$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$$\tan(u+v) = \frac{\sin(u+v)}{\cos(u+v)} = \frac{\sin u \cos v + \cos u \sin v}{\cos u \cos v - \sin u \sin v}$$

$$= \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan 2u = \tan(u+u) = \frac{2\tan u}{1-\tan^2 u}$$

---

(9) $\cos 2\theta = \frac{3}{5}$ $\qquad 90° \le \theta \le 180°$

half-angle formulas

$\sin^2\theta = 1 - \cos^2\theta$

$\cos 2\theta = \boxed{\cos^2\theta - \sin^2\theta}$

$= 1 - 2\sin^2\theta$ *

$= 2\cos^2\theta - 1$ **

double angle

* $\boxed{\cos 2\theta = 1 - 2\sin^2\theta}$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\boxed{\sin\theta = \pm\sqrt{\frac{1-\cos 2\theta}{2}}}$

power reducing

half angle

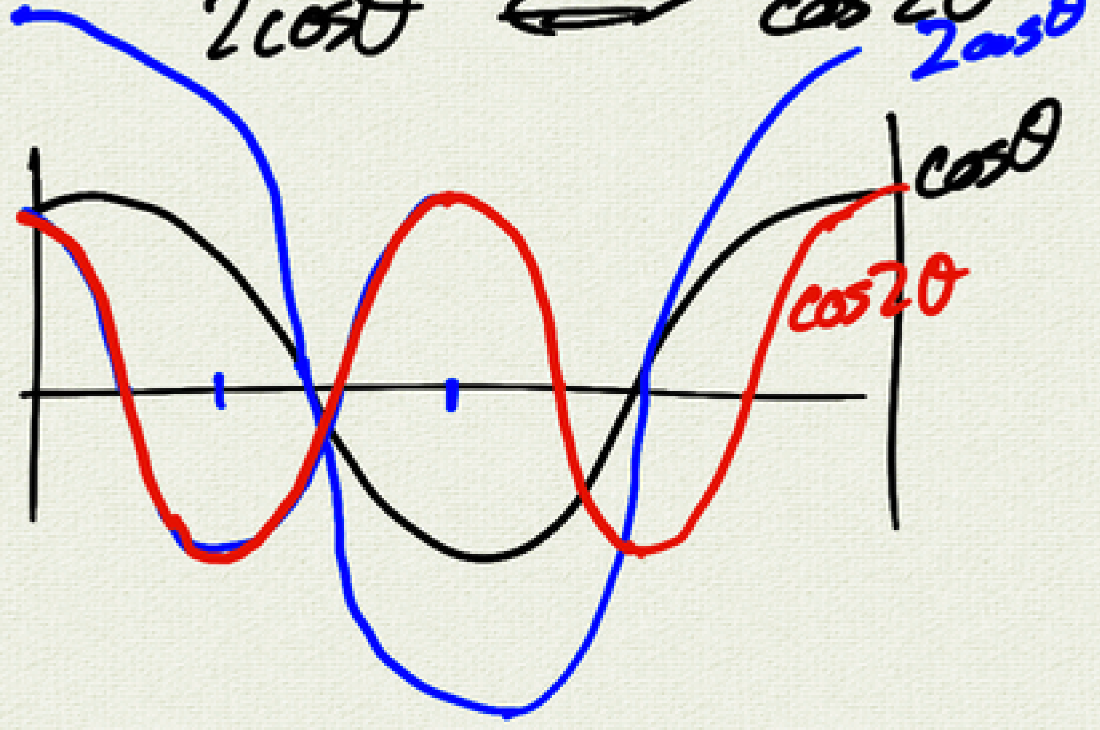
** $\boxed{\cos 2\theta = 2\cos^2\theta - 1}$

$2\cos^2\theta = 1 + \cos 2\theta$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\cos\theta = \pm\sqrt{\frac{1+\cos 2\theta}{2}}$

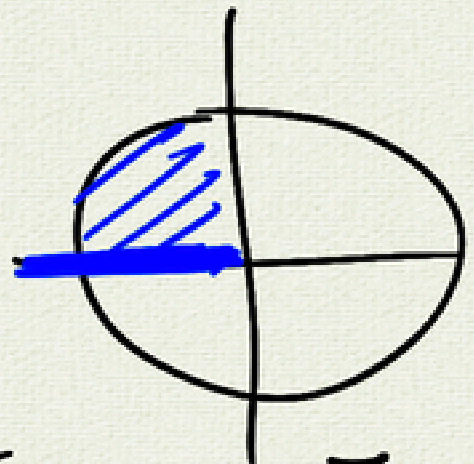
$2\cos\theta \longleftrightarrow \cos 2\theta$



(9) $\cos\theta = \sqrt{\dfrac{1+\cos 2\theta}{2}}$

$\sin\theta = \sqrt{\dfrac{1-\cos 2\theta}{2}}$

$\cos 2\theta = \frac{3}{5}$ $\qquad 90° \leq \theta \leq 180°$



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$\Rightarrow \cos\theta = \pm\sqrt{\dfrac{1+(3/5)}{2}} = \pm\sqrt{\dfrac{8}{10}} = -\sqrt{4/5}$

$\sin\theta = \pm\sqrt{\dfrac{1-(3/5)}{2}} = \sqrt{\dfrac{2}{10}} = +\sqrt{\dfrac{1}{5}}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sqrt{1/5}}{-\sqrt{4/5}} = -\frac{1}{2}$

$$\boxed{\begin{aligned} \sin\theta &= \sqrt{\frac{1-\cos 2\theta}{2}} \\ \sin\frac{\theta}{2} &= \sqrt{\frac{1-\cos\theta}{2}} \end{aligned}}$$

$\sin 15° = \sqrt{\dfrac{1-\cos 30°}{2}}$

---

(17) $\tan\left(\frac{\pi}{2} - x\right)$

$\tan\frac{\pi}{2}$ undefined

$= \cot x$

---

(13) $\sin\frac{\pi}{8} = \sqrt{\dfrac{1-\cos \pi/4}{2}}$

$= \sqrt{\dfrac{1-\sqrt{2}/2}{2} \cdot \dfrac{2}{2}}$

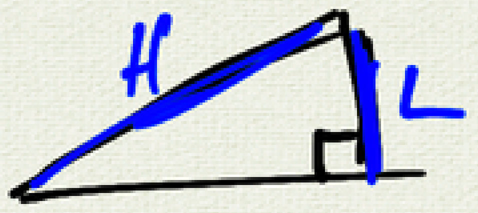
$= \sqrt{\dfrac{2-\sqrt{2}}{4}}$

$= \frac{1}{2}\sqrt{2-\sqrt{2}}$

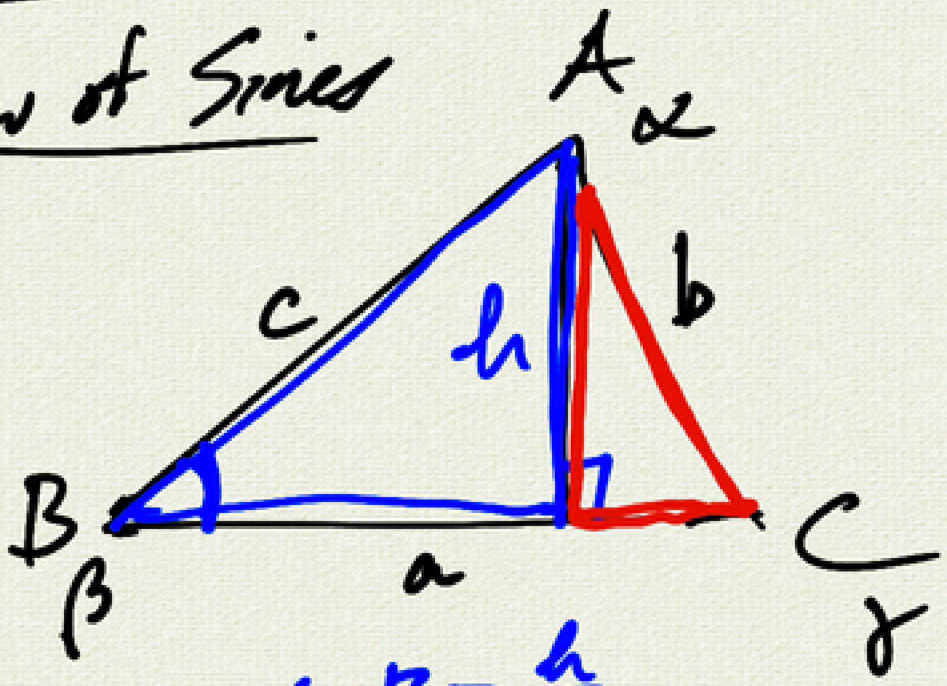
2.4 Law of Sines / Cosines

SAS SSS ASA ASS

← sometimes ?



Law of Sines



α alpha

β beta

γ gamma

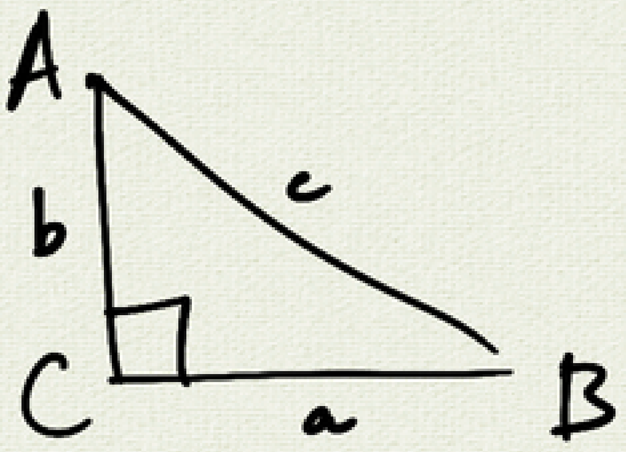
$\sin B = \frac{h}{c}$

$$h = c\sin B = b\sin C$$

$$\boxed{\frac{\sin B}{b} = \frac{\sin C}{c} \quad \left(= \frac{\sin A}{a} \right)}$$

Law of Sines

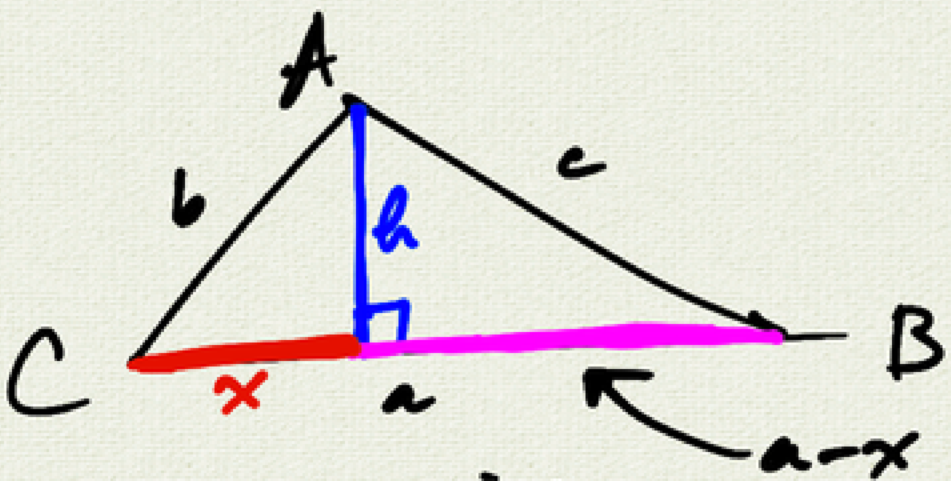
Law of Cosines

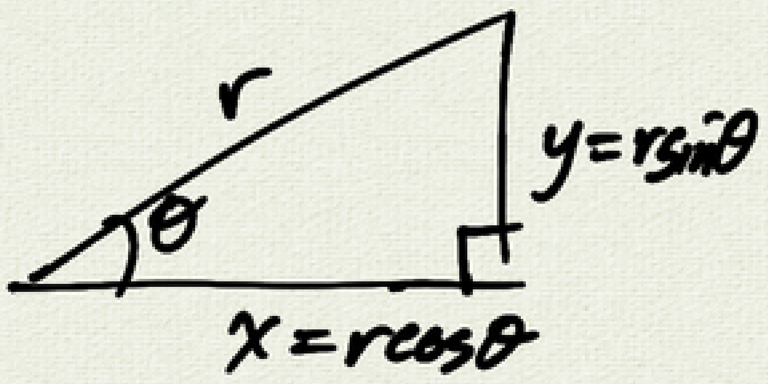


Pythagorean Thm:

$$c^2 = a^2 + b^2$$

$c = ?$





$h = b\sin C$

$x = b\cos C$

$\cos C = \frac{x}{b}$

$x = b\cos C$

$$c^2 = h^2 + (a-x)^2$$

$$= (b\sin C)^2 + (a - b\cos C)^2$$

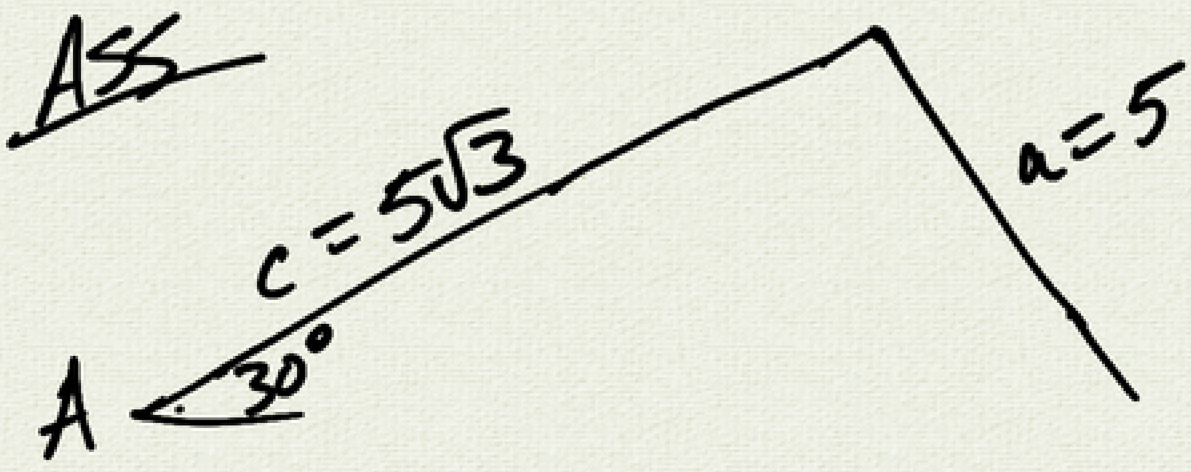
$$= b^2\sin^2 C + a^2 + b^2\cos^2 C - 2ab\cos C$$

$$= a^2 + b^2(\sin^2 C + \cos^2 C) - 2ab\cos C$$

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

Law of Cosines

ASS



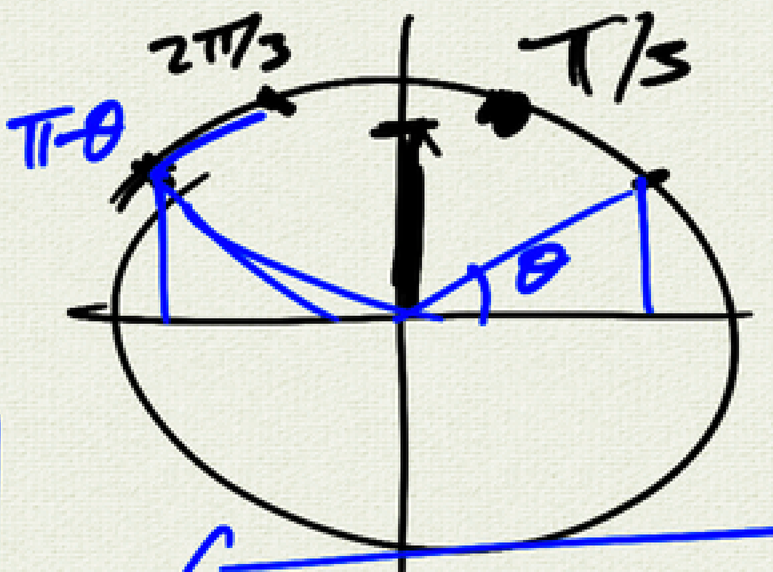
$A=30°$
$a=5$
$c=5\sqrt{3}$

solve the $\triangle$
(find b, B, C)

$$\frac{\sin A}{a}=\frac{\sin C}{c}$$

$$\sin C=\frac{c}{a}\sin A=\frac{5\sqrt{3}}{5}\sin 30°=\frac{\sqrt{3}}{2}$$

$$\Rightarrow C=\frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$



$$\boxed{\sin(\pi-\theta)=\sin\theta}$$

$$\sin(\pi-\theta)=\underbrace{\sin\pi}_{0}\cos\theta-\underbrace{\cos\pi}_{-1}\sin\theta=\sin\theta$$

challenge: prove with cofactor identities

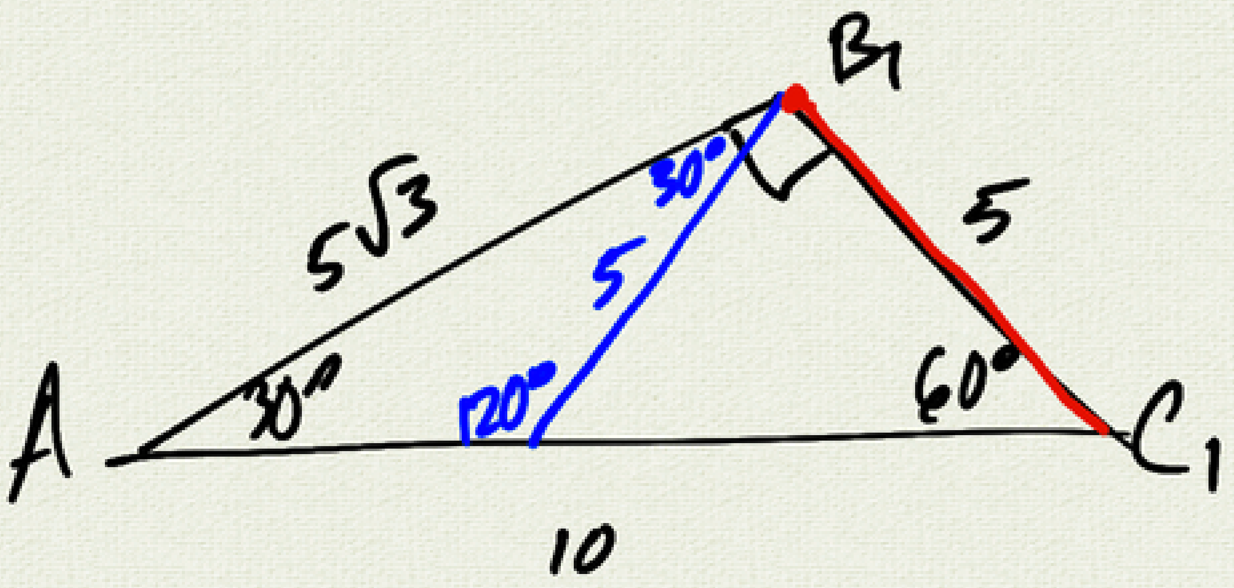
$C_1=\frac{\pi}{3}=60°$ $\quad$ $C_2=\frac{2\pi}{3}=120°$

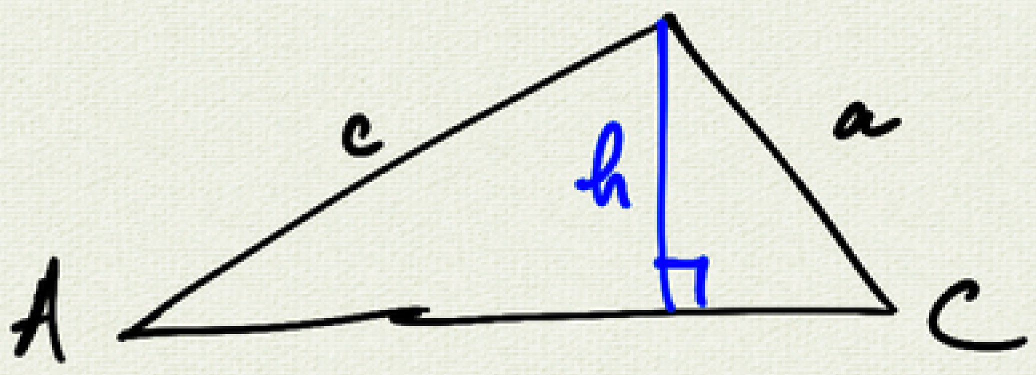
$B_1=90°$ $\quad$ $B_2=30°$

$$\frac{b}{\sin B}=\frac{a}{\sin A}$$

$$b=a\frac{\sin B}{\sin A}=5\cdot\frac{1}{(1/2)}=10$$

$$b=a\frac{\sin B}{\sin A}=5\cdot\frac{(1/2)}{(1/2)}=5$$





$h=c\sin A$

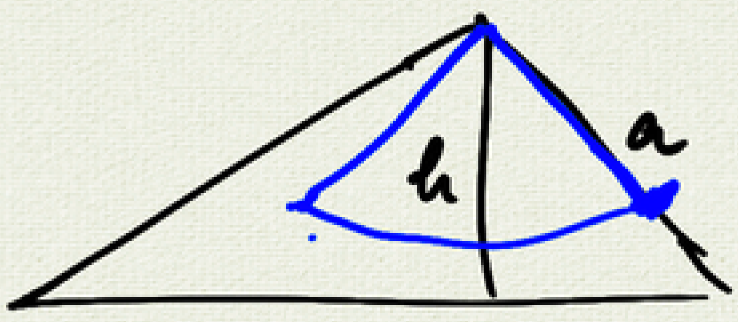
$$\frac{\sin C}{c}=\frac{\sin A}{a}$$

$$\sin C=\frac{\boxed{c\sin A}}{a}=\frac{h}{a}$$

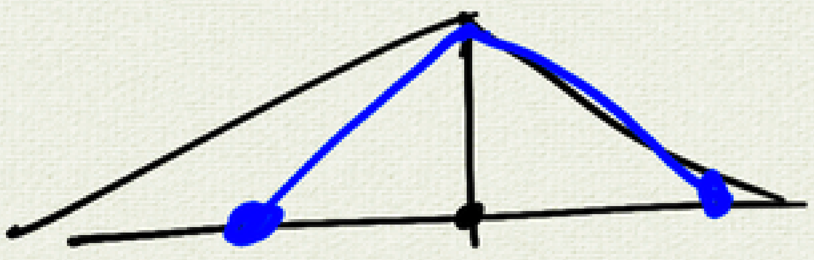
<1 2 possible angles

$=1$ $C=\pi/2$ 1 possibility

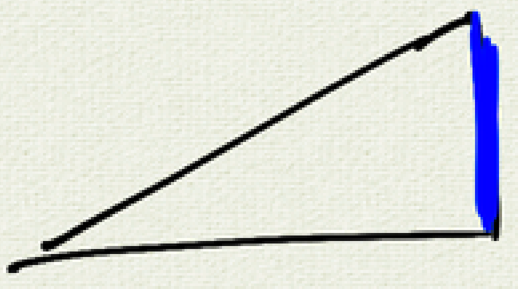
>1 no possible triangles



$\frac{h}{a}>1 \Rightarrow h>a$



$\frac{h}{a}<1 \Rightarrow h<a$



$\frac{h}{a}=1 \Rightarrow h=a$