limits:

1. plug in
2. $\frac{\square}{0} \longrightarrow \infty$ (check left/right)
3. $\frac{0}{0} \longrightarrow$
   a. cancel
   b. special limit

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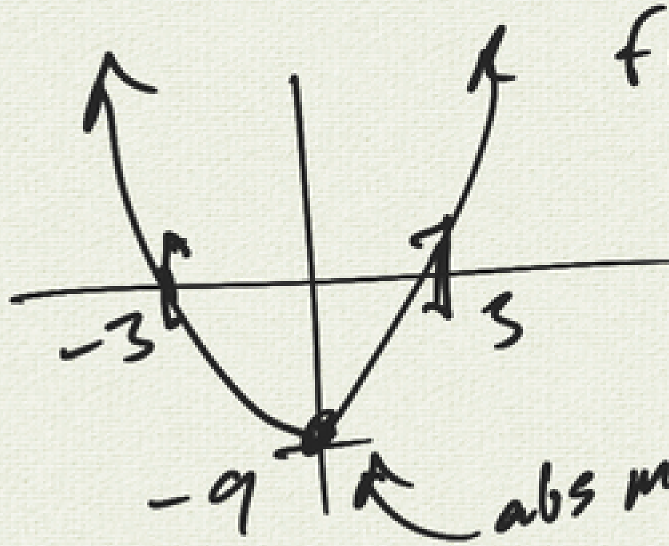
3 types of functions

1. polynomials & rational functions
2. trig functions
3. exp/log

(1)

$f(x) = x^2 - 9$



abs min

local min

EVT (closed finite interval, continuous function)

$\Rightarrow$ there exists an abs min + max on $[-3, 3]$

(1) end pt

or

(2) critical pt

critical pts:

$f'(x) = 2x$

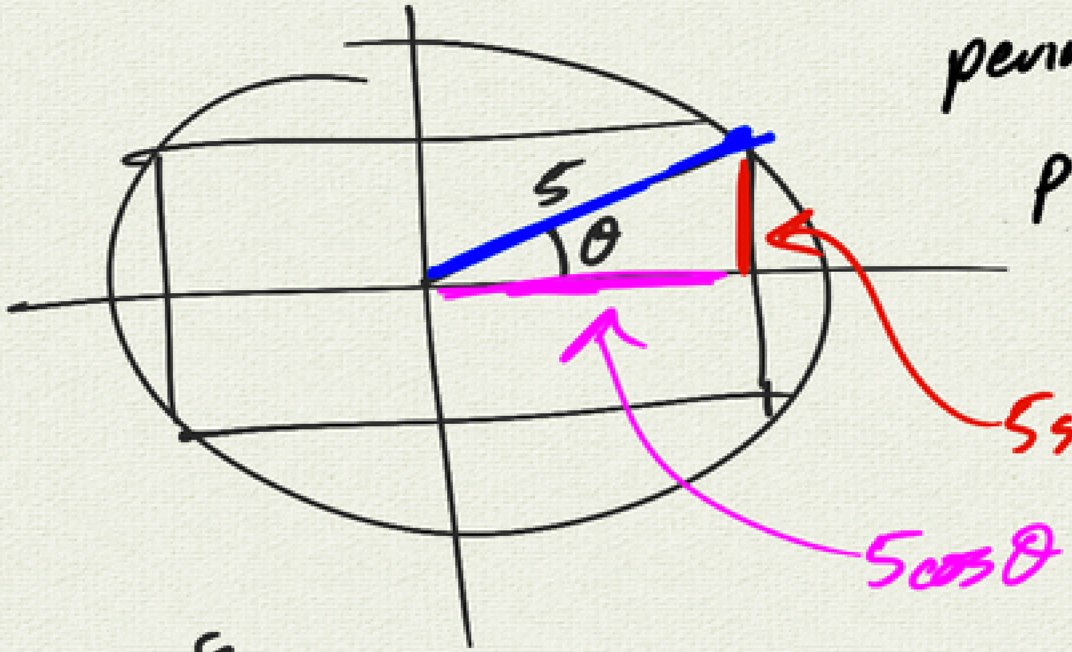
$f'(x) = 0 \Rightarrow x = 0$

$f''(x) = 2$

$f''(0) = 2 > 0$ local min (2nd deriv. test)

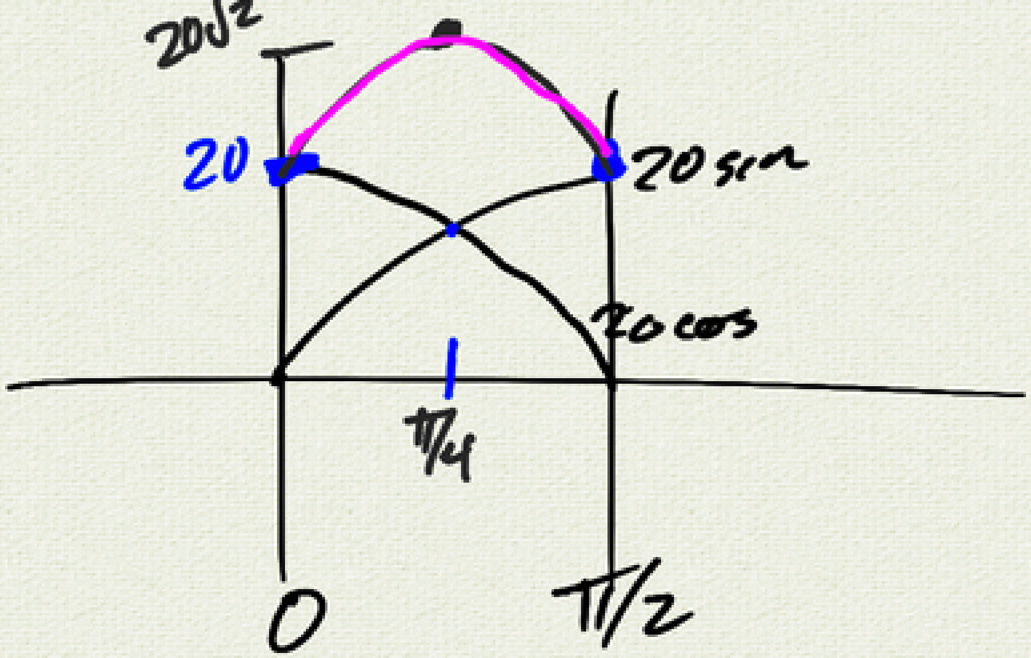
$(-3, 3)$ open interval

4



perimeter of $\square$

$$P(\theta) = 4\cdot 5\sin\theta + 4\cdot 5\cos\theta$$
$$= 20\sin\theta + 20\cos\theta$$



$$P(\pi/4) = 20\cdot\frac{\sqrt{2}}{2} + 20\,\frac{\sqrt{2}}{2}$$
$$= 20\sqrt{2}$$

critical pts:

$$P'(\theta) = 20\cos\theta - 20\sin\theta$$

$$P'(\theta) = 0 \Longrightarrow 20\cos\theta - 20\sin\theta = 0$$
$$\cos\theta = \sin\theta$$
$$\tan\theta = 1$$
$$\theta = \pi/4$$

$$P''(\theta) = -20\sin\theta - 20\cos\theta$$
$P''(\pi/4) < 0$ local max

EVT:

θ	$P(\theta)$
0	20
$\pi/4$	$20\sqrt{2}$
$\pi/2$	20

abs min at $x = 0,\ x = \pi/2$

abs max at $x = \pi/4$

projectiles

$x(t)$
$y(t)$ position

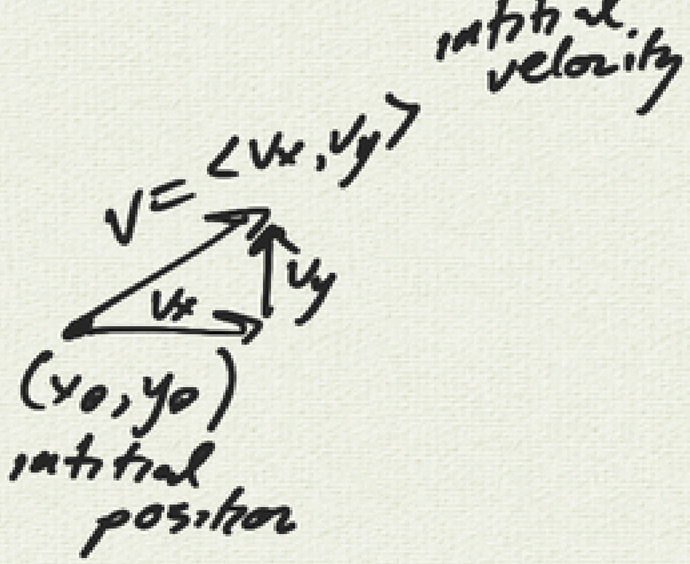
$x'(t)$
$y'(t)$ velocity

speed $= |\langle x', y' \rangle|$

$x''(t)$
$y''(t)$ acceleration

---

assume: acceleration $-32\ ft/s^2$



$x''(t) = 0$

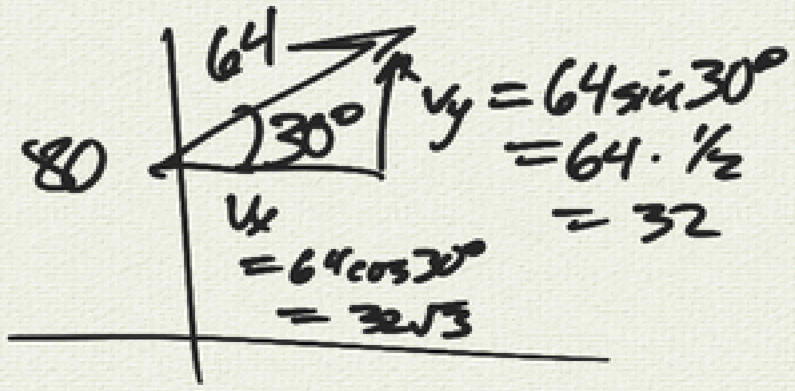
$y''(t) = -32$

$x'(t) = \text{const} = v_x$

$y'(t) = -32t + C_1$

$y'(0) = v_y = C_1$

$y'(t) = -32t + v_y$



$v_x = x'(0)$

$x(t) = v_x t + C_2 \Leftarrow x(0) = x_0$

$= v_x t + x_0$

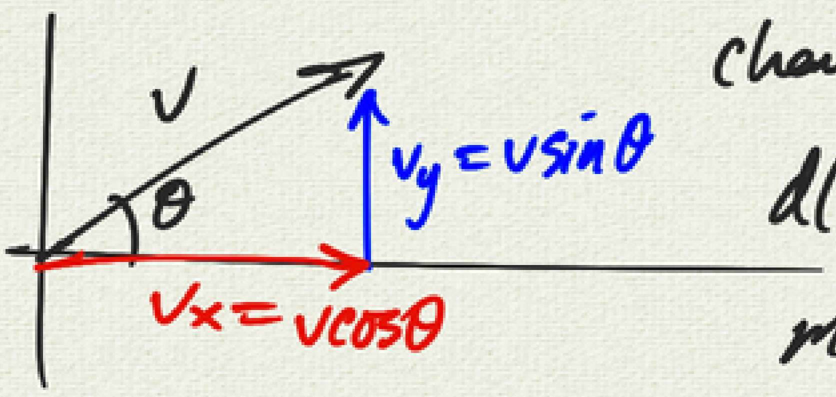
$y(t) = -\frac{32t^2}{2} + v_y t + C_3 \Leftarrow C_3 = y_0$

$= -16t^2 + v_y t + y_0$

$x(t) = x_0 + v_x t$

$y(t) = y_0 + v_y t - 16t^2$

projectile motion equations



fix v

change θ

$(x_0, y_0) = (0,0)$

$d(\theta)$ = distance of cannonball

maximize $d(\theta)$

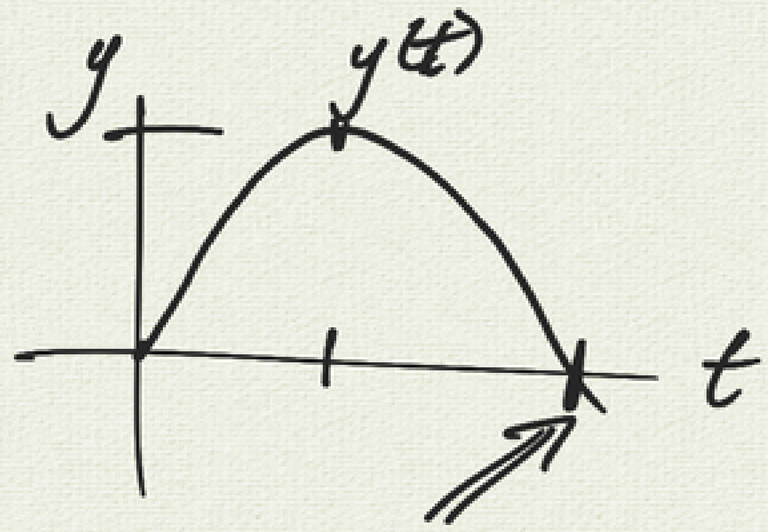
$x(t) = v_x t$

$y(t) = v_y t - 16t^2$

$\boxed{x(t) = (v\cos\theta)t}$

$y(t) = (v\sin\theta)t - 16t^2$

$= t\,(v\sin\theta - 16t)$



$v\sin\theta - 16t = 0$

$\boxed{t_{max} = \frac{v\sin\theta}{16}}$

$y'(t) = v\sin\theta - 32t$

$y'(t) = 0 \Rightarrow t = \frac{v\sin\theta}{32}$

$= t_{max}/2$

$\sin 2\theta = 2\sin\theta\cos\theta$

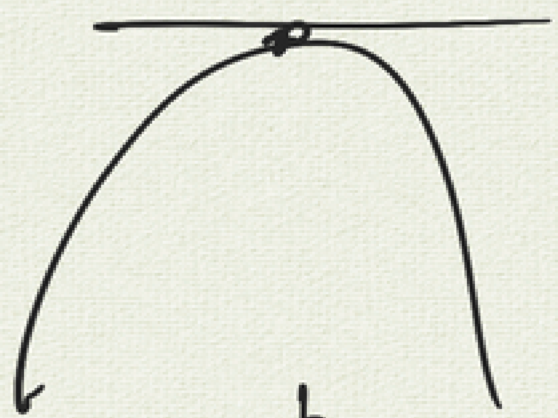
$\cos 2\theta = \cos^2\theta - \sin\theta$

$d(\theta) = \underbrace{x(t_{max})}_{\text{function}} = (v\cos\theta)t_{max}$

$= v\cos\theta\left(\frac{v\sin\theta}{16}\right)$

$d(\theta) = \frac{v^2}{16}\sin\theta\cos\theta$ $\quad \boxed{= \frac{v^2\, 2\sin\theta\cos\theta}{16\cdot 2}}$

$\boxed{d(\theta) = \frac{v^2}{32}\sin 2\theta}$ $\quad$ maximize $d(\theta)$



$d'(\theta) = \frac{v^2}{32}(\cos 2\theta)\cdot 2$

$= \frac{v^2}{16}\cos 2\theta$

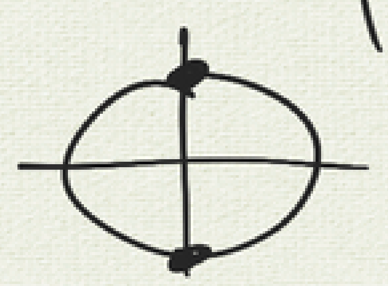
critical pts

$d'(\theta) = 0$

$\cos 2\theta = 0$

$2\theta = \pi/2 + \pi k$

$\theta = \pi/4 + \frac{\pi}{2}k$

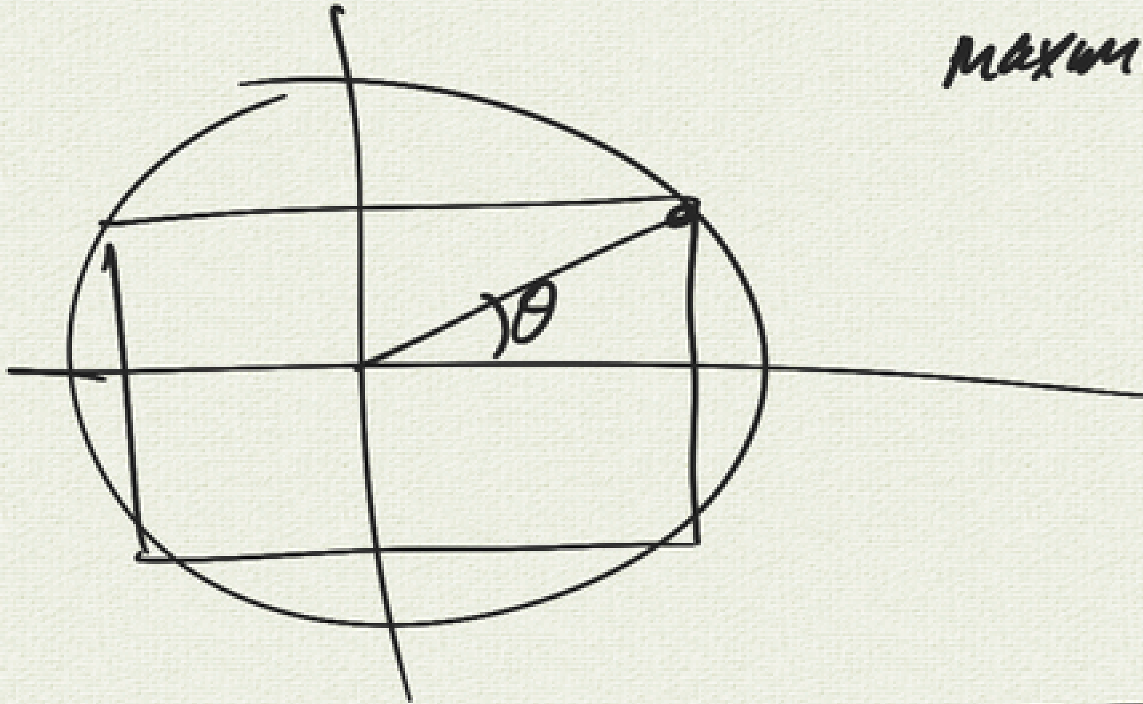


$d''(\theta) = \frac{v^2}{16}(-\sin 2\theta)(2)$

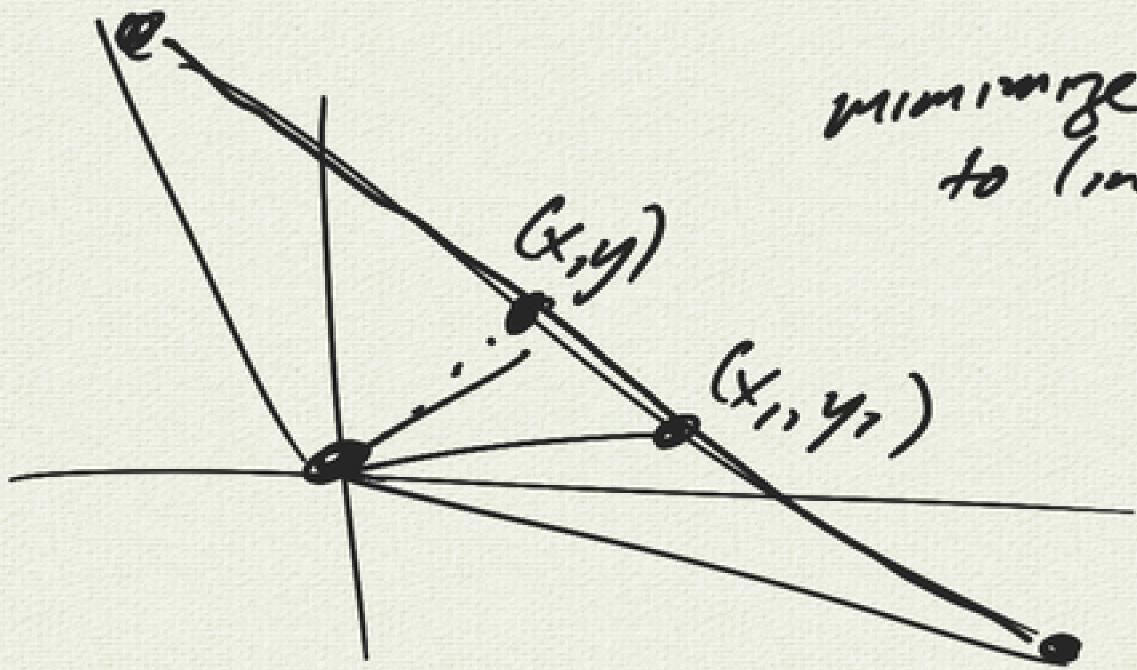
$= -\frac{v^2}{8}\sin 2\theta$

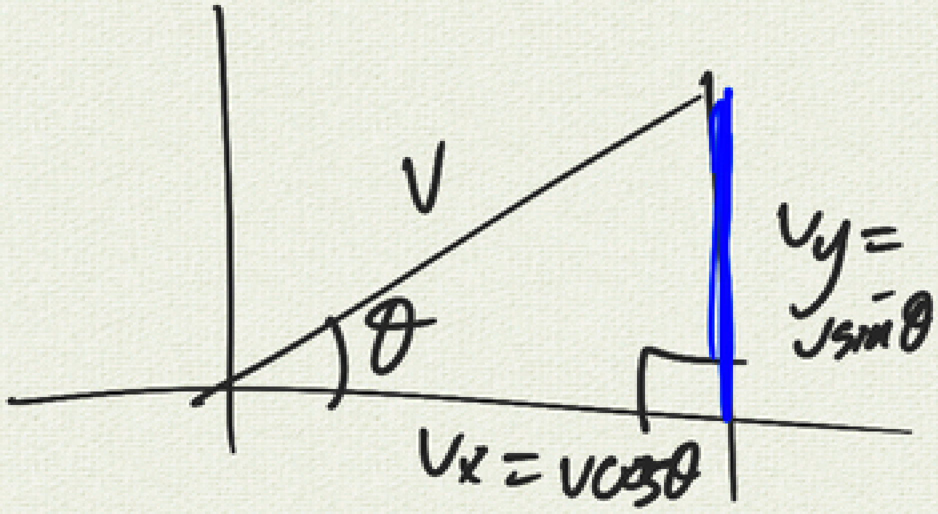
$d''\left(\frac{\pi}{4}\right) = -\frac{v^2}{8} < 0$ local max

maximize $A(\theta)$
area



minimize distance
to line





$\sin\theta = \frac{opp}{hyp}$

$\sin\theta = \frac{v_y}{v}$

$\Longrightarrow v_y = v\sin\theta$

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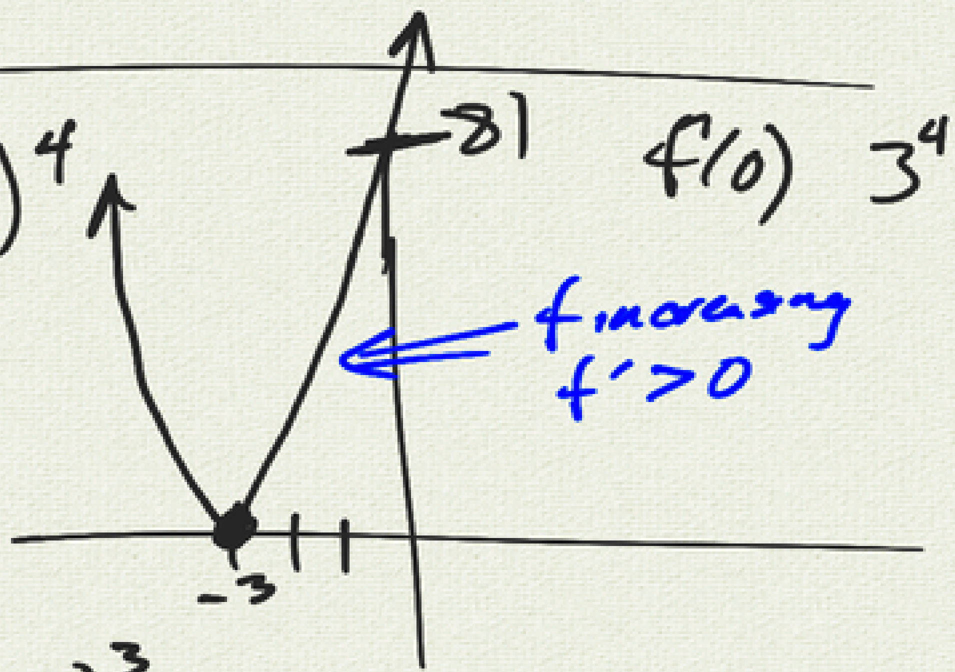
(3) $f(x) = (x+3)^4$



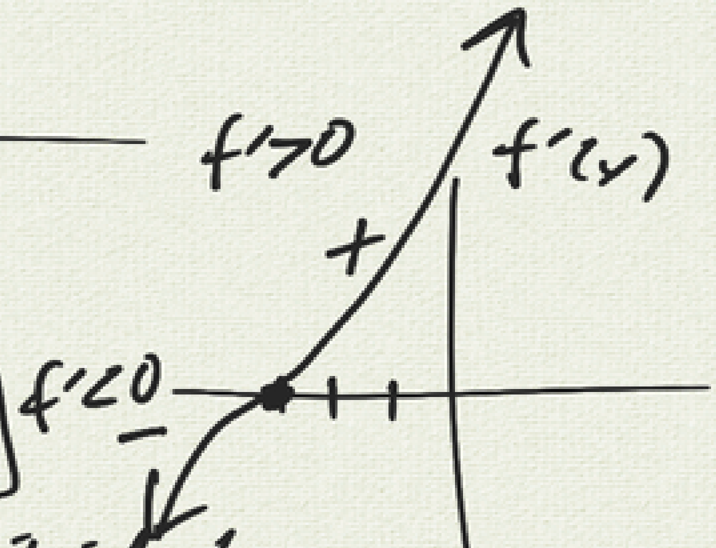
$f(0) \quad 3^4 = 9^2 = 81$

$f'(x) = 4(x+3)^3 \cdot 1$

$f'(x) = 0 \Longrightarrow \boxed{4(x+3)^3 = 0}$

$x+3=0$

$x=-3$



$f''(x) = 12(x+3)^2$

$f''(-3) = 0 \Longleftarrow$ 2nd deriv test inconclusive

1st deriv test

$\Downarrow$

f has local min at $x=-3$