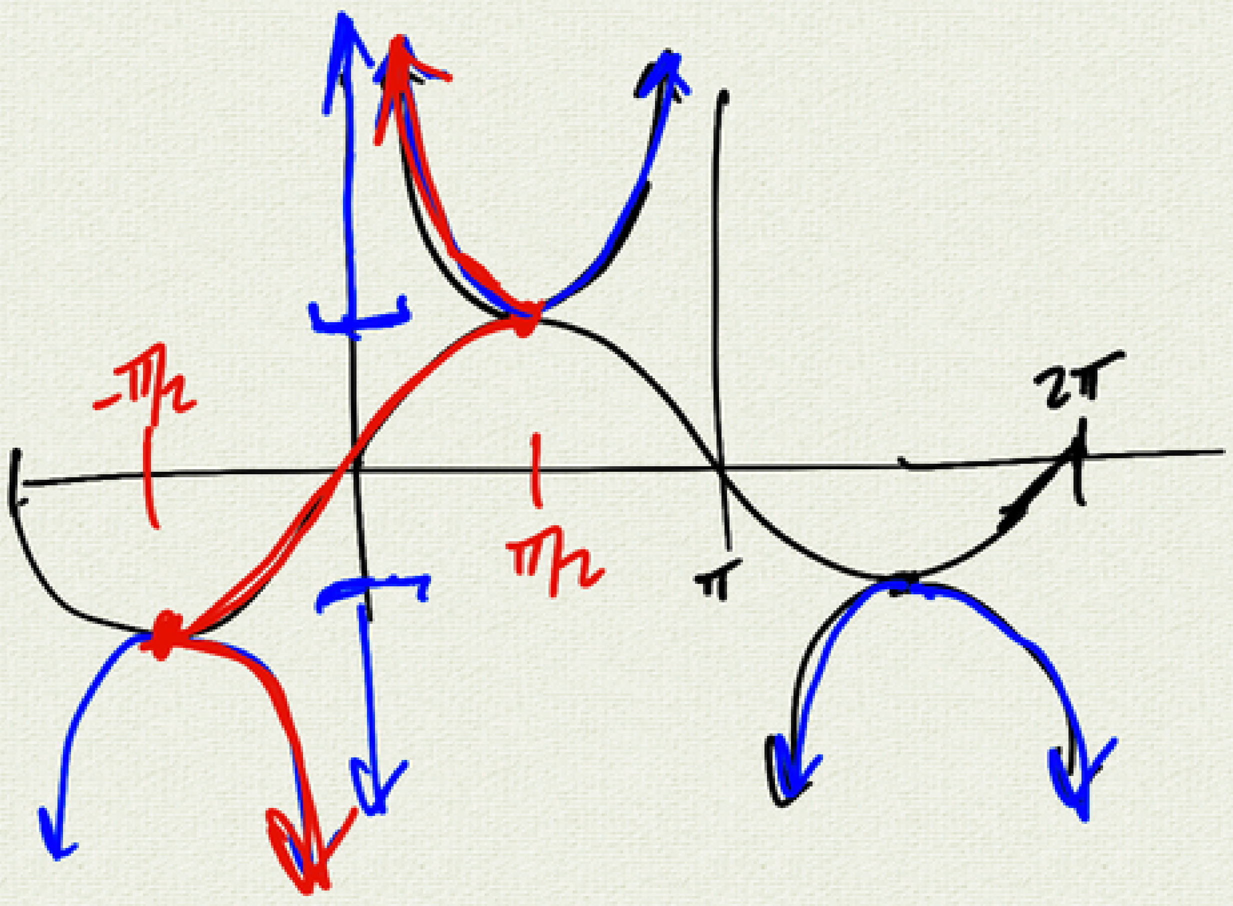
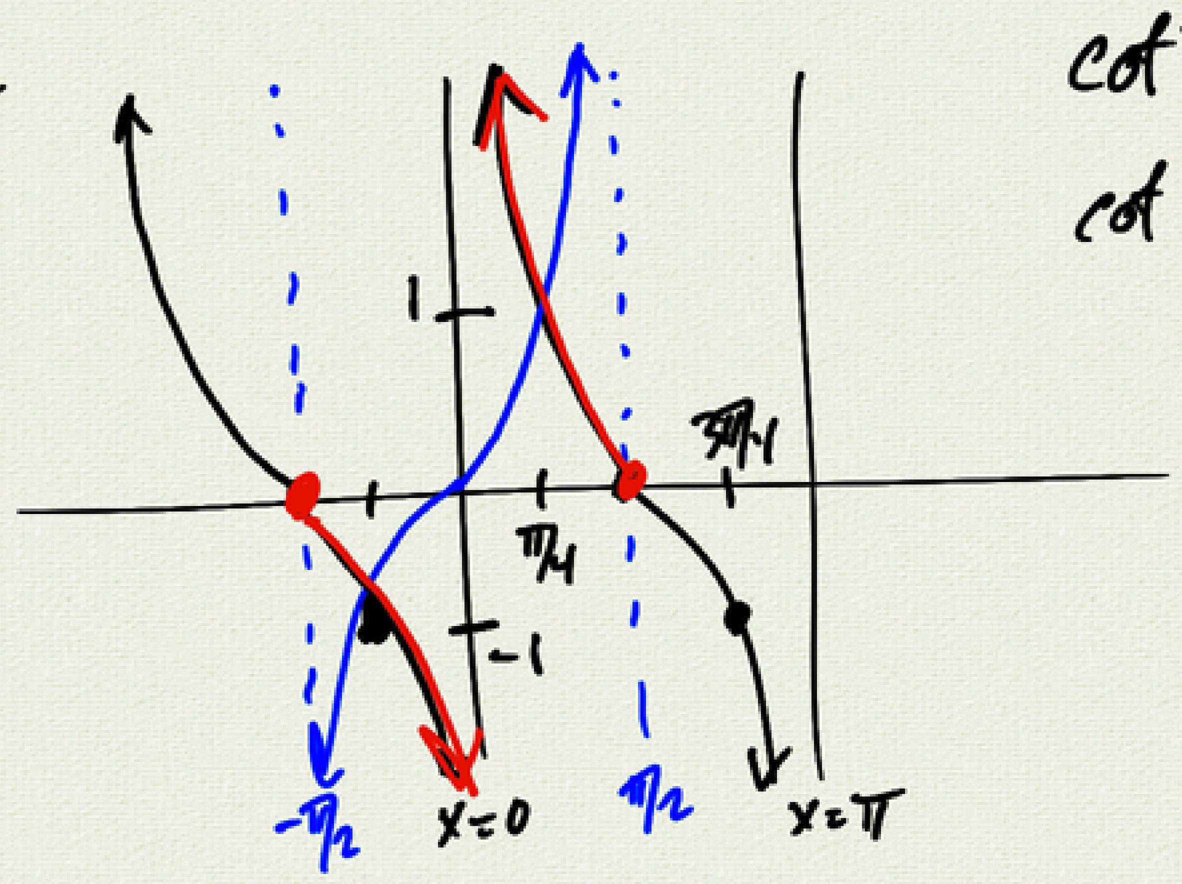


range(csc) $= [1, \infty) \cup (-\infty, -1]$

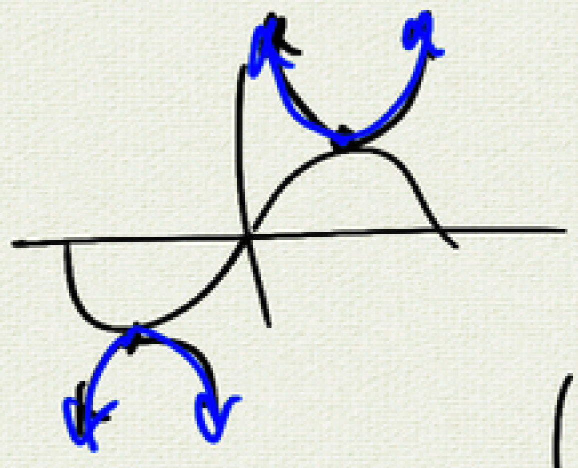
cot



$\cot^{-1}(1) = \frac{\pi}{4}$

$\cot^{-1}(-1) = \left.\begin{matrix} -\frac{\pi}{4} \\ \frac{3\pi}{4} \end{matrix}\right]$?

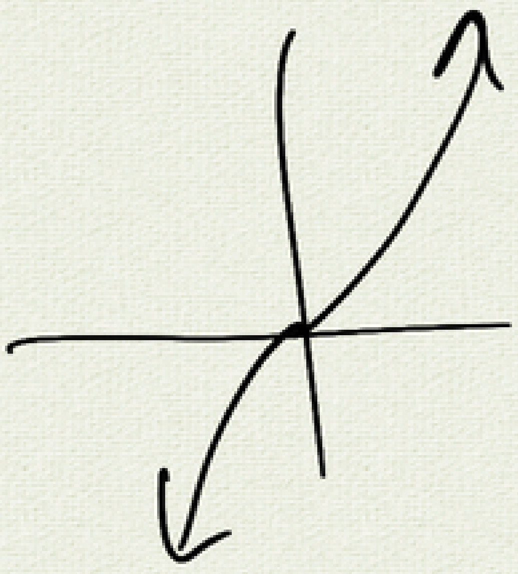
$\csc(-x) = \frac{1}{\sin(-x)} = \frac{1}{-\sin x} = -\csc(x)$ odd



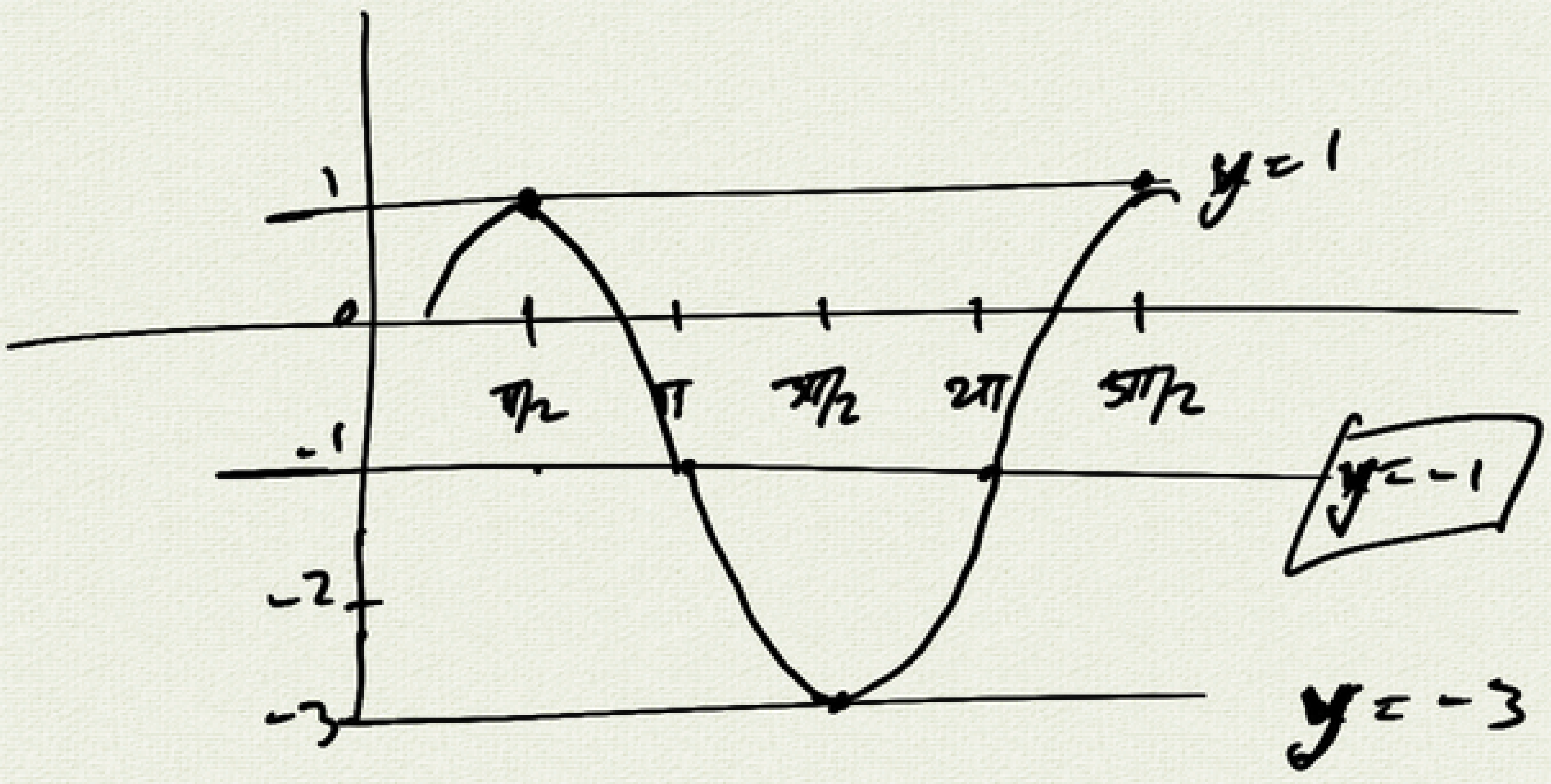
$\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x}$

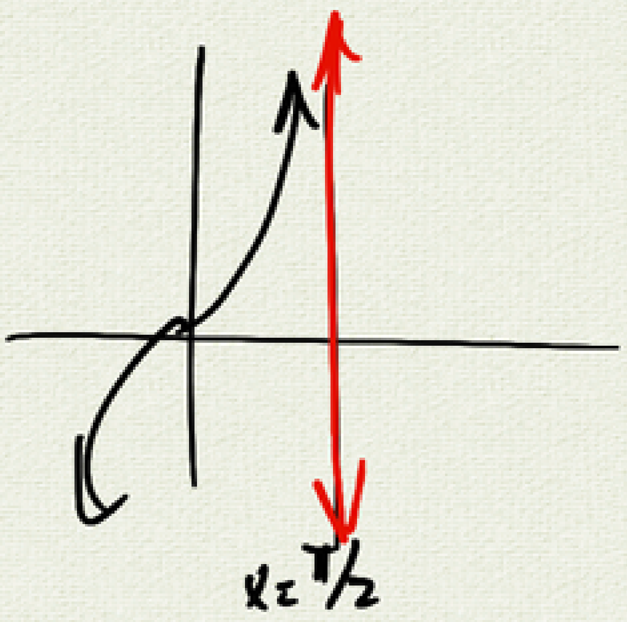
$= -\tan x$

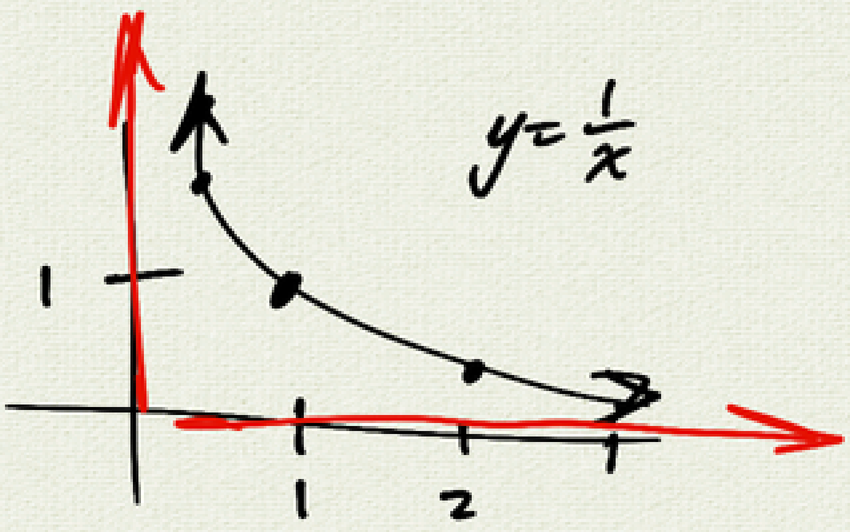
odd



$f(x) = 2\cos\left(x - \frac{\pi}{2}\right) - 1$ period 2π







(39) Show

$$\frac{1+\sin x}{\cos x} = \frac{\cos x}{1+\sin(-x)}$$

$$\frac{1+\sin x}{\cos x} = \frac{1+\sin x}{\cos} \cdot \frac{1-\sin x}{1-\sin x}$$

$$= \frac{1-\sin^2 x}{\cos x\,(1-\sin x)}$$

$$= \frac{\cos^2 x}{\cos x\,(1-\sin x)}$$

$$= \frac{\cos x}{1-\sin x}$$

$$= \frac{\cos x}{1+\sin(-x)}$$ ✓

Simplify

(13) $$\frac{\boxed{1+\tan^2\theta}}{\csc^2\theta} + \sin^2\theta + \frac{1}{\sec^2\theta}$$

$$= \frac{\boxed{\sec^2\theta}}{\csc^2\theta} + \boxed{\sin^2\theta + \cos^2\theta}$$

$$= \frac{(1/\cos^2\theta)}{(1/\sin^2\theta)} + 1$$

$$= \tan^2\theta + 1$$

$$= \sec^2\theta$$

Pythagorean:

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

(11) $$-\tan(-x)\cot(-x)$$

$$= -\frac{\sin(-x)}{\cos(-x)} \cdot \frac{\cos(-x)}{\sin(-x)}$$

$$= -1$$

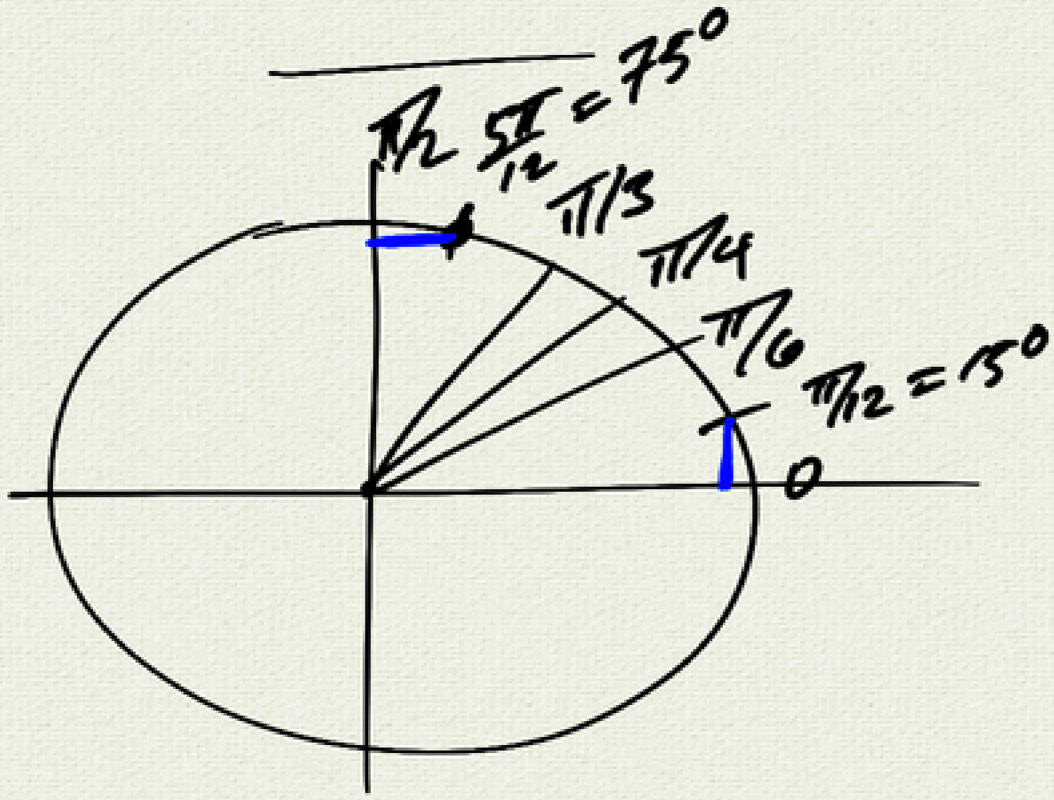
$$\cot(-x) = \frac{1}{\tan(-x)}$$

$$\cot(\theta) = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

(2.2) Sum identities

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$



$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

$$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\sin(u-v) = \sin(u+(-v))$$

$$= \sin u \underbrace{\cos(-v)}_{\cos v} + \cos u \underbrace{\sin(-v)}_{-\sin v}$$

$$= \sin u \cos v - \cos u \sin v$$

$$\boxed{\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v}$$

$$\boxed{\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v}$$

Sum/difference identities

$$\tan(u \pm v) = \frac{\sin(u \pm v)}{\cos(u \pm v)}$$

$$= \frac{\boxed{\sin u \cos v} \pm \cos u \sin v}{\cos u \cos v \mp \sin u \sin v} \qquad \frac{\frac{1}{\cos u \cos v}}{\frac{1}{\cos u \cos v}}$$

$$= \frac{\boxed{\tan u} \pm \tan v}{1 \mp \tan u \tan v}$$

2.3 Multiple Angle Identities

$$\boxed{\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v\end{aligned}}$$

$$\sin 2u = \sin(u+u)$$
$$= \sin u \cos u + \cos u \sin u$$
$$\boxed{\sin 2u = 2\sin u \cos u}$$ double angle identity

$$\cos 2u = \cos(u+u)$$
$$= \cos^2 u - \sin^2 u \quad (A)$$

$$\cos 2u = \cos^2 u - \sin^2 u \quad (A)$$
$$= 1 - 2\sin^2 u \quad (B)$$
$$= 2\cos^2 u - 1 \quad (C)$$

$$\sin^2 u + \cos^2 u = 1$$
$$\cos^2 u = 1 - \sin^2 u$$
$$\boxed{\sin^2 u = 1 - \cos^2 u}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$
$$= \cos^2 u - (1 - \cos^2 u)$$
$$= 2\cos^2 u - 1$$

(B)

$$\boxed{\cos 2u = 1 - 2\sin^2 u}$$

$$2\sin^2 u = 1 - \cos 2u$$

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$ power-reducing

$$\sin u = \pm\sqrt{\frac{1-\cos 2u}{2}}$$ half-angle

(C)

$$\boxed{\cos 2u = 2\cos^2 u - 1}$$

$$2\cos^2 u = 1 + \cos 2u$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\cos u = \pm\sqrt{\frac{1+\cos 2u}{2}}$$

---

example: $\sin 15^\circ = \sqrt{\frac{1-\cos 30^\circ}{2}}$
$$= \sqrt{\frac{1-\sqrt{3}/2}{2}}$$

$$\sin \frac{v}{2} = \pm\sqrt{\frac{1-\cos v}{2}}$$ half-angle

$$\left(\frac{v}{2} = u\right)$$

$\sin(u+v) = \sin u \cos v + \cos u \sin v$