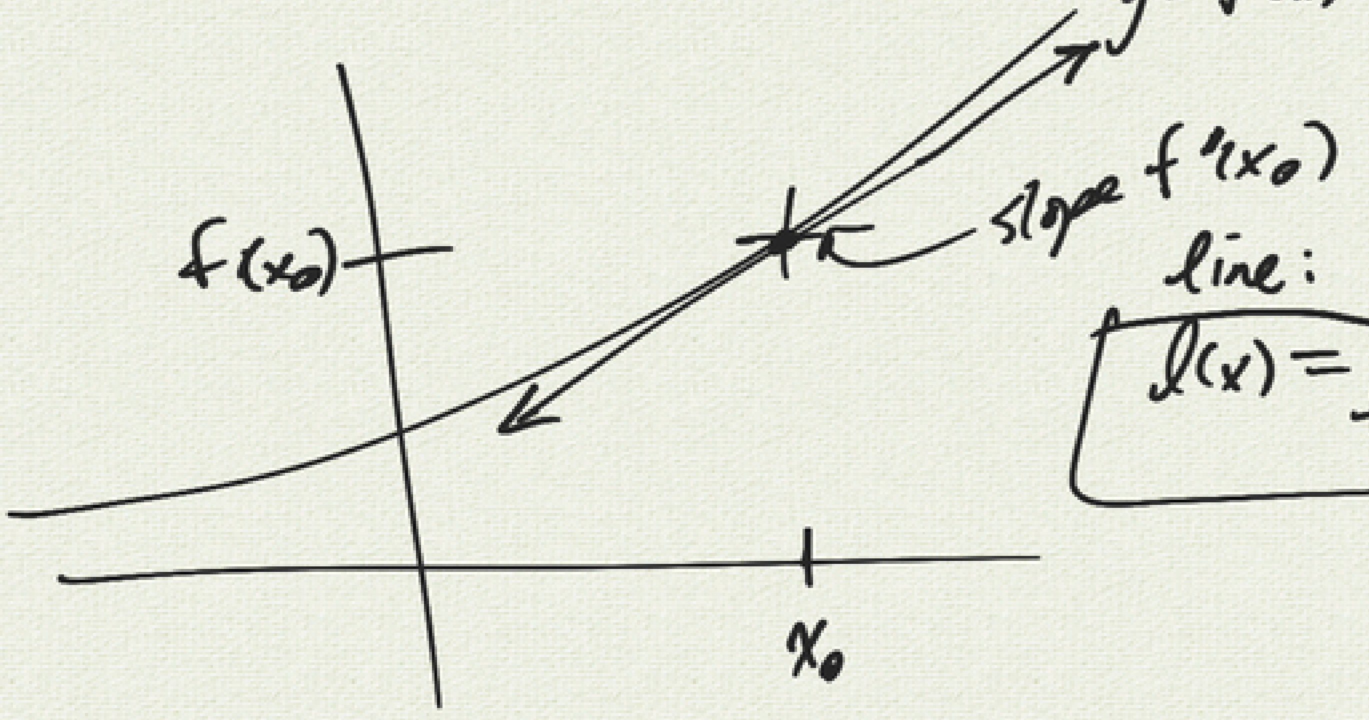
10.5 Linearization



line:

$$\boxed{\ell(x) = f(x_0) + \underbrace{f'(x_0)}_{\text{slope } f'(x_0)}(x - x_0)}$$

point $(x_0, f(x_0))$

point slope

$y - y_1 = m(x - x_1)$

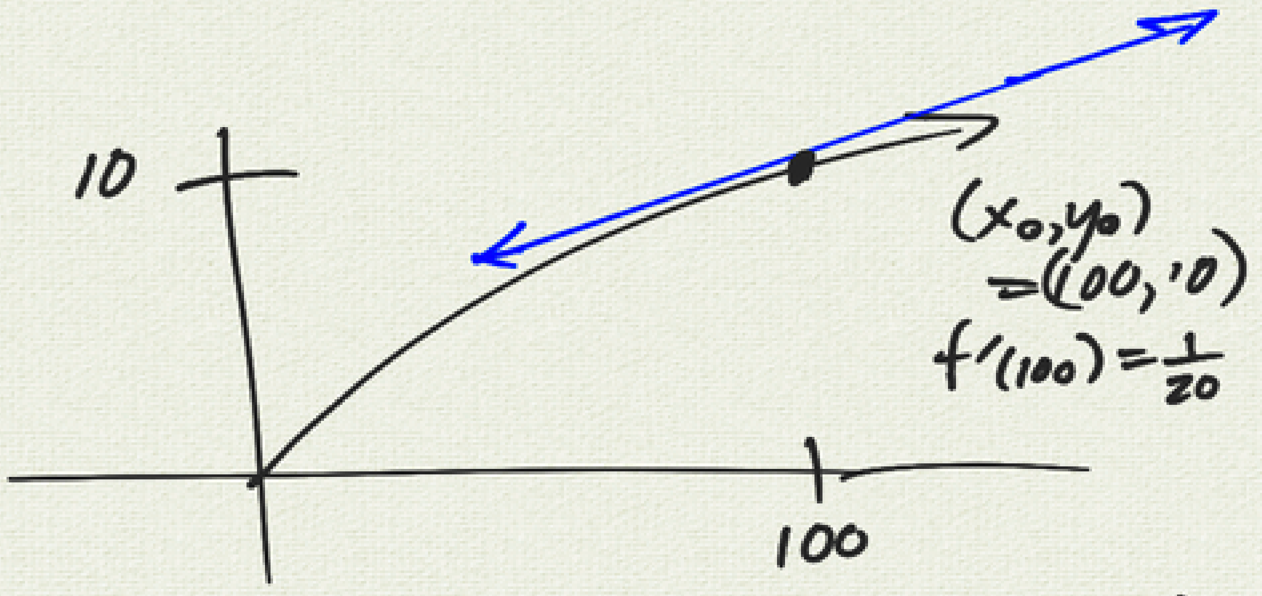
$y = y_1 + m(x - x_1)$

$y = y_0 + m(x - x_0)$

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$



$(x_0, y_0) = (100, 10)$

$f'(100) = \frac{1}{20}$

tangent line $\ell(x) = f(x_0) + f'(x_0)(x - x_0)$

$$\boxed{\ell(x) = 10 + \tfrac{1}{20}(x - 100)}$$

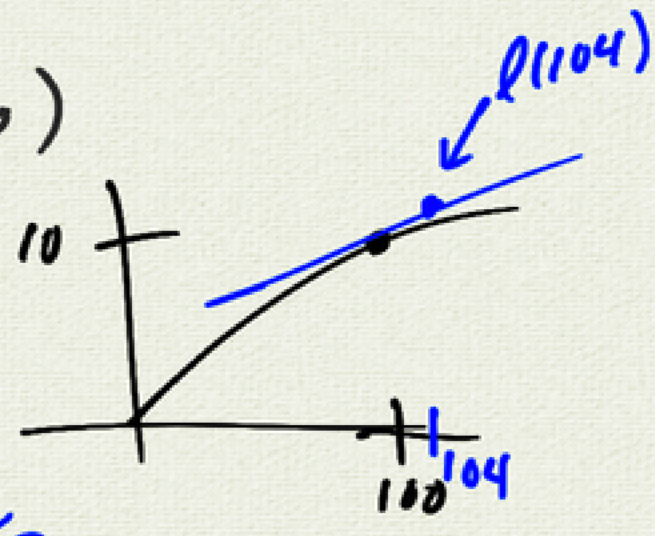
linearization: $f(x) \approx \ell(x)$

approximate f by its tangent line near $x = x_0$

$f(x) = \sqrt{x}$

$f(x) \approx f(100) + f'(100)(x - 100)$

$10 + \frac{1}{20}(x - 100)$



$f(104) \approx 10 + \frac{1}{20}\underbrace{(4)}_{}$

$\Delta x = x - x_0$

$= 104 - 100$

$= 10 + \frac{4}{20}$

$= 10 + \frac{1}{5}$

$= 10.2$

$\sqrt{104} \approx 10.2$

actual:

$\sqrt{104} \approx 10.198$

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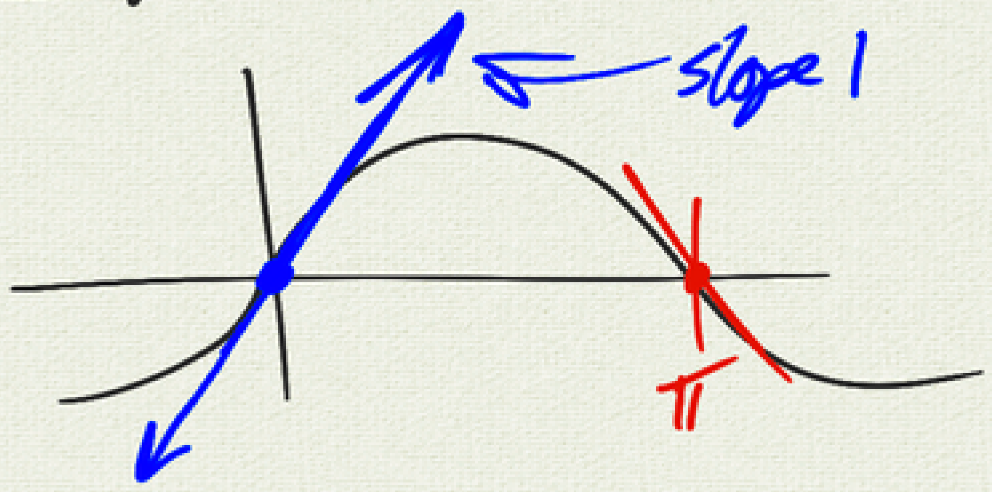
example: approximate $\sin x$ near 0



$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$f(x) = \sin x$

$f'(x) = \cos x$

$f'(0) = \cos 0 = 1$

$f(x) \approx \underbrace{f(0)}_{0} + \underbrace{f'(0)}_{1}(x - 0)$ ← $x_0 = 0$

$f(x) \approx x$

$$\boxed{\sin x \approx x}$$

near $x = 0$

near $x = \pi$: (x_0) $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$

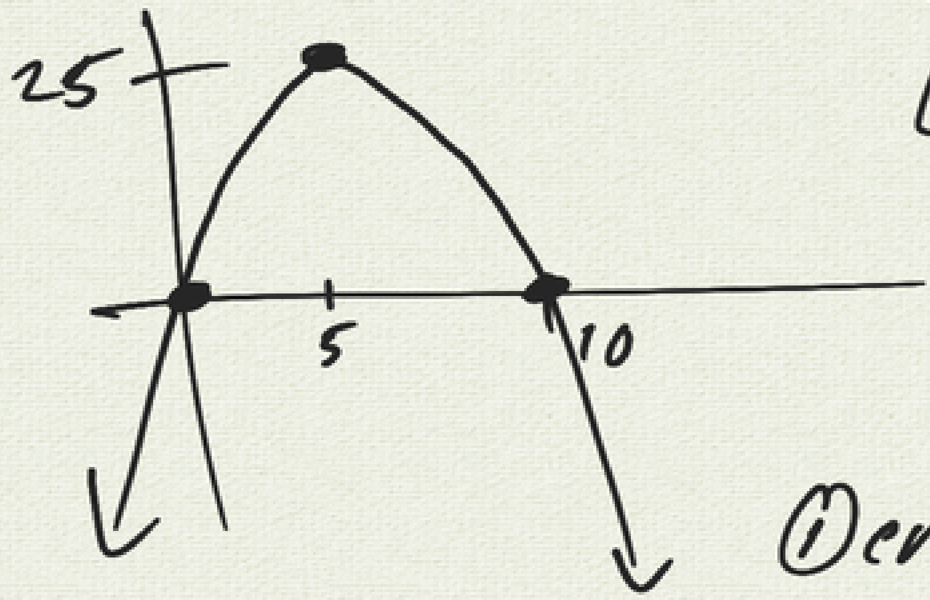
$= \underbrace{f(\pi)}_{0} + \underbrace{f'(\pi)}_{-1}(x - \pi)$

$\sin x \approx -(x - \pi)$

near $x = \pi$

337 $x+y=10 \Leftarrow$ constraint $y=10-x$

maximize/minimize xy



$f(x)=xy$

$\boxed{f(x)=x(10-x)}$

$= -x(x-10)$

$= -x^2+10x$

extreme value theorem

$\Rightarrow$ max/min exists on $[0,10]$

① critical pts

$f'(x) = -2x+10$

$f'(x)=0 \Rightarrow x=5$

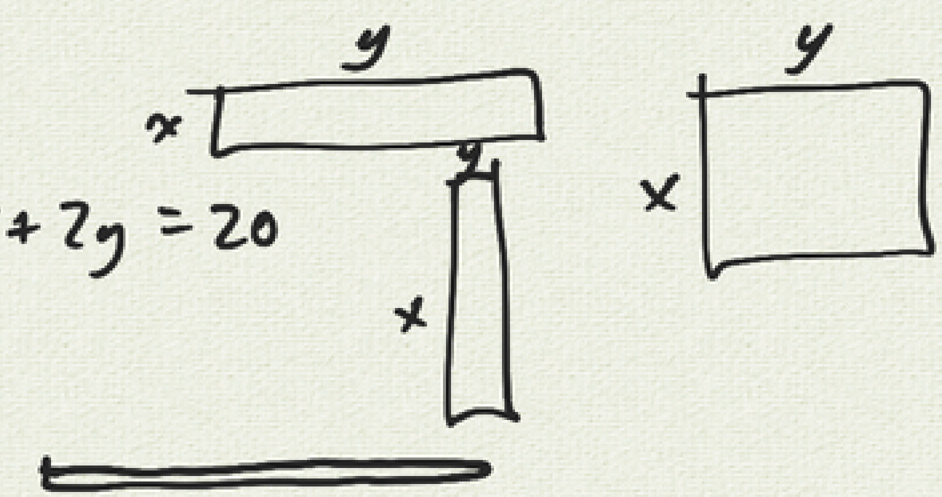
$f''(x)=-2<0$

local max

$f(5)=25$

② end pts: $f(0)=0=f(10)$

geometrically:



$2x+2y=20$

maximize/minimize

area $= xy$