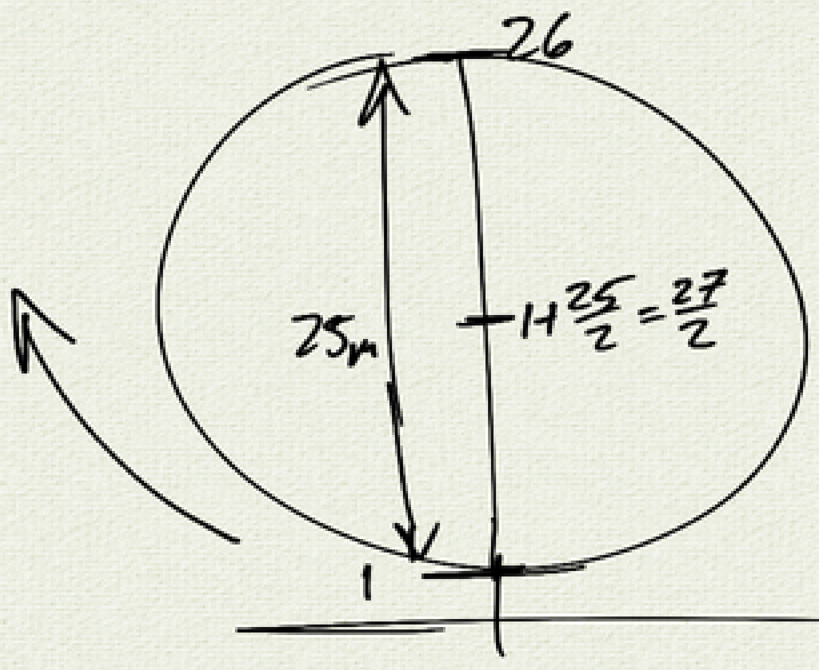
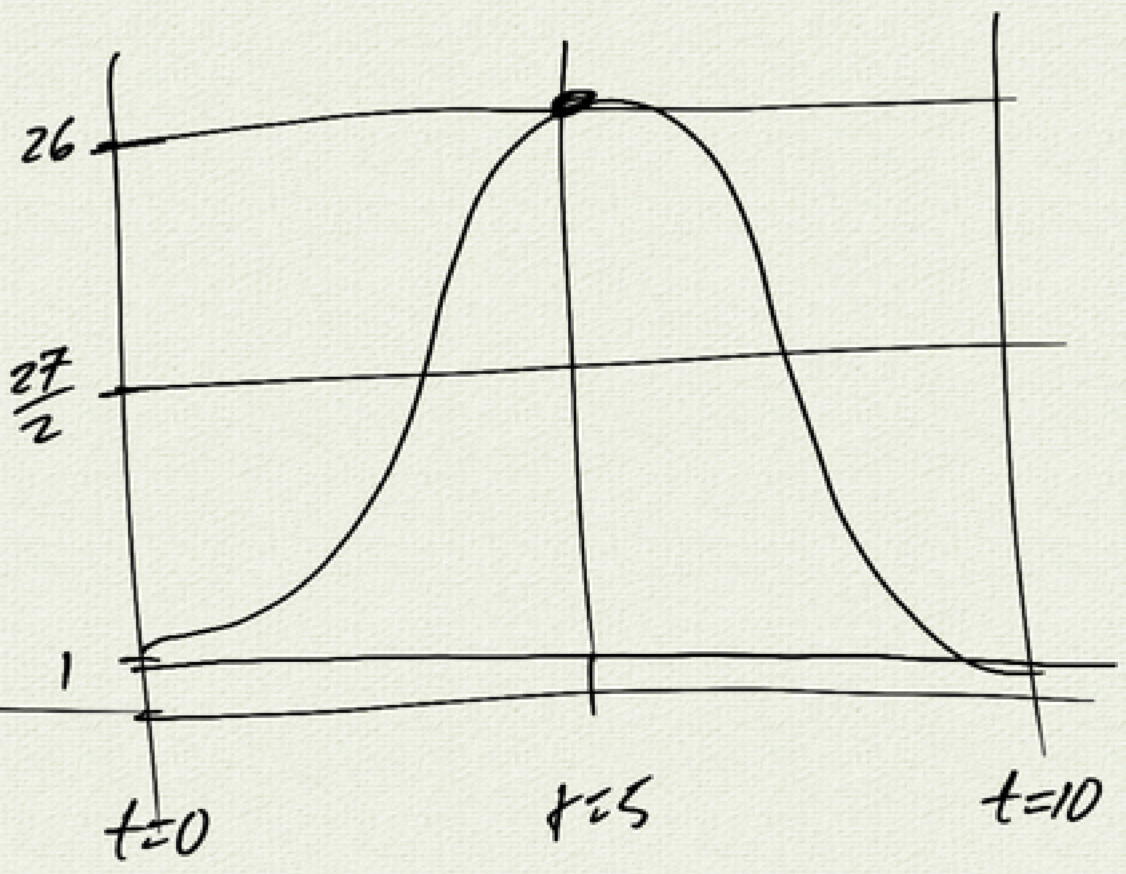
(48) Ferris wheel





$\boxed{\text{period } 10 \text{ min}}$

$\Longrightarrow 10 = \frac{2\pi}{b}$

$b = \frac{2\pi}{10}$

amplitude $\frac{25}{2}$

vertical shift $\frac{27}{2}$

$$h(t) = \boxed{y(t) = -\frac{25}{2}\cos\left(\frac{2\pi}{10}t\right) + \frac{27}{2}}$$

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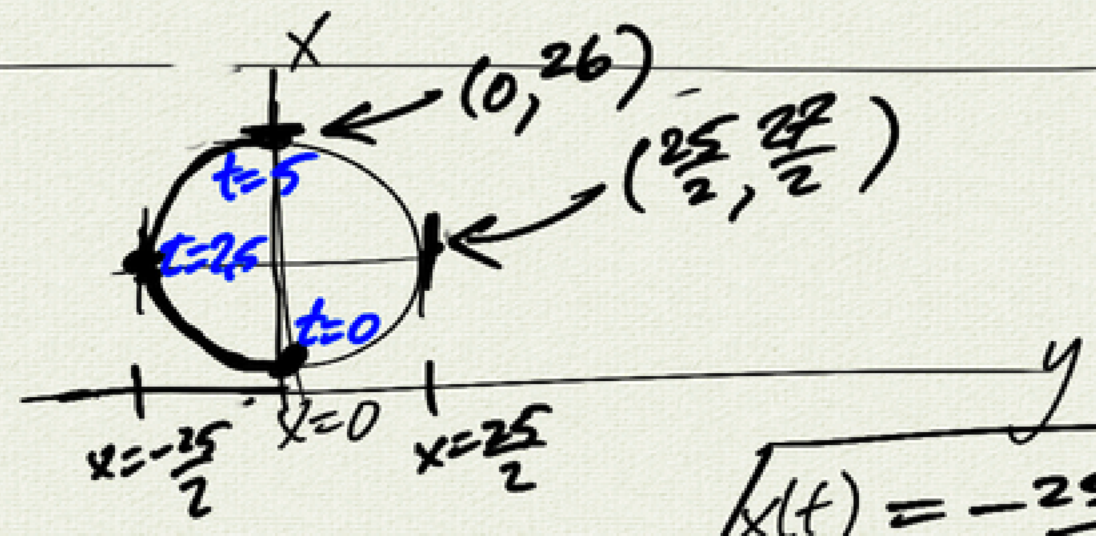
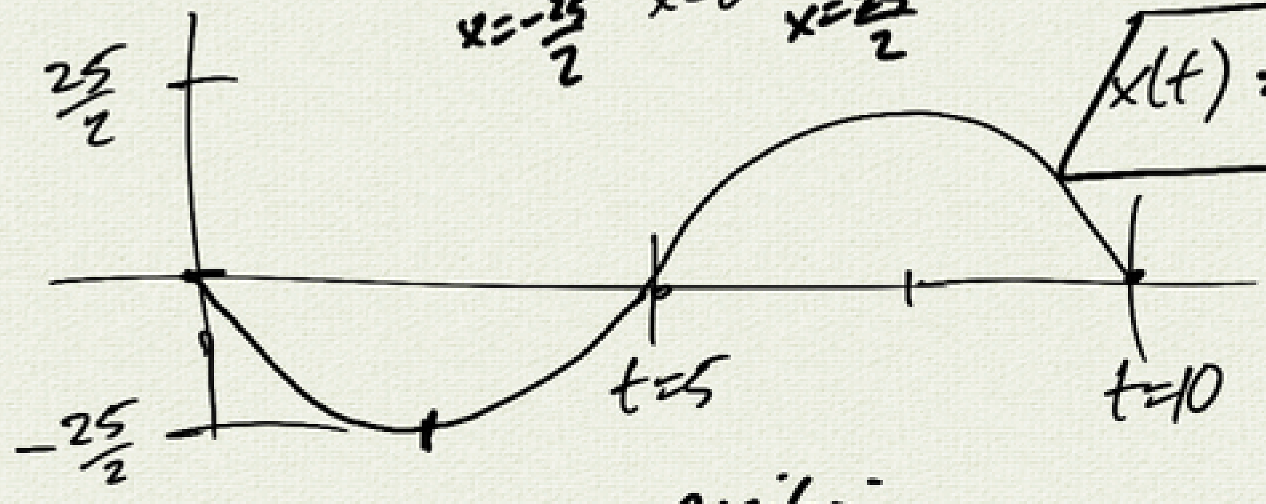
x- values :



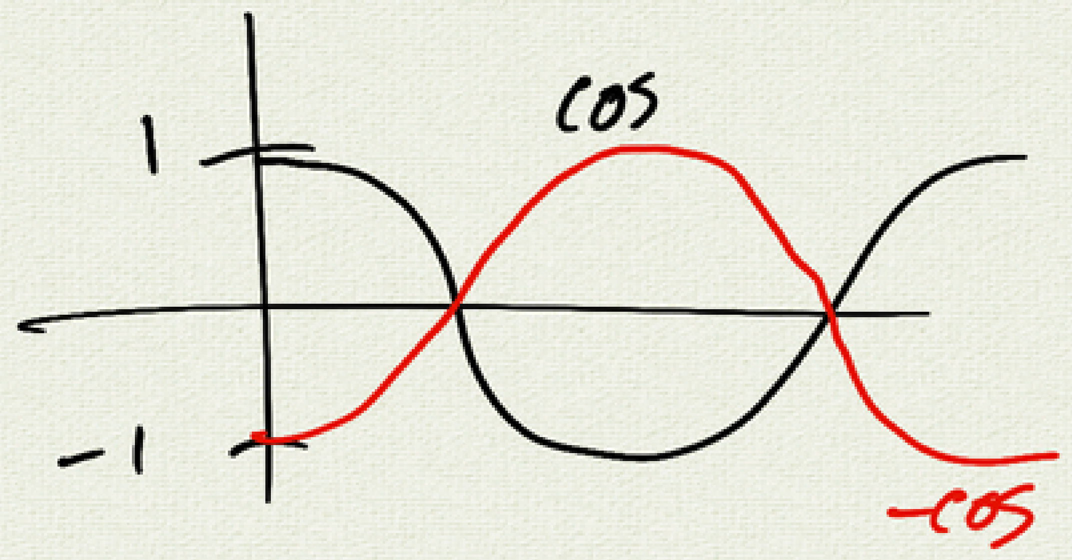
$$\boxed{x(t) = -\frac{25}{2}\sin\left(\frac{2\pi}{10}t\right)}$$



at $t=15$: $\overset{\text{position}}{(x(t), y(t))} = ?$

intuition $\Longrightarrow$ top of wheel

$$x(15) = -\frac{25}{2}\sin\left(\frac{2\pi}{10}\cdot 15\right)$$
$$= -\frac{25}{2}\sin(3\pi)$$
$$= 0$$
$$y(15) = -\frac{25}{2}\cos\Big(\underbrace{\frac{2\pi}{10}\cdot 15}_{3\pi}\Big) + \frac{27}{2}$$
$$= \frac{25}{2} + \frac{27}{2}$$
$$= 26$$



<u>sinusoid</u> (sin or cos)

$$f(t) = a\sin(b(t-h)) + k$$

amplitude

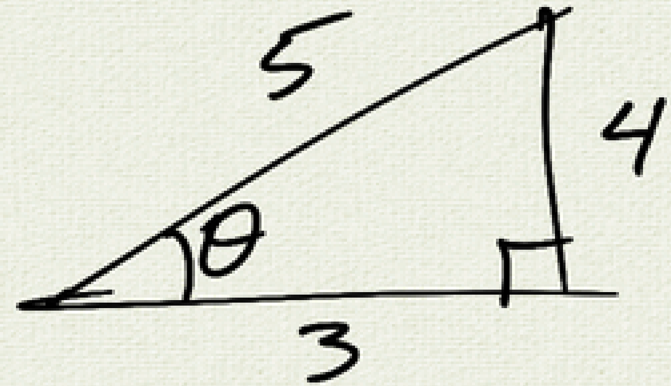
period $\frac{2\pi}{b}$

$(h, k) =$ "center"

horizontal shift

vertical shift

33 $\sin\left(\boxed{\underbrace{\cos^{-1}\left(\frac{3}{5}\right)}_{\theta}}\right)$

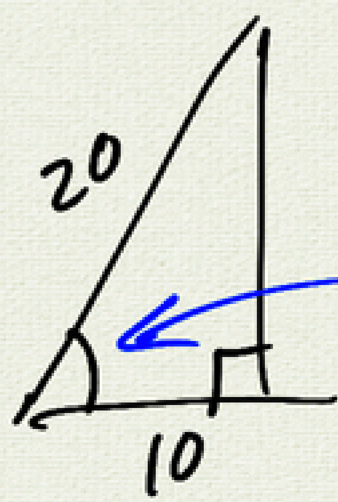


let $\theta = \cos^{-1}\left(\frac{3}{5}\right)$

$$\cos\theta = \frac{3}{5}$$

$$\Rightarrow \sin\theta = \frac{4}{5}$$

61



angle of elevation

60°

2.1 Trig Identities

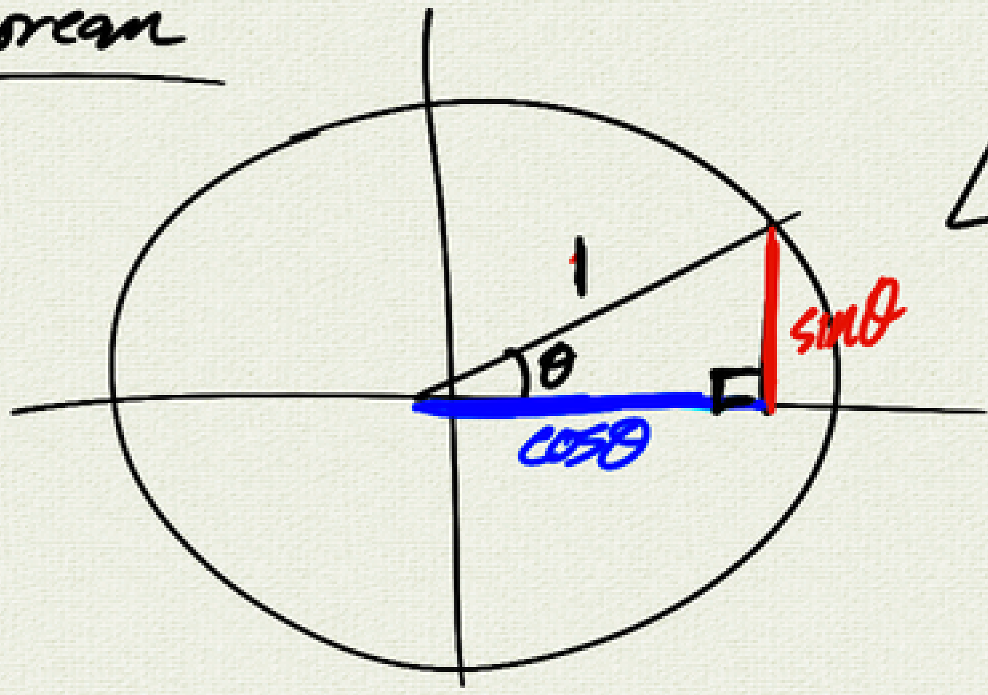
sin, cos defined from unit circle

basic

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

Pythagorean



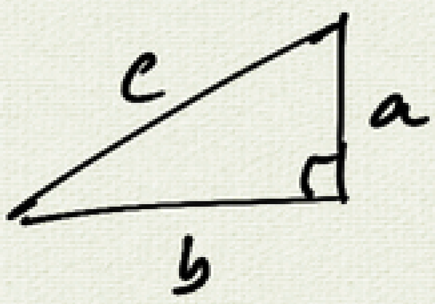
$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

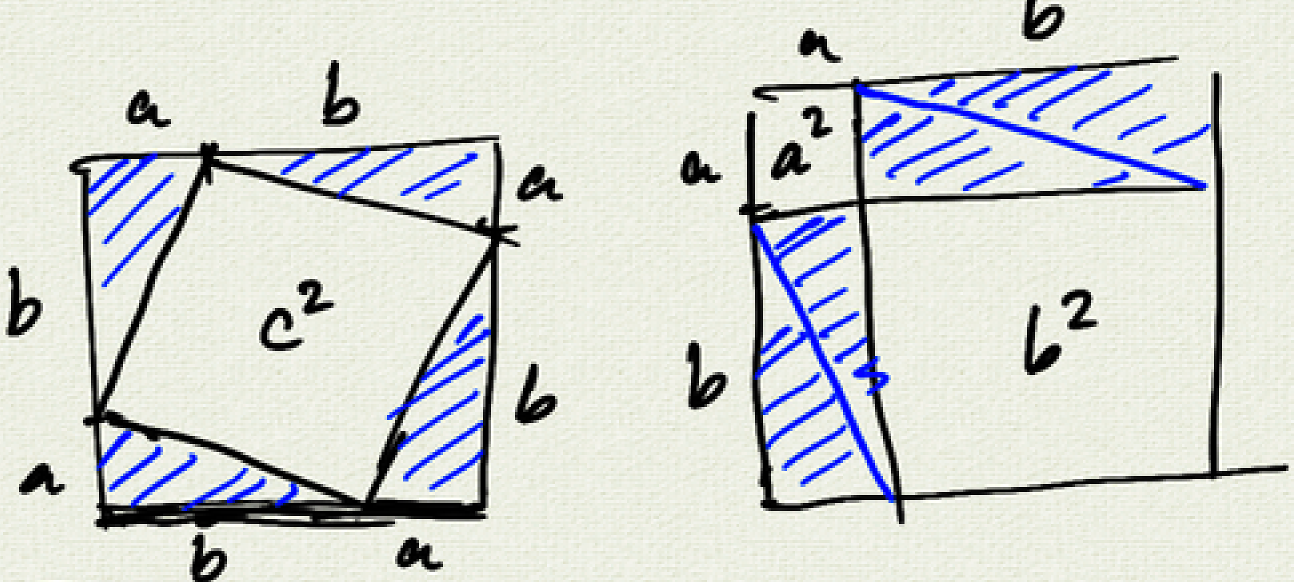
notation:

$$\sin^2\theta = (\sin\theta)^2$$

remember:

$$\underset{\text{inverse function}}{\sin^{-1}\theta} \neq \frac{1}{\sin\theta}$$





$$\boxed{\sin^2\theta + \cos^2\theta = 1} \implies \begin{array}{l} \sin^2\theta = 1 - \cos^2\theta \\ \cos^2\theta = 1 - \sin^2\theta \end{array}$$

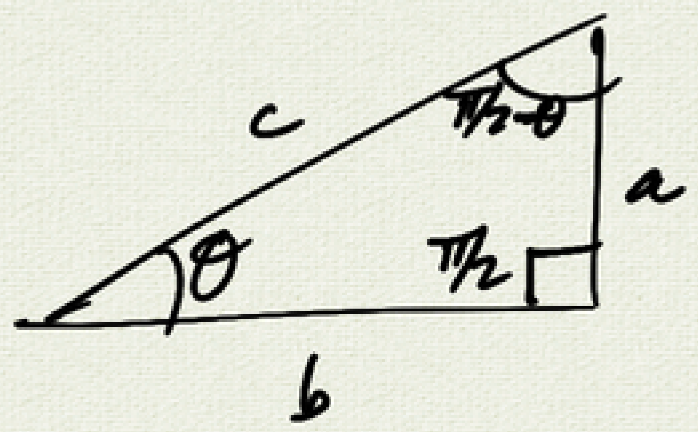
$$\implies \frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

cofactor identities

$$\boxed{\sin\Big(\underbrace{\tfrac{\pi}{2}-\theta}_{\text{complement of }\theta}\Big) = \cos\theta}$$



also: $\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$

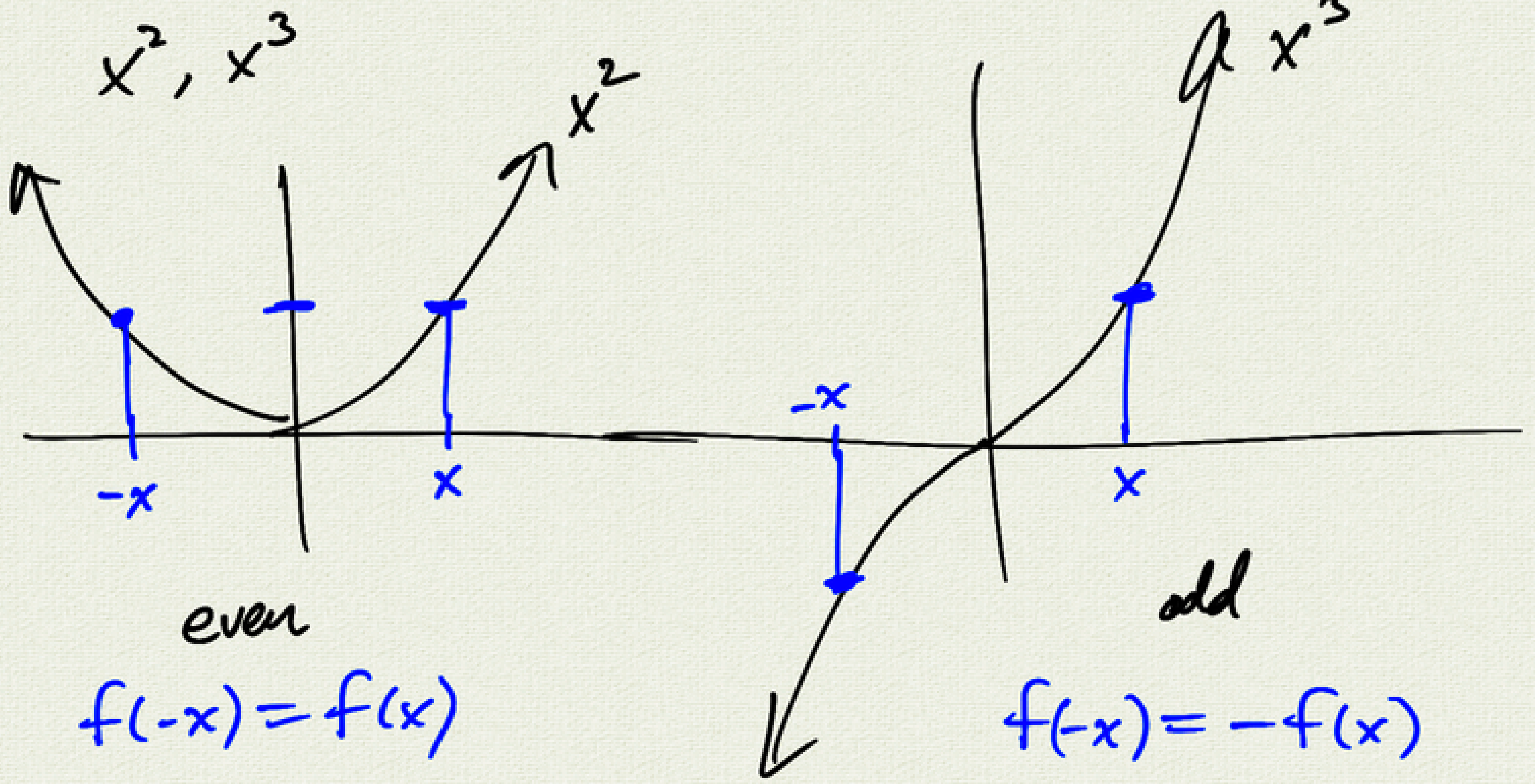
$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$

$\sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$

$$\sin\left(\frac{\pi}{2}-\theta\right) = \frac{b}{c} = \cos\theta$$

odd/even identities

x^2, x^3

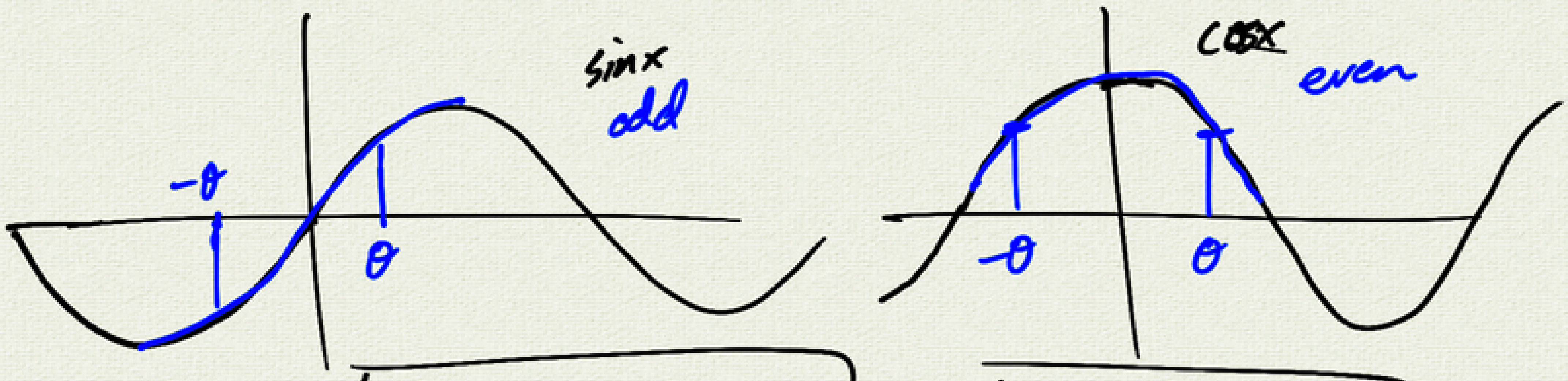


even

$f(-x) = f(x)$

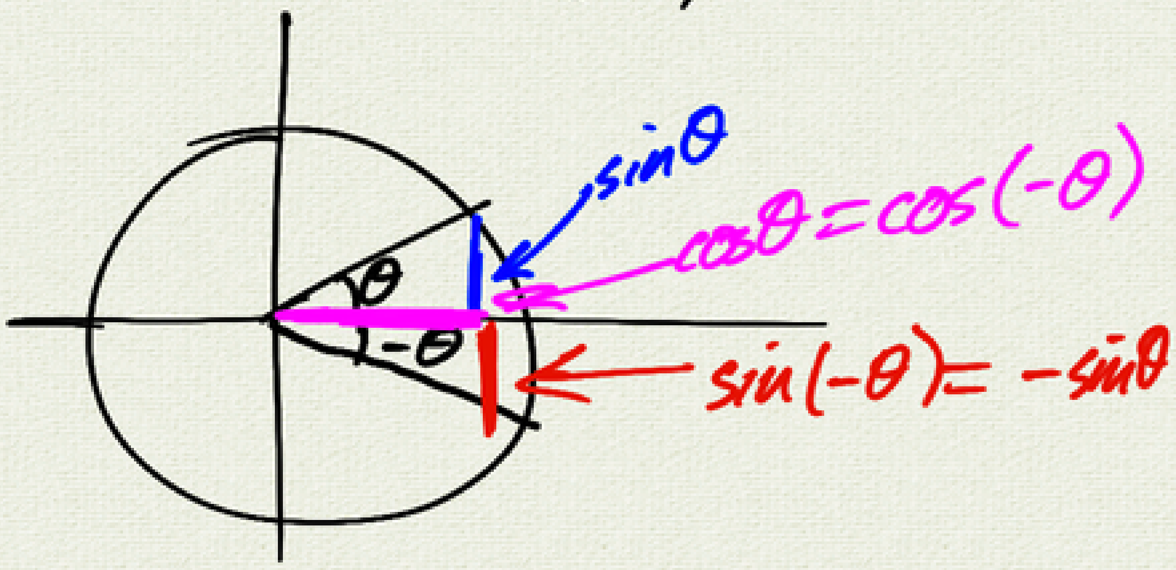
odd

$f(-x) = -f(x)$

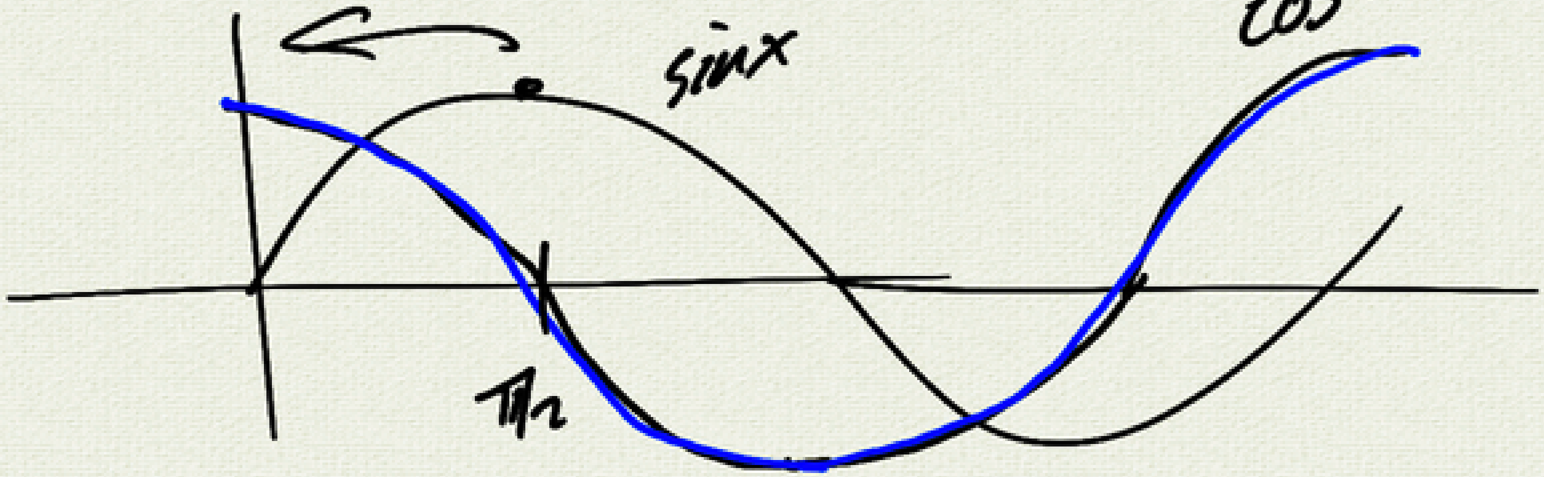


$\boxed{\sin(-\theta) = -\sin\theta}$ $\boxed{\cos(-\theta) = \cos\theta}$

odd/even identities



examples



Prove: $\boxed{\cos x = \sin\left(x+\frac{\pi}{2}\right)}$

cofactor
odd/even
Pythag.

$$\sin\left(x+\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}+x\right)$$
$$= \sin\left(\frac{\pi}{2}-(-x)\right)$$
$$= \cos(-x) \quad \text{(cofactor)}$$
$$= \cos x \quad \text{(cos is even)}$$

✓

we proved this identity
$\sin\left(x+\frac{\pi}{2}\right) = \cos x$
for any x

Prove: $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$

$$\frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{\cos x\,(1+\sin x)}{1-\sin^2 x}$$

$$= \frac{\cos x\,(1+\sin x)}{\cos^2 x} \qquad \text{(Pythag.)}$$

$$= \frac{1+\sin x}{\cos x}$$

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \sec x + \tan x \quad \checkmark$$

2.2 Sum identities

$$(a+b)^2 \stackrel{?}{=} a^2+b^2$$

in general $f(a+b) \neq f(a)+f(b)$

(challenge: for what functions is this true?)

$$\sin(u+v) \neq \sin u + \sin v$$

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sum identity:

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin(75°) = \sin(30°+45°)$$
$$= \sin(\pi/6 + \pi/4)$$
$$= \sin\tfrac{\pi}{6}\cos\pi/4 + \cos\pi/6 \sin\pi/4$$
$$= \frac{1}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$