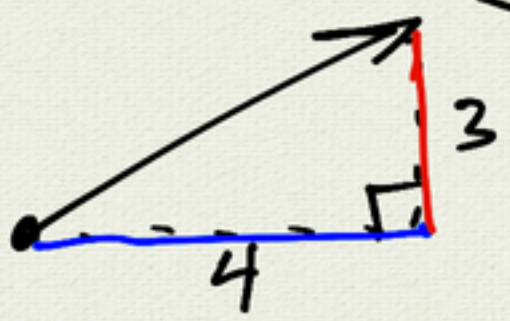
3.1 Vectors



$\langle 4, 3 \rangle$

↖ "special" ordered pair

basic operations:

addition $\langle 4, 3 \rangle + \langle 1, 2 \rangle = \langle 5, 5 \rangle$

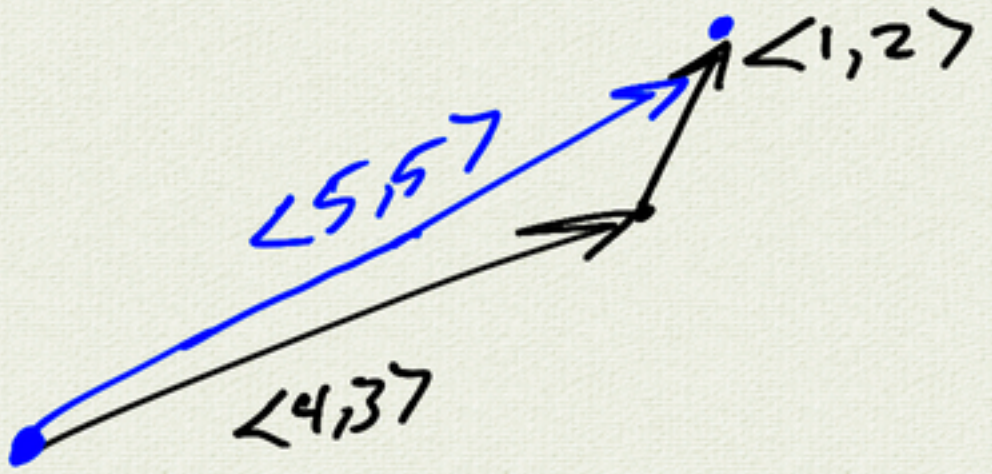
4+1 3+2

Scalar multiplication

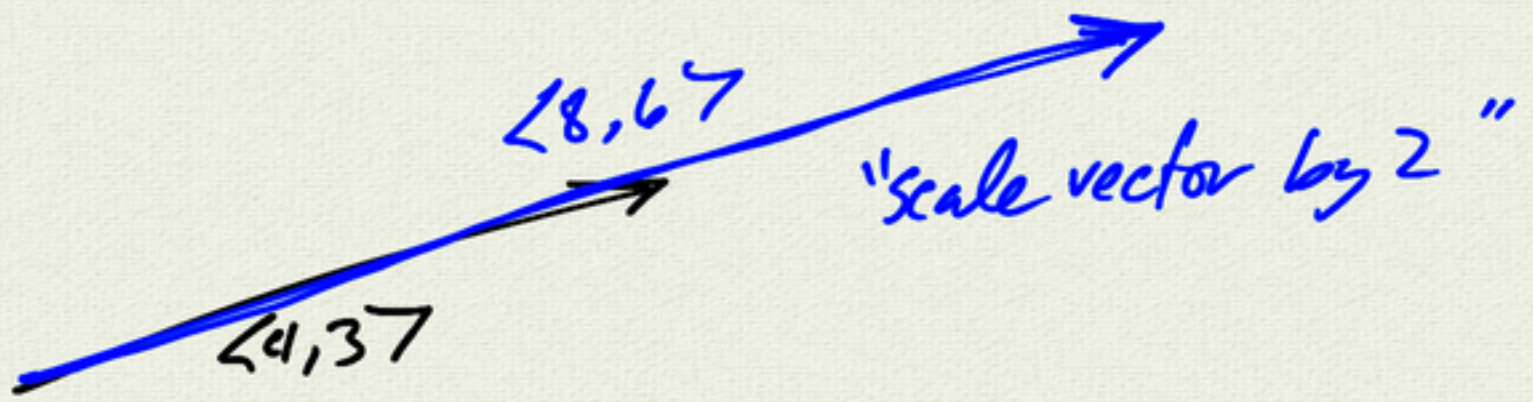
$2\langle 4, 3 \rangle = \langle 8, 6 \rangle$

"scalar" = number (not vector)

vector vector



geometric interpretation of vector addition



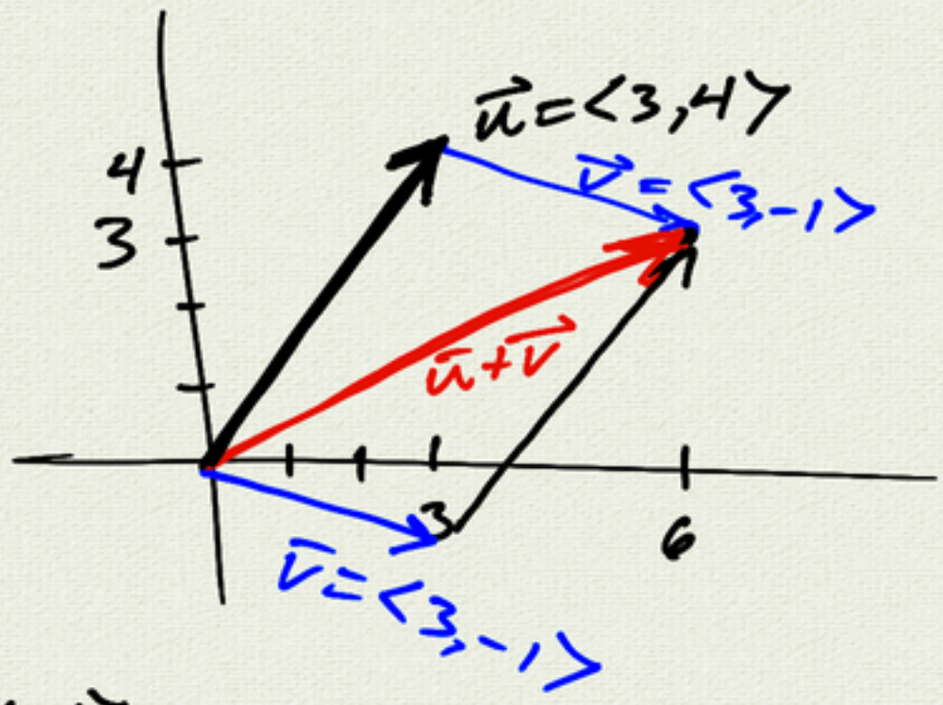
"scale vector by 2"

$\vec{u} = \langle 3,4 \rangle$

$\vec{v} = \langle 3,-1 \rangle$

$\vec{u}+\vec{v} = \langle 6,3 \rangle$

$\bar{u} = \vec{u}$



magnitude of $\bar{u} = \langle x,y \rangle$

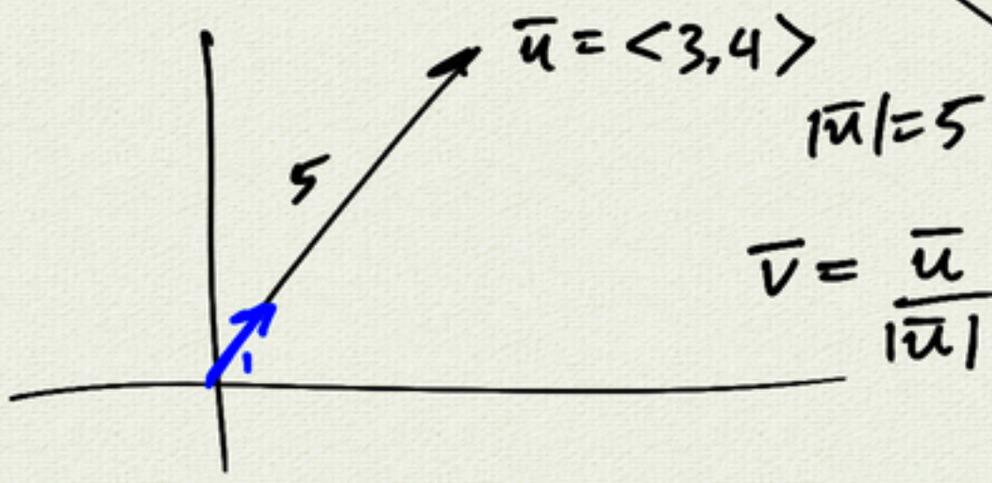
$= |\vec{u}| = \sqrt{x^2+y^2}$

(length of vector)

$|\langle 3,4 \rangle| = \sqrt{3^2+4^2} = 5$

$\vec{u} = \langle x, y \rangle$

↗ x component ↖ y component



$|\bar{u}| = 5$

length = 1

$\bar{v} = \dfrac{\bar{u}}{|\bar{u}|} = \frac{1}{5}\langle 3,4 \rangle = \langle \frac{3}{5}, \frac{4}{5} \rangle$

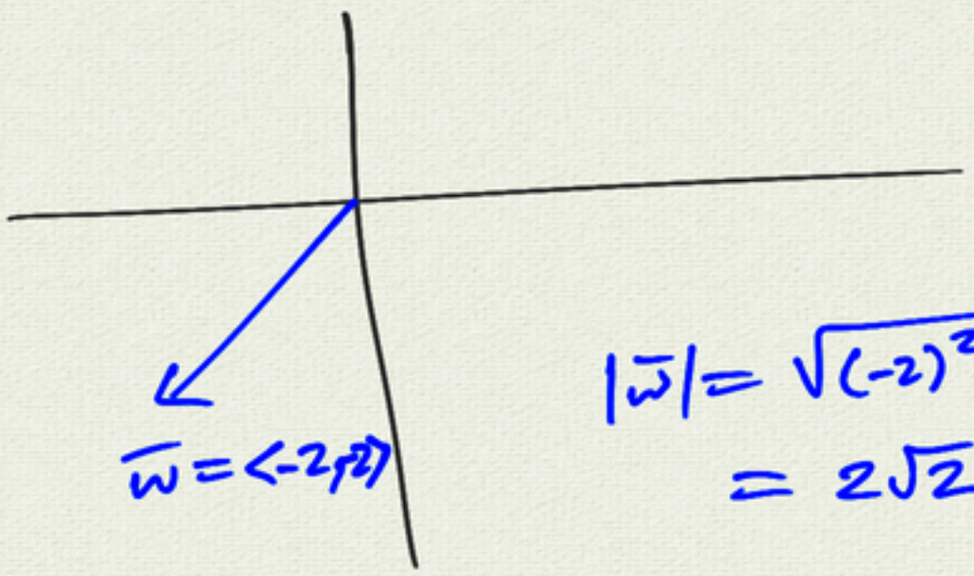
unit vector in the direction of $\bar{u}$

check $|\bar{v}| = 1$:

$|\bar{v}| = \sqrt{(3/5)^2 + (4/5)^2}$

$= \sqrt{\frac{9+16}{25}}$

$= 1$



$|\bar{w}| = \sqrt{(-2)^2+(2)^2}$

$= 2\sqrt{2}$

$\bar{u}$ is a <u>unit vector</u> if $|\bar{u}| = 1$

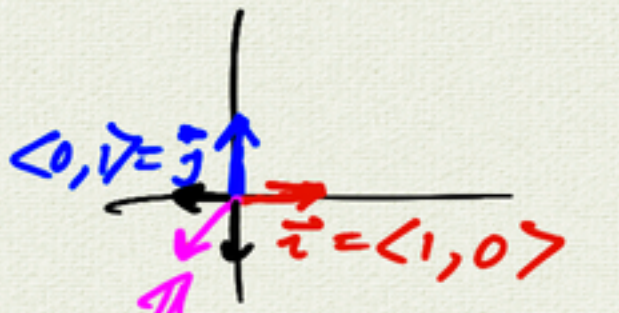
<u>examples:</u>

$\langle \frac{3}{5}, \frac{4}{5} \rangle$

$\langle 0, 1 \rangle = \vec{j}$

$\langle 1, 0 \rangle = \vec{i}$

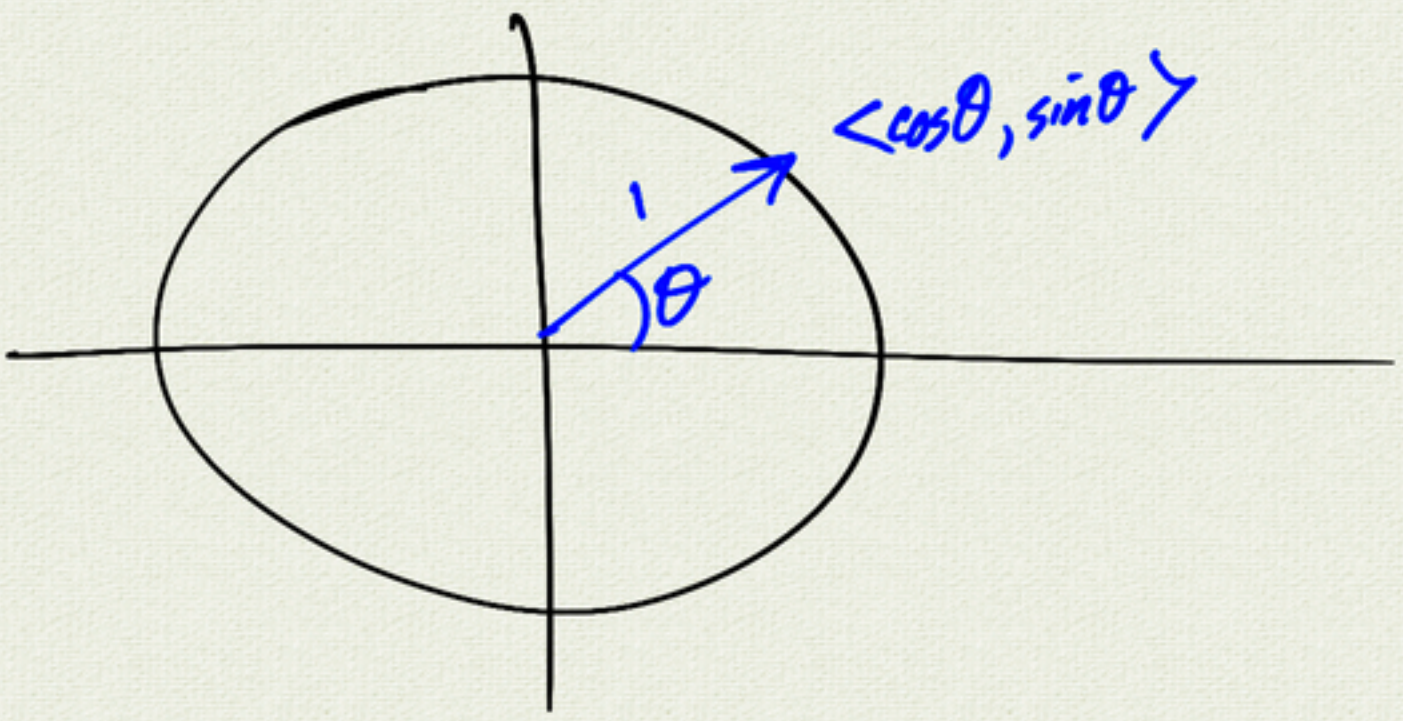
$\langle 0, -1 \rangle \quad (= -\vec{j})$

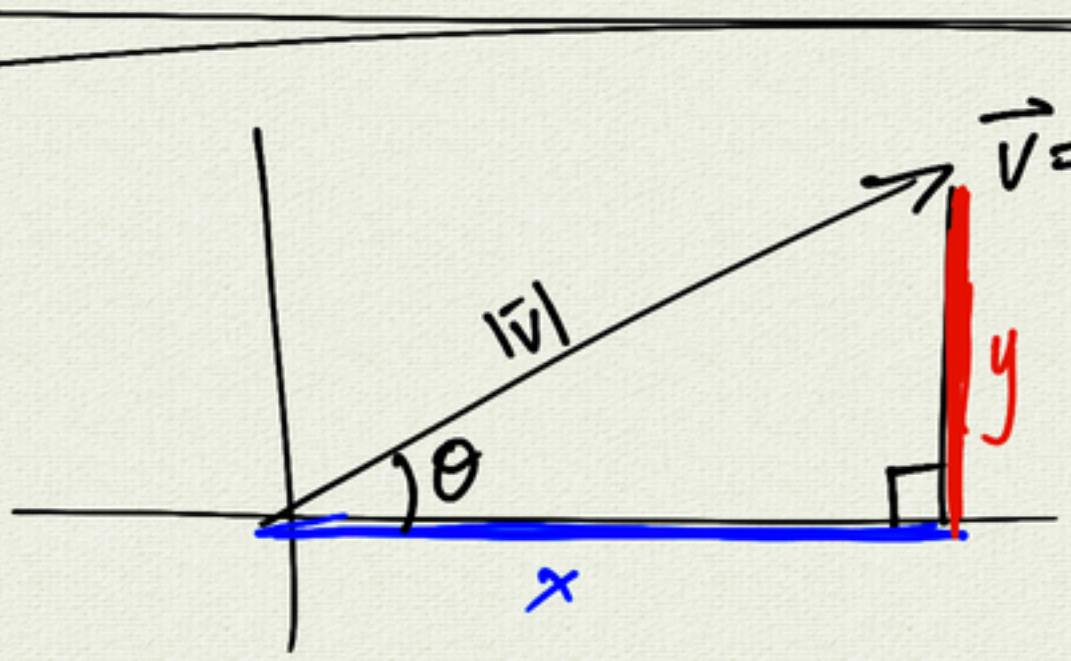


$\langle -1, -1 \rangle$ not a unit vector

$|\langle -1, -1 \rangle| = \sqrt{2}$

$\langle -1, 0 \rangle \quad (= -\vec{i})$





$|\bar{v}| = \sqrt{x^2 + y^2}$ magnitude

$\tan\theta = y/x$ direction

given $|\bar{v}|, \theta$:

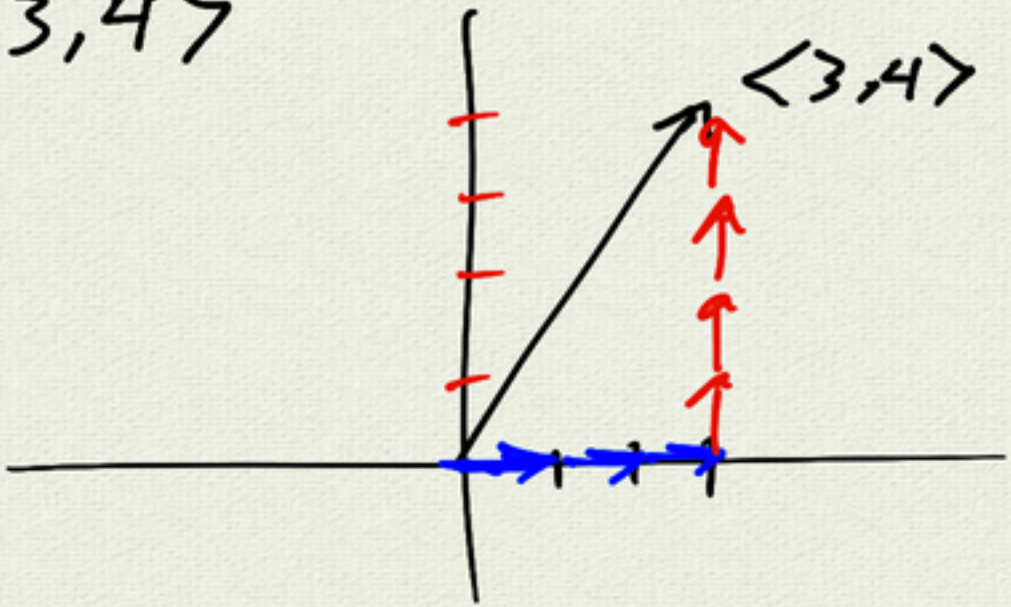
$\sin\theta = \frac{y}{|\bar{v}|} \Rightarrow y = |\bar{v}| \sin\theta$

$x = |\bar{v}| \cos\theta$

} components

$$3\vec{i} + 4\vec{j} = 3\langle 1,0\rangle + 4\langle 0,1\rangle$$
$$= \langle 3,0\rangle + \langle 0,4\rangle$$
$$= \langle 3,4\rangle$$

linear combination of $\vec{i}$ and $\vec{j}$



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